17-0: Distributed Problem Solving

- Last week, we talked about *loosely coupled* distributed problems
  - Primarily distributed search

- A large data set is divided across clients.

- Clients interact only with a central server; no client-client communication

17-1: “Medium-coupled” problems

- In medium-coupled problems, each node can do a substantial amount of computing on its own

- A final solution will require communication between nodes.

- Actions taken by one node can affect other nodes.

17-2: Example: scheduling classes

- Before the semester starts, Wolber sends every teacher an email: When and where do you want your classes?

- Each professor has their own constraints, and can come up with choices locally:
  - Brooks: no classes before 10 am, in Kudlick
  - Wolber: No classes on Friday
  - Galles: Mornings, nothing on Lone Mountain.

- Brooks is able to eliminate some possible schedules without consulting with anyone else.

17-3: Example: scheduling classes

- Some choices require communication with a center (Wolber)
  - Only one class in Kudlick at a time.
  - Some classes may be constrained by other departments’ choices
    - CS 110 and Calculus I should be at different times.

- Nodes start with a large set of constraints; some are eliminated by the center.

- Hopefully at least one viable schedule remains.
  - If not, someone is “encouraged” to relax their constraints.

17-4: Constraint Satisfaction

- A constraint satisfaction problem is one of assigning values to variables so as to satisfy a set of constraints.

- Typically, any solution that satisfies the constraints is equally acceptable.

- Toy problems:
  - N-queens
  - Map coloring

- Real problems:
  - Scheduling CS classes
  - Building a car
  - Register allocation
More formally, a CSP consists of:
- a set of variables \( \{x_1, x_2, \ldots, x_n\} \)
- Each variable as a domain of possible values \( D_1, D_2, \ldots, D_n \)
- and a set of constraints \( C_1, C_2, \ldots \)
  - Unary constraints: \( x < 10, \mod{x}{2} = 0, \text{ etc} \)
  - Binary constraints: \( x < y, x + y < 50, \text{ etc} \)
  - N-ary constraints: \( x_1 + x_2 + \ldots + x_n = 75 \)
  - (a problem with N-ary constraints can be transformed into a binary constraint problem, with an increase in the number of variables)

An assignment of values to variables that satisfies all constraints is called a consistent solution.

We might also have an objective function \( y = f(x_1, \ldots, x_n) \) that lets us compare solutions.

If the domain of all variables is continuous (i.e. real numbers) and constraints are all linear functions, we can use linear programming to solve the problem.

This is actually the easy version (in P).
- Express the problem as a system of equations
- If variables are discrete, the problem is harder.
- We can use dynamic programming.

Often, variables have complex or symbolic values.

In the most general case, we can express a CSP as a search problem.

We can use depth-first search to solve a CSP
- Begin with an initial state: no values assigned to \( x_1, \ldots, x_n \)
- Sequentially assign values to variables.
- We are done if we find an assignment to each variable such that all constraints are met.

Since CSPs are commutative (for a solution, it doesn’t matter which order values are assigned) we can consider one variable at a time.

With CSPs, the challenge is deciding how to proceed when a constraint is violated.
- Some assignment of values to variables must be undone, but which?
- This decision is called backtracking
- Standard DFS undoes the most recently assigned value.
- This is called chronological backtracking
- Easy to implement
- Problem: an early assignment may have doomed us to an inconsistent solution.

Three-coloring the map of Australia

Assigning Q = red, NSW = green, V = blue, T = red cannot lead to a solution.

Different values for T will not change this.

V needs a different value.
17-11: Distributed CSP

- In some cases, the variables in a CSP may be distributed over multiple nodes or processes or agents.
  - Natural division of problem (e.g., scheduling a meeting, coordinating research teams)
  - Information may be private, or difficult to quantify and share.
- Constraints may be intra-agent or inter-agent.
- Agents communicate by message passing with asynchronous communication.

17-12: Algorithms for Distributed CSP

- For purposes of demonstration, we’ll make the following assumptions:
  - Each process has a single variable
  - All variables have binary values
  - All constraints are binary.
- Note: this is just to keep the examples simple; the algorithms work fine without these assumptions.

17-13: Asynchronous Backtracking

- Asynchronous backtracking extends normal backtracking search to a distributed environment.
- Each process has a priority $p_i$.
- Each agent chooses a value for its variable and send this value to all processes that it shares a constraint with.
  - No defined order in which messages are sent (asynchronous communication)
  - All agents are selecting values simultaneously.
- If a node cannot find a consistent value, it generates a nogood.

17-14: Asynchronous Backtracking Example

- $x_1$ and $x_3$ can be either 1 or 2.
- $x_2$ can only be 2.
- $x_1 \neq x_3$, $x_2 \neq x_3$
- (we can see that the only solution is $x_1 = x_2 = 2$, $x_3 = 1$)

17-15: Asynchronous Backtracking Example

- Each node sends messages to other nodes that it shares a constraint with.
  - Includes all known assignments
- For example, $x_1$ sends an $(ok?, (x_1, 1))$ to $x_3$.
- $x_2$ sends $(ok?, (x_2, 2))$ to $x_3$.
- $x_3$ constructs a local view from this: $((x_1, 1), (x_2, 2))$
- $x_3$ cannot choose a value consistent with this local view.

