Automata Theory

CS411-2015F-05

Deterministic Finite Automata vs. Non-Deterministic Finite Automata

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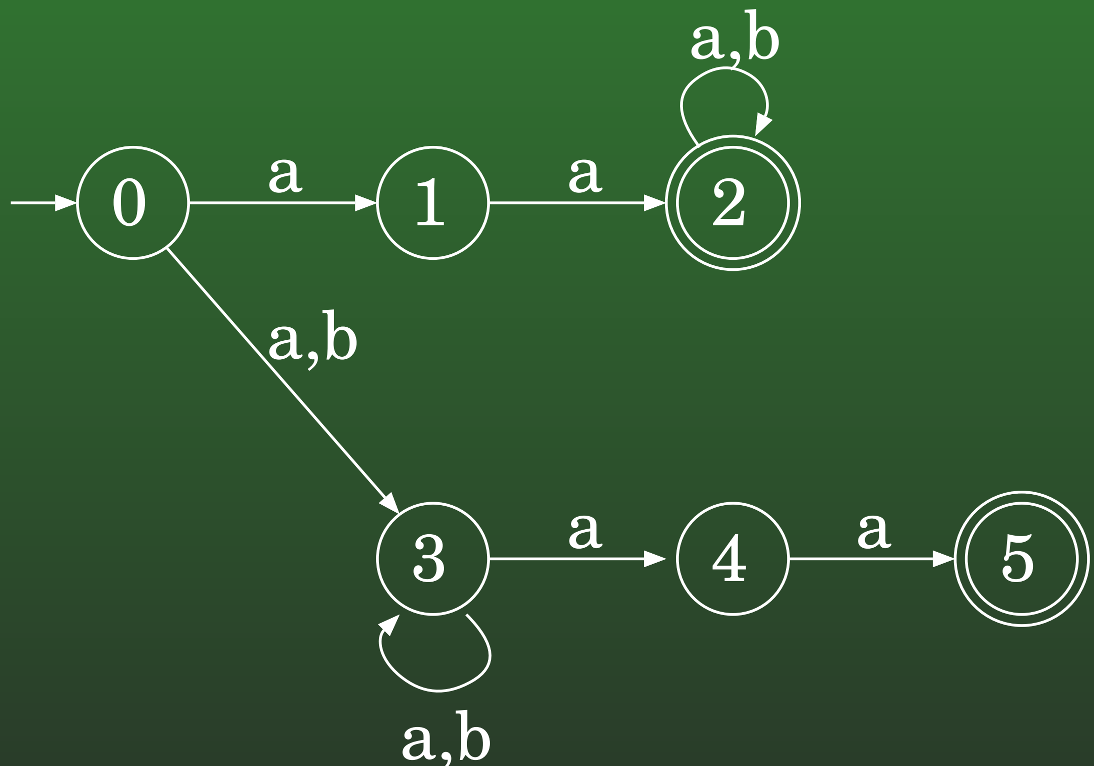
05-0: YANFAE[1]

- All strings over {a,b} that begin or end with aa

[1] Yet Another NFA Example

05-1: YANFAE[1]

