August 19, 2014

Note Title 8/19/2014

- LOCKING SEQUENCE THEOREM

IF $\varphi \in \mathcal{F}$ IDENTIFIES $L \in RE$ THEN THERE IS A $\sigma \in SEQ$

(i) $rng(\sigma) \subseteq L$

(ii) $\varphi(\sigma)$ = INDEX FOR L. $(W_{\varphi(\sigma)} = L.)$

(iii) $\forall \tau \in SEQ$ IF $rng\, \tau \subseteq L$ THEN $\varphi(\sigma \wedge \tau) = \varphi(\sigma)$.

- LET $\mathcal{S} \subseteq \mathcal{F}$. THEN $[\mathcal{S}] = \{\mathcal{L} \subseteq RE \mid \exists f \in \mathcal{S}$ s.t. f ID's $\mathcal{L}\}$.

EXAMPLES: $RE_{FIN} \cup \{N\} \notin [\mathcal{F}]$.

$\mathcal{L}_2 = \{\mathbb{N} - \{x\} \mid x \in \mathbb{N}\} \cup \{\mathbb{N}\} \notin [\mathcal{F}]$.

L IS INFINITE, THEN $RE_{FIN} \cup \{L\} \notin [\mathcal{F}]$.

PROP. $\mathcal{L}_2 \notin [\mathcal{F}]$.

PROOF. SUPPOSE φ IDENTIFIES $\mathcal{L}_2$.

LET σ BE A LOCKING SEQUENCE FOR $\mathbb{N}$.

$\Rightarrow \forall \tau \in SEQ$, IF $rng\, \tau \subseteq \mathbb{N}$, $\varphi(\sigma \wedge \tau) = \varphi(\sigma)$ = INDEX FOR $\mathbb{N}$.

Let x = least number $\notin rng\, \sigma$.

THEN $t = \sigma \wedge 0 \wedge 1 \wedge 2 \wedge \ldots \wedge x-1 \wedge x+1 \wedge \ldots$

IS A TEXT FOR $\mathbb{N} - \{x\}$.

BUT $\varphi(\bar{t}_m)$ IS AN INDEX FOR $\mathbb{N}$ FOR ANY

$m > \ln \sigma$. $\rightarrow\leftarrow$. ∎

NOTE: IF $\mathcal{L} \notin [\mathcal{F}]$ THEN NEITHER IS ANY $\mathcal{L}' \supseteq \mathcal{L}$.

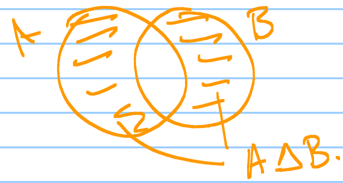
COR: $RE \notin [\mathcal{F}]$. $RE_{REC} \notin [\mathcal{F}]$.

THM (WEIHAGEN, '78) $\exists \mathcal{L} \subseteq RE$ s.t.

(i) $\forall L \in RE\ \exists L' \in \mathcal{L}$ THAT IS A FINITE VARIANT OF L

(ii) $\mathcal{L}$ IS IDENTIFIABLE.

A & B ARE FINITE VARIANTS IN $\text{card}(A \,\Delta\, B)$ IS FINITE.



SO, RE IS ALMOST IDENTIFIABLE.

NEXT GOAL: $[\mathcal{F}^{\text{REC}}] \subsetneq [\mathcal{F}]$

$$\mathcal{L} = \{ \mathbb{K} \cup \{x\} \mid x \in \mathbb{N} \}$$

NON-COMPUTABLE

LEMMA 1 $\mathcal{L} \in [\mathcal{F}]$.

pf

$$f(\sigma) = \begin{cases} \text{LEAST INDEX FOR } \mathbb{K} & \text{IF } \operatorname{rng}\sigma \subseteq \mathbb{K} \\ \text{LEAST INDEX FOR } \mathbb{K} \cup \{x\} & \text{WHERE } x \text{ is least number in } \operatorname{rng}\sigma \notin \mathbb{K}. \end{cases}$$

LEMMA 2 $\mathcal{L} \notin [\mathcal{F}^{\text{REC}}]$

PROOF SUPPOSE φ ID'S $\mathcal{L} = \{ \mathbb{K} \cup \{x\} \mid x \in \mathbb{N} \}$.

THEN φ ID'S $\mathbb{K}$ (since $\mathbb{K} \cup \{x\} = \mathbb{K}$ FOR $x \in \mathbb{K}$).

LET σ BE A LOCKING SEQUENCE FOR φ AND $\mathbb{K}$, SO $\varphi(\sigma)$ IS AN INDEX FOR $\mathbb{K}$, AND IF $\operatorname{rng}\tau \subseteq \mathbb{K}$, $\varphi(\sigma^\wedge \tau) = \varphi(\sigma)$.

LET $k_0, k_1, k_2, \ldots$ BE A COMPUTABLE ENUMERATION OF $\mathbb{K}$.

FOR EACH x, LET $t^x = \sigma^\wedge x^\wedge k_0{}^\wedge k_1{}^\wedge \ldots$

IF $x \in \overline{\mathbb{K}}$, THEN $\exists n > lh(\sigma)$ S.T. $\varphi(\overline{t^x_n}) \neq \varphi(\sigma)$

BECAUSE φ ID'S $\mathbb{K} \cup \{x\}$ AND t^x IS A TEXT FOR THIS SET, AND $\varphi(\sigma)$ IS AN INDEX FOR $\mathbb{K}$.

IF $x \in \mathbb{K}$, THEN $\forall n > lh(\sigma)$ $\varphi(\overline{t^x_n}) = \varphi(\sigma)$

BECAUSE t^x IS JUST ANOTHER TEXT FOR $\mathbb{K}$ IF $x \in \mathbb{K}$.

