August 18, 2014
Note Title 8/18/2014

WE'VE SEEN LOTS OF EXAMPLES OF ID'BLE CLASSES OF LANGUAGES, ALL COMPUTABLY ID'BLE.

TODAY — SEE SOME $\mathcal{L}$ THAT ARE <u>NOT</u> ID'BLE.

- RECALL : $\mathbb{N} - \{x\}$ OR $\mathbb{N}$.
- HOW TO SHOW THAT SOME $\mathcal{L}$ IS <u>NOT</u> ID'BLE?
  - WE MUST SHOW EVERY $f \in \mathcal{F}$ FAILS TO IDENTIFY $\mathcal{L}$.
  - WE WILL DO THIS BY ASSUMING

    $\exists f \in \mathcal{F}$ THAT ID'S $\mathcal{L}$.

    ⋮ ← HOW TO DO THIS? USE LOCKING SEQUENCE THEOREM!

    $\rightarrow \leftarrow$.

NOTATION: IF $\sigma, \tau \in SEQ$, THEN $\sigma^\wedge\tau$ IS JUST σ FOLLOWED BY τ.

ex. $\sigma = (2,17,0)$, $\tau = (19,3)$, $\sigma^\wedge\tau = (2,17,0,19,3)$

IF $(\sigma^1, \sigma^2, \sigma^3, \ldots)$ AND $\forall i < j$, $\underbrace{\sigma^i \subseteq \sigma^j}$

ex. $((2,0), (2,0,14,3), (2,0,14,3,5), \ldots)$

σ^i IS AN INITIAL SEGMENT OF σ^j.

AND $\forall n \in \mathbb{N}\ \exists m \in \mathbb{N}$ S.T. $\ln(\sigma^m) > n$.

THEN $t = \lim_{i \to \infty} \sigma^i = \bigcup_{i=1}^{\infty} \sigma^i$.

LOCKING SEQUENCE THEOREM.

ASSUME $f \in \mathcal{F}$ THAT IDENTIFIES $L \in RE$.
THEN $\exists \sigma \in SEQ$ SUCH THAT

SUCH A σ IS A LOCKING SEQUENCE FOR f AND L.

(i) $\operatorname{rng} \sigma \subseteq L$

(ii) $W_{f(\sigma)} = L$.

(iii) $\forall \tau \in SEQ$, IF $\operatorname{rng} \tau \subseteq L$ THEN $f(\sigma^\wedge\tau) = f(\sigma)$.

PROP $\mathcal{L}_1 = RE_{FIN} \cup \{\mathbb{N}\}$. $\mathcal{L}_1$ IS NOT IDENTIFIABLE.

PROOF ASSUME $f \in \mathcal{F}$ IDENTIFIES $\mathcal{L}_1$.

THEN f IDENTIFIES $\mathbb{N}$. LET σ BE A LOCKING SEQUENCE FOR f AND $\mathbb{N}$.

NOTE THAT $\mathrm{rng}\,\sigma \in RE_{FIN}$, SAY $\mathrm{rng}\,\sigma = \{s_1, s_2, \ldots s_k\}$.

THEN $\sigma^\frown \underbrace{s_1{}^\frown s_1{}^\frown s_1}_{\tau} \ldots$ IS A TEXT FOR $\mathrm{rng}\,\sigma$.

$\mathrm{rng}\,\tau \subseteq \mathbb{N}$.

BUT $f(\sigma^\frown\tau) = f(\sigma)$, SINCE σ IS L.S. FOR f AND $\mathbb{N}$

AND THIS IS TRUE FOR ANY $\tau = s_1{}^\frown s_1{}^\frown \cdots {}^\frown s_1$

OF ANY LENGTH, AND THIS AN INDEX FOR $\mathbb{N}$.

BUT THEN f DOESN'T ID THE FINITE SET $\mathrm{rng}\,\sigma$.

$\rightarrow\leftarrow$

PROOF (OF LOCKING SEQ THM). BWOC, SUPPOSE THM IS FALSE.

THEN FOR ANY $\eta \in SEQ$, IF $\mathrm{rng}\,\eta \subseteq L$ AND $W_{f(\eta)} = L$ THEN $\exists \tau \in SEQ$ S.T. $\mathrm{rng}(\tau) \subseteq L$ AND $f(\eta^\frown\tau) \neq f(\eta)$. *

WE SHOW THAT $* \rightarrow$ THERE IS A TEXT, t, FOR L THAT f FAILS TO IDENTIFY. (CONTRADICTS THAT f ID'S L).

LET $s = s_0, s_1, s_2, \ldots$ BE A TEXT FOR L.

WE BUILT t IN STAGES (BY FINITE EXTENSION).

AT STAGE n, WE'LL HAVE DEFINED σ^n, AND IN THE END $t = \bigcup_{n=1}^{\infty} \sigma^n$.

CONSTRUCTION:

STAGE 0: LET $\sigma^0 \in SEQ$ S.T. $\mathrm{rng}\,\sigma^0 \subseteq L$ AND $W_{f(\sigma^0)} = L$.

STAGE $n+1$: HAVE σ^n. DEFINE σ^{n+1} BASE ON THESE 2 CASES.

CASE 1: IF $W_{f(\sigma^n)} \neq L$, SET $\sigma^{n+1} = \sigma^n \wedge s_n$.

CASE 2: IF $W_{f(\sigma^n)} = L$, THEN BY $*$ WE CAN FIND A $\tau \in SEQ$ S.T. $\mathrm{rng}\,\tau \subseteq L$ BUT

$$f(\sigma^n \wedge \tau) \neq f(\sigma^n).$$

SET $\sigma^{n+1} = \sigma^n \wedge \tau \wedge s_n$.

LET $t = \bigcup \sigma^n$

END CONSTRUCTION.

NOTE THAT $\forall n$, $\mathrm{rng}\,\sigma^n \subseteq L$, AND t IS A TEXT FOR L.

AND f DOES NOT CONVERGE ON t. $\rightarrow\leftarrow$

So, THEOREM MUST HOLD. ∎

NOTATION: $[\mathcal{F}]$ = SET OF ALL CLASSES OF LANGUAGES THAT ARE IDENTIFIABLE BY SOME $f \in \mathcal{F}$.

$[\mathcal{F}^{REC}]$ = SET OF ALL CLASSES OF LANGUAGES ID'BLE BY SOME $f \in \mathcal{F}^{REC}$.

EXERCISES:

1. $L_x = \mathbb{N} - \{x\}$. LET $\mathcal{L}_2 = \{L_x \mid x \in \mathbb{N}\} \cup \{\mathbb{N}\}$.

   $\mathcal{L}_2$ IS NOT ID'BLE.

2. LET L BE ANY INFINITE RE SET. THEN $RE_{FIN} \cup \{L\}$ IS NOT ID'BLE.

3. IF $\mathcal{L}$ IS NOT ID'BLE, NEITHER IS ANY SUPERSET OF $\mathcal{L}$.