Computer Science 411
Homework 5: Regular Language Closure, CFG
Fall 2014
Due Friday, October 2nd, 2015

1. Let L_R be some regular language, and let L_{NR} be some non-regular language. Prove or disprove each of the following statements (use a counter-example to disprove):

   (a) (3 points) If $L \subseteq L_R$, then L *must* be regular.

   (b) (3 points) If $L \subseteq L_{NR}$, then L *cannot* be regular.

   (c) (3 points) If $L \subseteq L_{NR} \cap L_R$, then L *must* be regular.

   (d) (3 points) If $L \subseteq L_{NR} \cap L_R$, then L *cannot* be regular.

   (e) (3 points) If $L = \overline{L_{NR}}$ (that is, L is the complement of L_{NR}), then L *cannot* be regular.

   (f) (3 points) If L_1 is regular, then the language $L_2 = \{xy : x \in L_1 \text{ and } y \notin L_1\}$ is also regular.

2. For each of the following languages, give a context-free grammar

   (a) (4 points) $\{a^{2n}b^{3n} : n > 0\}$

   (b) (4 points) $\{a^n x : n \geq 0, x \in (a+b)^*, |x| = n\}$

   (c) (4 points) $L = \{a^i b^j c^k : k \neq i + j\}$

3. Give a Regular Expression equivalent to each of the following CFGs. Think carefully about the language described by the CFG, and create an equivalent regular expression.

   (a) (3 points)

   $S \to ABA$
   $A \to aA$
   $A \to \epsilon$
   $B \to bb$

   (b) (3 points)

   $S \to AB$
   $A \to aAa|bAb|a|b$
   $B \to aB|bB|\epsilon$

   (c) (3 points)

   $S \to AA$
   $A \to AAA$
   $A \to a$