- All strings over {a,b} that begin or end with aa



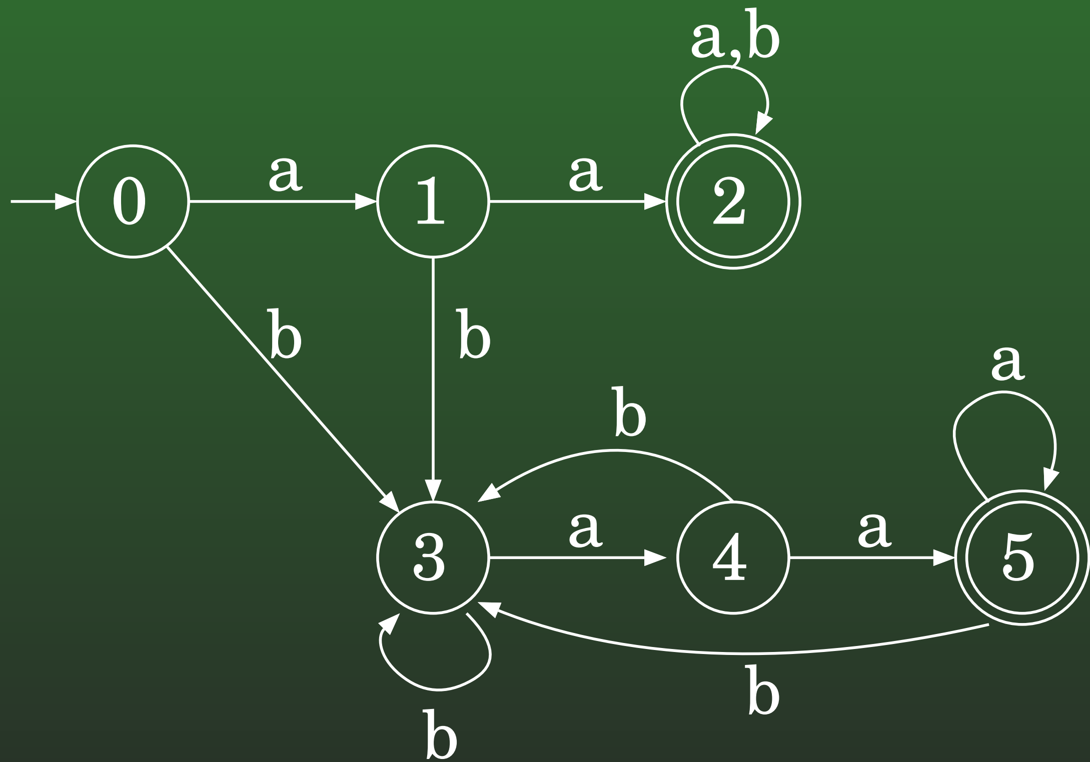
[1] Yet Another NFA Example

05-2: NFA $\rightarrow$ DFA

- Can we create a DFA for the same langauge?
- All strings over {a,b} that begin or end with aa

05-3: NFA $\rightarrow$ DFA

- Can we create a DFA for the same langauge?
- All strings over {a,b} that begin or end with aa



05-4: L_{NFA} vs L_{DFA}

- What is the relationship between L_{NFA} and L_{DFA}?
  - $L_{DFA} \subseteq L_{NFA}$
  - Why?

05-5: L_{NFA} vs L_{DFA}

- What is the relationship between L_{NFA} and L_{DFA}?
  - $L_{DFA} \subseteq L_{NFA}$
  - Every DFA is also an NFA

05-6: L_{NFA} vs L_{DFA}

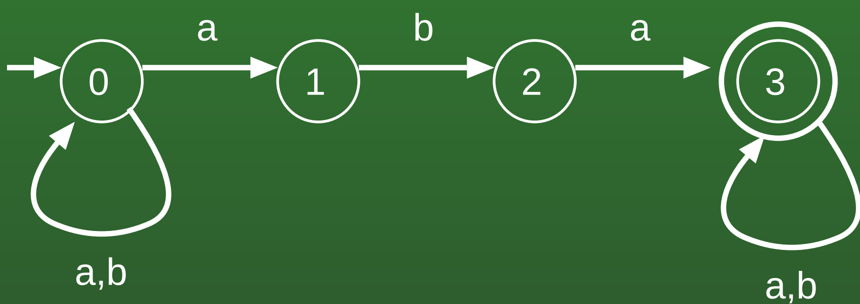
- What is the relationship between L_{NFA} and L_{DFA}?
  - $L_{DFA} \subset L_{NFA}$?
  - $L_{DFA} \subseteq L_{NFA} \wedge L_{NFA} \subseteq L_{NFA} (L_{NFA} = L_{DFA})$?
- Given any NFA M, can we create a DFA M' such that $L[M] = L[M']$?