17-16: Asynchronous Backtracking Example

- $x_3$ sends a nogood to the lowest-priority process in its local view. ($x_2$).
  - $(nogood(x_1, 1), (x_2, 2))$
  - Because of asynchronous communication, $x_3$ must include the entire local view.
  - $x_2$ may have already changed its value.
- Since $x_2$ does not have a link to $x_1$, it adds $x_1$ as a neighbor.
- Requests $x_1$, current value.
17-17: Asynchronous Backtracking

Example

- $x_1$ returns $(x_1, 1)$, which $x_2$ adds to its local view.
- $x_2$ checks its current assignment and its local view against the list of nogoods.
  - $(x_1, 1), (x_2, 2)$ is a nogood.
- Therefore, $x_2$ can’t find a consistent value, and so it sends $(\text{nogood}, (x_1, 1))$ back to $x_1$.
- $x_1$ receives the nogood. Since there are no other processes included, $x_1$ knows that it must change its value.
- It chooses $(x_1, 2)$ and resends $(\text{ok?}, (x_1, 1))$ to $x_2$ and $x_3$.
- $x_2$ can safely also choose 2. When $x_3$ gets the message, it sends $(\text{ok?}, (x_1, 2), (x_3, 2))$ to $x_2$.

17-18: Asynchronous backtracking

- Guaranteed to terminate and find a solution if one exists (complete).
- Same process as single-processor backtracking
- If an variable can’t be assigned a value, undo the next-most-recent variable.
- More communication, due to asynchronicity.

17-19: Asynchronous Weak-commitment Search

- A problem with asynchronous backtracking is the statically defined priorities:
  - In the example, a poor choice by $x_1$ influenced everything.
  - A bad choice by a high-priority agent might cause a great deal of needless resetting of variables.
- Rather than just sending a nogood to the lowest-priority process, we use a heuristic.

17-20: Asynchronous Weak-commitment Search

- Each process gets a priority value assigned dynamically.
- When an agent can’t find a value that satisfies all constraints, it:
  - chooses a value that minimizes constraint violations
  - increases its priority to $k + 1$, where $k$ is the highest nogood received.
  - Sends nogoods to all lower-priority processes.

17-21: Asynchronous Weak-commitment Search Example

- One agent per row
- Initially all priorities are 0.
- All agents send ok? messages to each other.
- $a_4$ is unable to find an assignment consistent with its local view.

17-22: Asynchronous Weak-commitment Search Example

- $a_4$ sends $(\text{nogood}(a_1, 1), (a_2, 4), (a_3, 2), (a_4, 4))$ messages to all other agents.
- $a_4$ increments its priority.
- Chooses a new value that minimizes constraints, given its local view.
  - $a_4 = 3$
  - $a_4$ sends ok? messages to all other processes.
17-23: Asynchronous
Weak-commitment Search Example

a3 receives a4's ok? and tries to find a new value.

No consistent value
- Selects 1 to minimize conflicts
- Increments priority to 2 (since a4 was 1).
- Sends ok? to all lower priorities.
- a2 and a4 are fine with this choice.

17-24: Asynchronous
Weak-commitment Search Example

a1 has a conflict with a3, and moves to 2.
- Consistent, so no need to send nogoods.
- The solution satisfies all constraints.
- Notice that the original problem stemmed from a bad choice by a1.
- Unlike asynchronous backtracking, this algorithm doesn't exhaustively search a2's choices before changing a1.

17-25: Breakout

Backtracking and weak-commitment are examples of algorithms that construct consistent partial solutions.
- Each node tries to select values that are consistent with some other subset of nodes.
- We can also use hill-climbing to solve this problem
  - Sometimes called iterative improvement.
- Premise start with a complete but flawed solution and try to apply improvements.

17-26: Breakout

We can use the min-conflict heuristic to guide hill-climbing.
- Start with a random assignment of values to variables.
- foreach variable:
  - Choose the value that minimizes the number of conflicts
- Repeat until:
  - A solution is found
  - A local optimum is reached.

17-27: Breakout

Breakout helps us jump out of local optima.
- Each constraint starts with a weight of 1.
- When a local optimum is found, all currently-violated constraints have their weights increased by 1.
  - This makes nearby states more appealing, and "pushes" the hillclimber out of the optimum.
- Like all hill-climbing algorithms, this is not complete, but can be tuned to work well in practice.

17-28: Distributed Breakout

We can also construct a distributed version of breakout.
- Neighbors exchange possible assignments, and the agent whose assignment produces maximal improvement is chosen.
- Agents detect that subgroups are trapped in a quasi-local minimum. (no need to solve consensus to detect a local minimum).
  - $x_i$ is in a quasi-local minimum if it is violating a constraint, and the improvement of all of its neighbors is 0.
  - In other words, no hill can be climbed in the agent's immediate vicinity.
**17-29: Distributed Breakout**

- Agents exchange ok? and improve? messages.
- When an ok? message is received, an agent evaluates currently-violated constraints.
- The value that minimizes the agents’ constraints is sent in an improve? message.
- Once replies from all neighbors are received, if this agent’s new value maximizes improvement, send the new value in an ok? message to all neighbors.
- Else send the old value in an ok? message.

**17-30: Distributed Breakout Example**

- Map-coloring problem with 2 colors.
- Initially, everyone sends an ok? message to their neighbors.
- All weights set to 1.
- Initially, no agent has a local improvement that will reduce the number of conflicts.

**17-31: Distributed Breakout Example**

- Weights on current constraints are increased:
  - nogood: (x1 = x6 = white): 2
  - nogood: (x3 = x4 = white): 2
  - nogood: (x2 = x5 = black): 2
- Now, x1, x3, x5, x6 have an improve value of 1.
- x1 and x3 have the right to change (lower number)

**17-32: Distributed Breakout Example**

- Now x2 has an improvement value of 4, and all other agents have improves of 0.
- So x2 changes to white, and the problem is solved.

**17-33: Summary**

- DCSP is a nice example of a medium-coupled problem
  - Each agent has local constraints that it must satisfy.
  - Constraints also exist with other agents.
  - Variables are iteratively changed until no constraints are violated.
- Many problems fit into this framework
  - Scheduling, planning, coordination
- Challenges: expressing complex constraints, intelligent heuristics.