Department of Computer Science **University of San Francisco**

Computer Science 411
Homework 7: Context-Free and Non-Context Free Languages
Fall 2015
Due 10/23/2015

1. Problem 3.3.4 from the text. Let $M = (K, \Sigma, \Gamma, \Delta, s, F)$ be a pushdown automaton. The **languages accepted by M by empty store** is defined as follows:

   $L_e(M) = \{w \in \Sigma^* : (s, w, e) \vdash_M^* (q, \epsilon, \epsilon) \text{ for some } q \in K\}$

   (a) (6 points) Show that there is a pushdown automaton M' such that $L_e(M') = L(M)$. Note that I want a formal contsruction for this problem!

   (b) (6 points) Show that there is a pushdown automaton M'' such that $L(M'') = L_e(M)$. Note that I want a formal contsruction for this problem!

   (c) (3 points) Show by a counterexample that it is not necessarily the case that $L_e(M) = L(M) \cup \{\epsilon\}$

2. For each language L, if:

   - L is regular, give a regular expression **and** a finite automaton (either deterministic or non-deterministic) for L
   - L is context-free (but not regular), prove that L is not regular using the pumping lemma, **and** give a Context-Free Grammar for L **and** give a push-down automata for L
   - L is not context-free, prove that L is not Context-Free using the context-free pumping lemma (and possibly closure properties).

   (a) (6 points) All strings over $\{a, b, c\}$ that have more a's than b's, and more a's than c's. So $abbaac, babbaccaaa, aaa, aabbaa \in L$, while $abc, aabc, ccaabab \notin L$

   (b) (6 points) All strings over $\{0, 1\}$ that are binary representations of the number $2^x - 1$ for some positive integer x

   (c) (6 points) $\{a^{2n} : n > 0\}$

   (d) (6 points) $\{a^{2^n} : n > 0\}$