05-7: L_{NFA} vs L_{DFA}

- What is the relationship between L_{NFA} and L_{DFA}?
  - $L_{DFA} \subseteq L_{NFA} \wedge L_{NFA} \subseteq L_{NFA} (L_{NFA} = L_{DFA})$
- Given any NFA M, we *can* create a DFA M' such that $L[M] = L[M']$

05-8: NFA $\rightarrow$ DFA

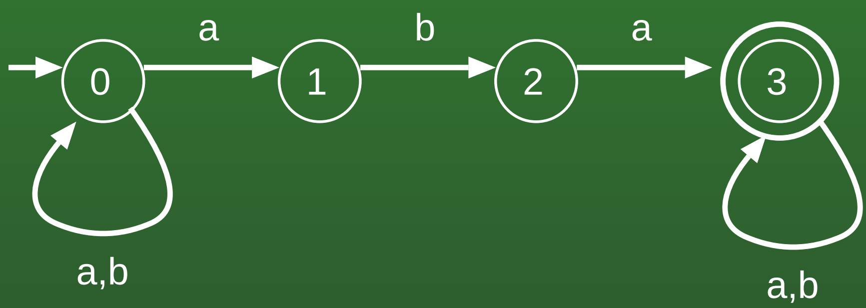
- NFA for all strings over {a,b} containing aba



- Trace abab

05-9: NFA $\rightarrow$ DFA

- NFA for all strings over {a,b} containing aba

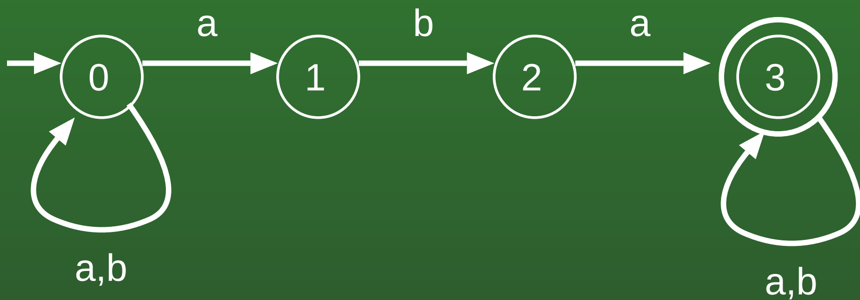


- Trace abab

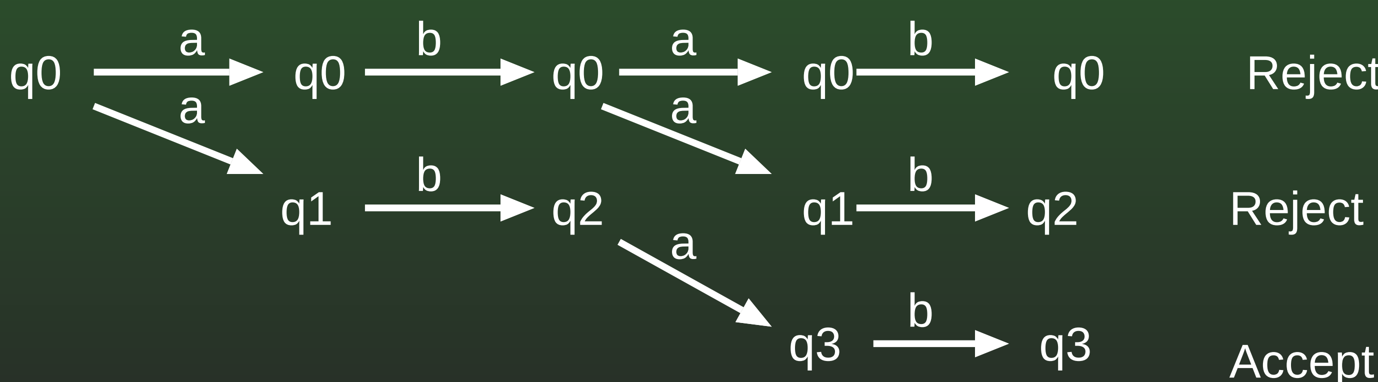
q0, abab →	q0, bab →	q0, ab →	q0, b →	q0,ε	Reject
	q1, bab →	q2, ab	q1, b →	q2,ε	Reject
			q3, b →	q3,ε	Accept