CONSIDER THE PROGRAM:

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     INPUT x, SET n = lh σ.
1.   IF φ(t^x_n) ≠ φ(σ)
       THEN OUTPUT 1.
       ELSE, SET n=n+1, GOTO 1.
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THIS PROGRAM HAS DOMAIN $\overline{\mathbb{K}}$.

BUT $\overline{\mathbb{K}}$ IS NOT C.E.!

$\rightarrow\leftarrow$.

RE_{SVT} IS IDENTIFIABLE, BUT NOT BY A COMPUTABLE LEARNING FUNCTION.

DEFNS LET $f: \mathbb{N} \to \mathbb{N}$. DEFINE $\text{graph} f = \{ \langle x,y \rangle \mid f(x) = y \}$.

$$RE_{SVT} = \{ \text{graph} f \mid f \text{ IS A COMPUTABLE FUNCTION} \}.$$

LEMMA $RE_{SVT} \in [\mathcal{F}]$.

NOT COMPUTABLE FUNCTION.

PROOF $\varphi(\sigma) =$ least index for A SET IN RE_{SVT} THAT IS CONSISTENT WITH σ.

$f(x) = x^2$, $t = \langle 0,0 \rangle ^\wedge \langle 1,1 \rangle ^\wedge \langle 2,4 \rangle ^\wedge \langle 3,9 \rangle ^\wedge \ldots$

$\langle 0,0 \rangle \longrightarrow$ INDEX FOR $f(x) = x$

$\langle 0,0 \rangle ^\wedge \langle 1,1 \rangle \longrightarrow f(x) = x$

$\langle 0,0 \rangle ^\wedge \langle 1,1 \rangle ^\wedge \langle 2,4 \rangle \longrightarrow$

LEMMA $RE_{SVT} \notin [\mathcal{F}^{REC}]$.

PROOF IDEA: ASSUME φ ID'S RE_{SVT}. BUILD $L \in RE_{SVT}$ AND t FOR L S.T. φ CHANGES ITS MIND INFINITELY OFTEN ON t.

BUILD t BY FINITE EXTENSION:

- $t = \bigcup \sigma^s$, BUILD $\sigma^s \in SEQ$ AT STAGE s.
- $\sigma^s = \langle 0, x_0 \rangle ^\wedge \langle 1, x_1 \rangle ^\wedge \cdots ^\wedge \langle n, x_n \rangle$

  $x_i \in \{0,1\}$.

BEFORE CONSTRUCTION,

A CLAIM: ASSUME φ ID'S RE_{SVT}.

LET $\sigma = \langle 0, x_0 \rangle ^\wedge \langle 1, x_1 \rangle ^\wedge \ldots ^\wedge \langle n, x_n \rangle$.

THEN $\exists j$ AND $k \in \mathbb{N}$. SUCH THAT

$\tau = \sigma ^\wedge \langle n+1, 0 \rangle ^\wedge \langle n+2, 0 \rangle ^\wedge \cdots ^\wedge \langle n+j, 0 \rangle$.

$\tau' = \tau ^\wedge \langle n+j+1, 1 \rangle ^\wedge \langle n+j+2, 1 \rangle ^\wedge \cdots ^\wedge \langle n+j+k, 1 \rangle$

NOTE: τ IS START OF A TEXT FOR A L THAT REPRESENTS A FUNCTION THAT IS EVENTUALLY $= 0$.

τ' is " " " " " " "
" " " " " " = 1.

$$\varphi(\tau) \neq \varphi(\tau').$$ ⊠

CONSTRUCTION:

STAGE 0: $\sigma^0 = \langle 0,0 \rangle$.

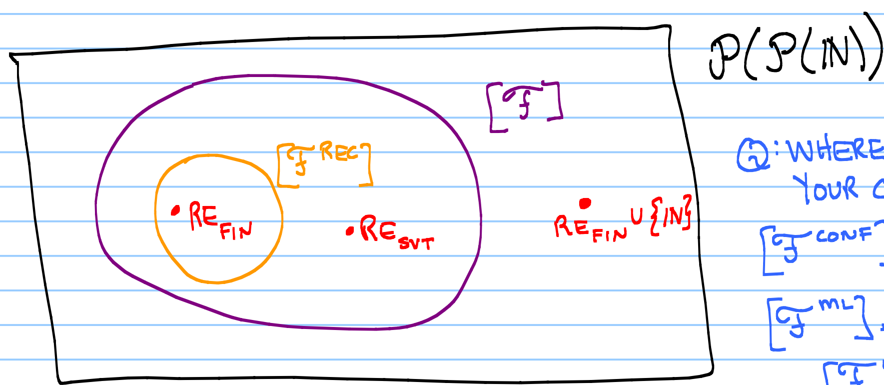
STAGE s+1: HAVE σ^s. APPLY CLAIM TO σ, FIND j AND k, τ AND τ'.

SET $\sigma^{s+1} = \tau' = \sigma^\frown\langle n,0\rangle \dots \langle n+j,0\rangle^\frown\langle n+j+1,1\rangle^\frown \dots \langle n+j+k,1\rangle$.

$t = \bigcup \sigma^s$

END CONSTRUCTION.

φ DOES NOT CONVERGE ON t, BUT $\operatorname{rng} t \in RE_{SVT}$. →←.



Q: WHERE IS YOUR CLASS?

$[\mathcal{F}^{CONF}]$, $[\mathcal{F}^{ML}]$, $[\mathcal{F}^{CONS}]$, $[\mathcal{F}^{ENUM}]$