Computer Science 245

Homework 3

Algorithm Analysis III

Due Wednesday, February 15th, 2017

1. Use the substitution method (that is, proof by induction) to prove the following bounds:

   (a) (3 points) $O(n \lg n)$ bound for:

   $$\begin{aligned} T(0) &= C_1 \\ T(1) &= C_1 \\ T(n) &= 4T(n/4) + C_2 n \end{aligned}$$

   (b) (3 points) $O(2^n)$ bound for:

   $$\begin{aligned} T(0) &= C_1 \\ T(1) &= C_1 \\ T(n) &= 2T(n-1) + C_2 \end{aligned}$$

   (Careful! You may need to subtract out a lower order term for this!)

   (c) (3 points) $O(n)$ bound for:

   $$\begin{aligned} T(0) &= C_1 \\ T(1) &= C_1 \\ T(n) &= T(n/2) + 2T(n/4) + C_2 \end{aligned}$$

   (Careful! You may need to subtract out a lower order term for this one as well!)

2. Use the master method to find Θ bounds for the following recurrence relations:

   (a) (1 point)

   $$\begin{aligned} T(0) &= C_1 \\ T(1) &= C_1 \\ T(n) &= 8T(n/3) + n^2 \end{aligned}$$

   (b) (1 point)

   $$\begin{aligned} T(0) &= C_1 \\ T(1) &= C_1 \\ T(n) &= 16T(n/2) + n^4 + 2n^2 + n \end{aligned}$$

   (c) (1 point)

   $$\begin{aligned} T(0) &= C_1 \\ T(1) &= C_1 \\ T(n) &= 4T(n/2) + n \end{aligned}$$

(d) (1 point)

$$\begin{aligned} T(0) &= C_1 \\ T(1) &= C_1 \\ T(n) &= 4T(n/2) + n^2 + n \lg n \end{aligned}$$