05-10: NFA $\rightarrow$ DFA

- NFA for all strings over {a,b} containing aba

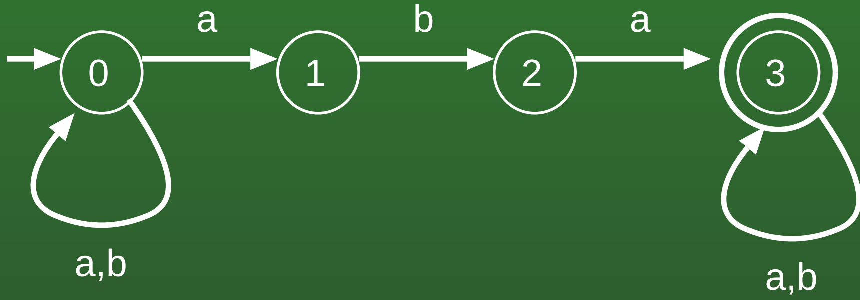


- Trace abab

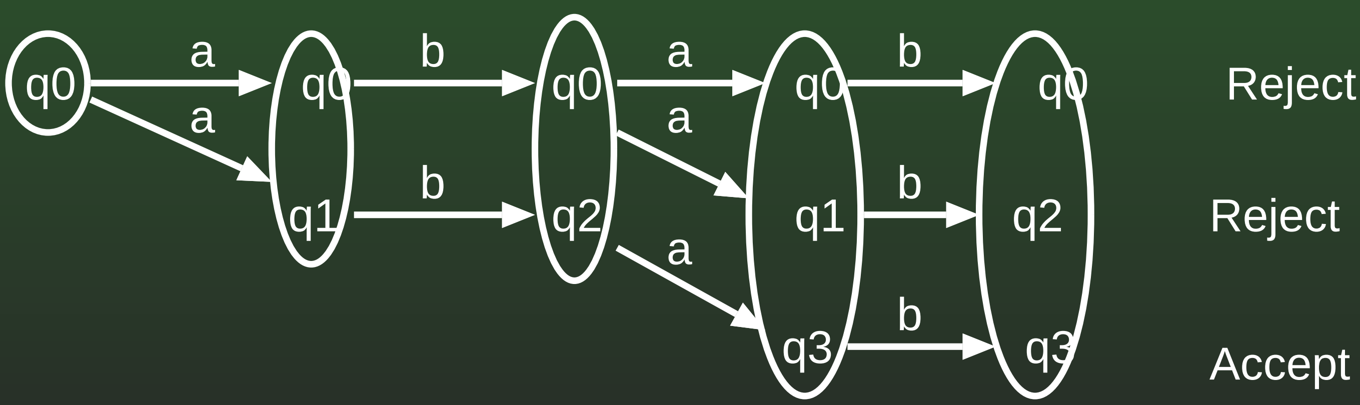


05-11: NFA $\rightarrow$ DFA

- NFA for all strings over {a,b} containing aba

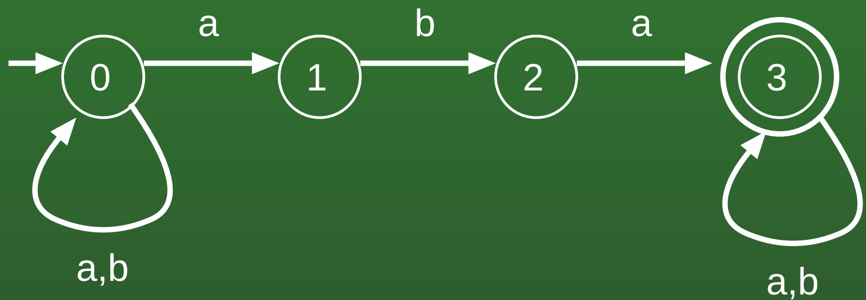


