Computer Science 245

Homework 2

Algorithm Analysis II

Due Wednesday, February 8th, 2017

1. For each of the following recursive functions:

   - (1 point) Describe what the function computes (careful, some of these are tricky!)
   - (1 point) Give a recurrence relation that describes the running time of the function (Give both base and recursive cases)
   - (2 point) Solve the recurrence to get a Θ running time for the function. Use either the repeated substitution method, or the recursion tree method (which is essentially the same as the repeated substitution method, just a little more graphical). *Do not* use the master method for this question (you will have a chance to use the master method on later questions!)

   (a)
   ```
   int recursive1(int n)
   {
     if (n == 0)
       return 0;
     else
       return 1 + recursive1(n-1);
   }
   ```

   (b)
   ```
   int recursive2(int n)
   {
     if (n == 0)
         return 0;
     return recursive1(n) + recursive2(n-1);
   }
   ```

   Note that we are making both a recursive call *and a non recursive call* in this function! recursive1 is the function defined in question 1b.

   (c)
   ```
   int recursive3(int n)
   {
     if (n == 0)
       return 1;
     else
       return recursive3(n-1) +  recursive3(n-1);
   }
   ```

   (d)
   ```
   int recursive4(int n)
   {
     if (n == 0)
       return 1;
     else
       return 2 * recursive4(n-1);
   }
   ```

(e)
```
int recursive5(int n)
{
  if (n <= 1)
    return  n;

  int dummy = 0;
  for (int i = 0; i < n; i++)
    dummy++;

  if (n % 2 != 0)
     return 1 + recursive5(n-1);

  return recursive5(n/2) + recursive5(n/2);

}
```

For this question, you should calculate what the function returns for all cases. For the runtime analysis, you may assume that n is a power of 2, so that you are always in the 2nd recursive case.

2. Use the substitution method (that is, proof by induction) to prove the following bounds:

   (a) (4 points) $O(n^2)$ bound for:

$$
\begin{aligned}
T(0) &= C_1 \\
T(1) &= C_1 \\
T(n) &= T(n-2) + C_2 n
\end{aligned}
$$