- Trace abab

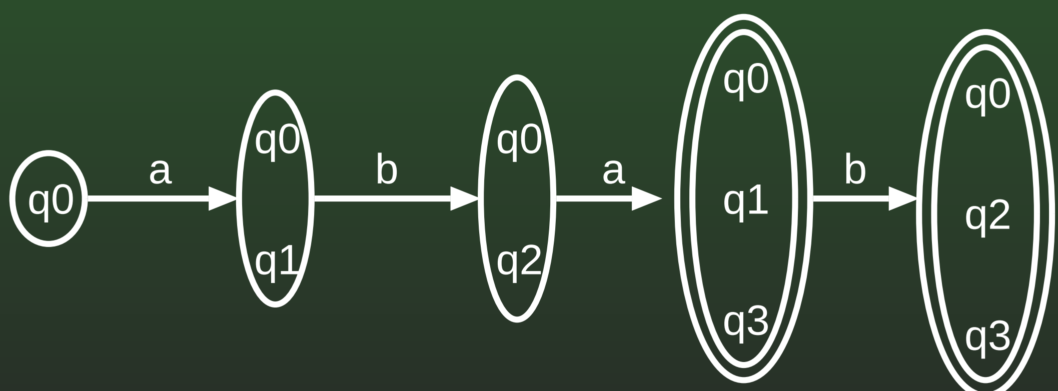


05-12: NFA → DFA

- NFA for all strings over {a,b} containing aba

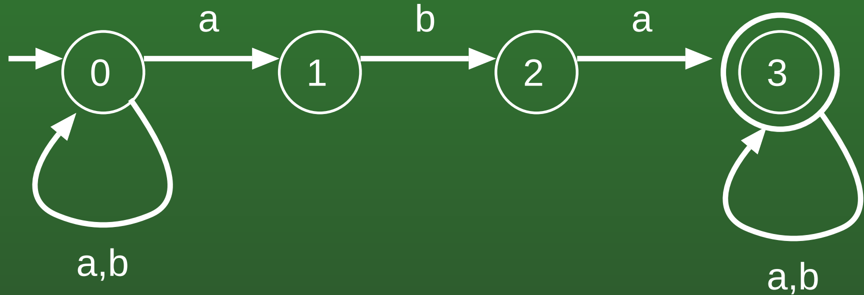


- Trace abab



05-13: NFA $\rightarrow$ DFA

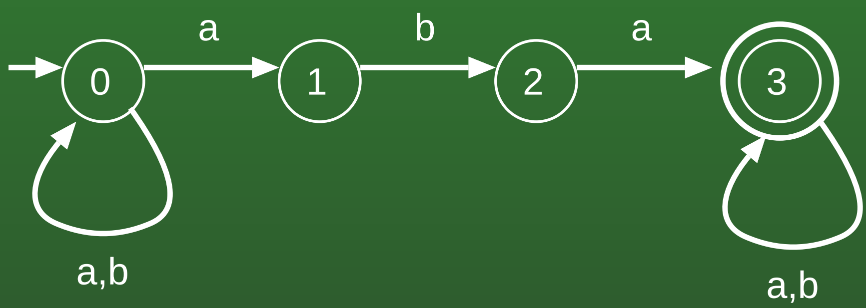
- NFA for all strings over {a,b} containing aba



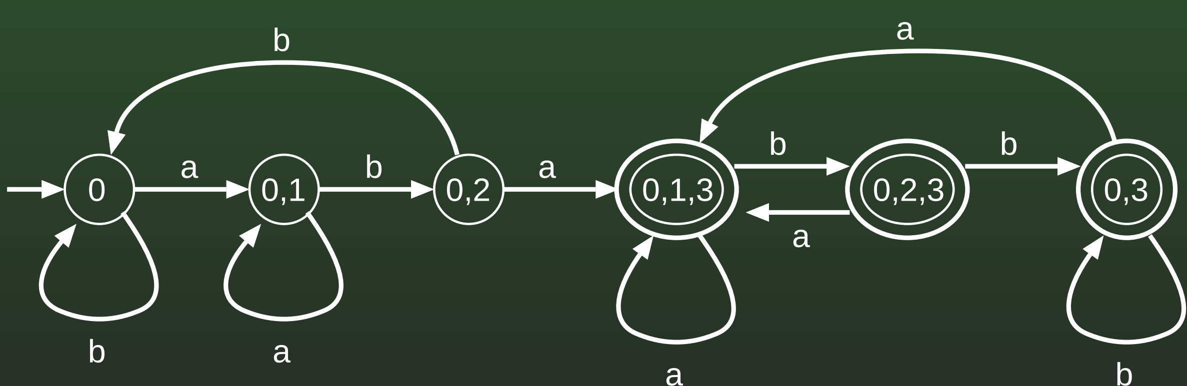
- Build Equivalent DFA

05-14: NFA $\rightarrow$ DFA

- NFA for all strings over {a,b} containing aba

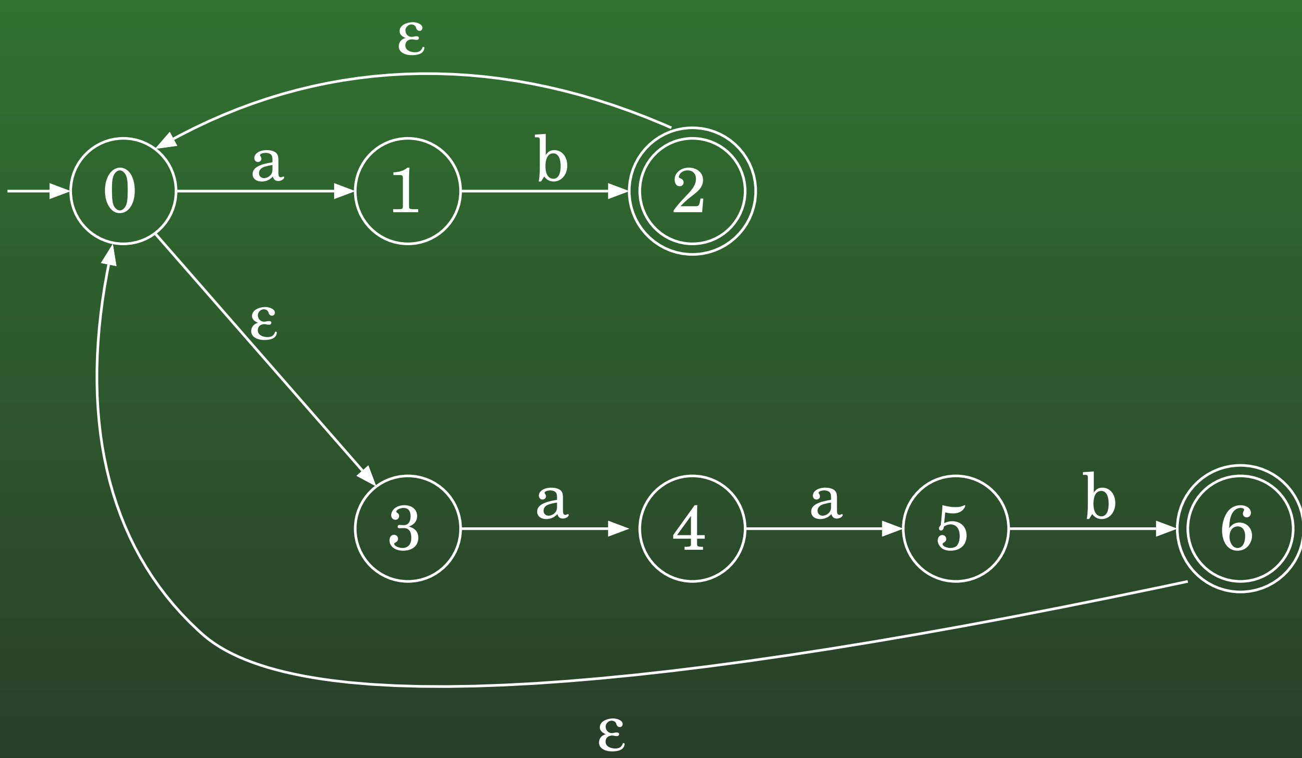


- Build Equivalent DFA



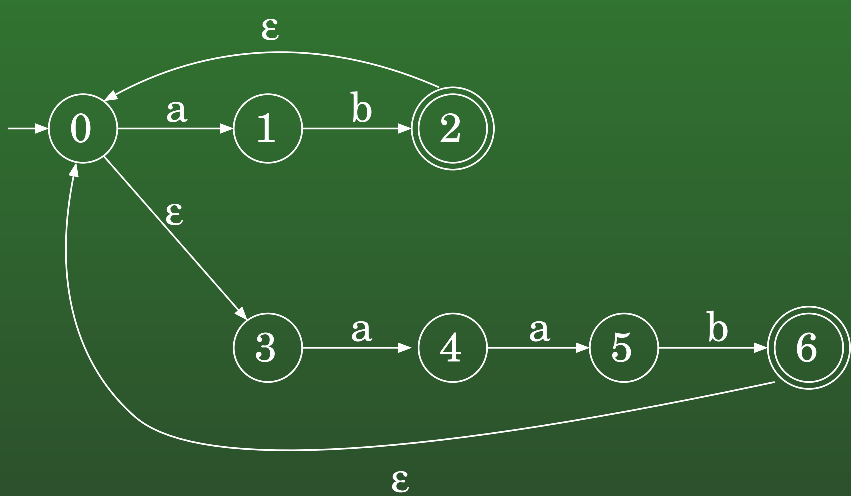
05-15: NFA $\rightarrow$ DFA

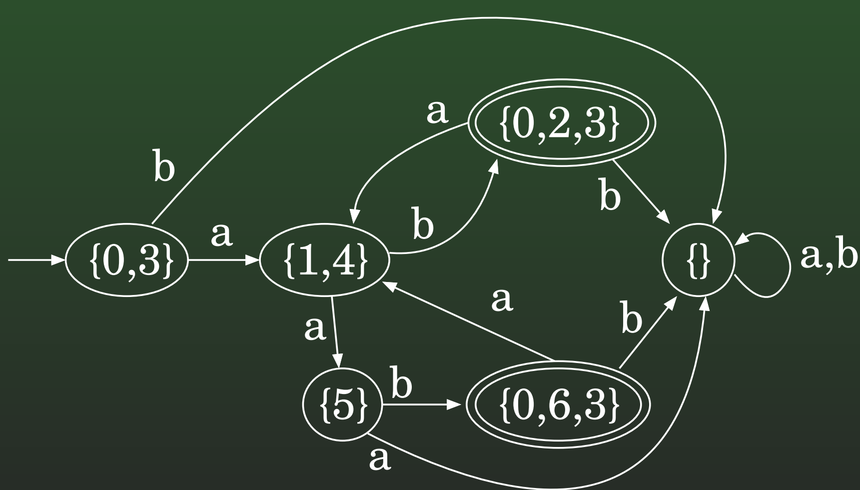
- What about ϵ-transitions?



05-16: NFA $\rightarrow$ DFA

- What about ϵ-transitions?





05-17: NFA $\rightarrow$ DFA

- Example $\neq$ Proof!
- Need to show, given *any* NFA M, we can create a DFA M' such that $L[M] = L[M']$
  - Constructive Proof

05-18: **Proof:** $L_{NFA} \subseteq L_{DFA}$

- Given NFA $M = (K, \Sigma, \Delta, s, F)$
- Create DFA $M' = (K', \Sigma', \delta', s', F')$
  - Such that $L[M'] = L[M]$

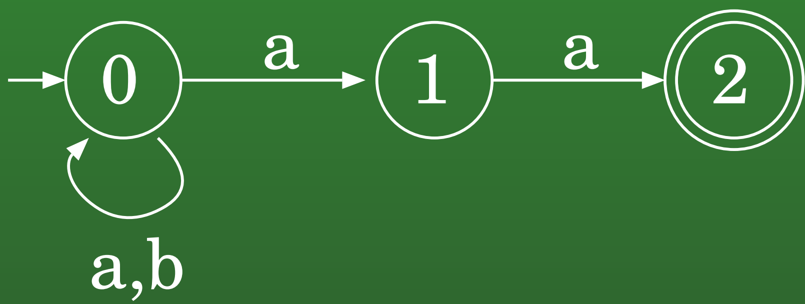
05-19: Proof: $L_{NFA} \subseteq L_{DFA}$

- NFA $M = (K, \Sigma, \Delta, s, F)$
- DFA $M' = (K', \Sigma', \delta', s', F')$
  - $K' =$
  - $\Sigma' =$
  - $\delta' =$
  - $s' =$
  - $F' =$

05-20: Proof: $L_{NFA} \subseteq L_{DFA}$

- NFA $M = (K, \Sigma, \Delta, s, F)$
- DFA $M' = (K', \Sigma', \delta', s', F')$
  - $K' = 2^K$
  - $\Sigma' = \Sigma$
  - $\delta' = \{((q_1, a), q_2) : q_1 \in K', a \in \Sigma,$
    $q_2 = \epsilon$-closure $(\{q : (q_3 \in q_1) \wedge ((q_3, a), q) \in \Delta\})$
  - $s' = \epsilon$-closure(s)
  - $F' = \{Q : Q \in 2^K \wedge Q \cap F \neq \emptyset\}$

05-21: Example: $L_{NFA} \subseteq L_{DFA}$



- $K = \{q_0, q_1, q_2\}$
- $\Sigma = \{a, b\}$
- $\Delta = ((q_0, a), q_0), ((q_0, a), q_1), ((q_0, b), q_0), ((q_1, a), q_2)\}$
- $s = q_0$
- $F = \{q_2\}$

05-22: Example: $L_{NFA} \subseteq L_{DFA}$

- $K' =$
  $\{\{\}, \{q_0\}, \{q_1\}, \{q_2\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$
- $\Sigma' = \{a, b\}$
- $\delta' = \{((\{\}, a), \{\}), ((\{\}, b), \{\}), ((\{q_0\}, a), \{q_0, q_1\}),$
  $((\{q_0\}, b), \{q_0\}), ((\{q_1\}, a), \{q_2\}), ((\{q_1\}, b), \{\}),$
  $((\{q_2\}, a), \{\}), ((\{q_2\}, b), \{\}),$
  $((\{q_0, q_1\}, a), \{q_0, q_1, q_2\}), ((\{q_0, q_1\}, b), \{q_0\}),$
  $((\{q_0, q_2\}, a), \{q_0, q_1\}), ((\{q_0, q_2\}, b), \{q_0\}),$
  $((\{q_1, q_2\}, a), \{q_2\}), ((\{q_1, q_2\}, b), \{\}),$
  $((\{q_0, q_1, q_2\}, a), \{q_0, q_1, q_2\}), ((\{q_0, q_1, q_2\}, b), \{q_0\})$
- $s' = \{q_0\}$
- $F' = \{\{q_2\}, \{q_0, q_2\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$

05-23: Example: $L_{NFA} \subseteq L_{DFA}$