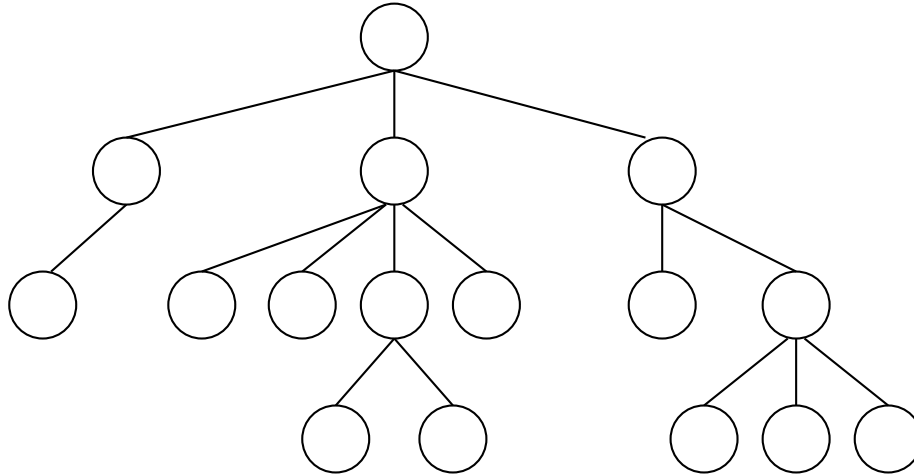


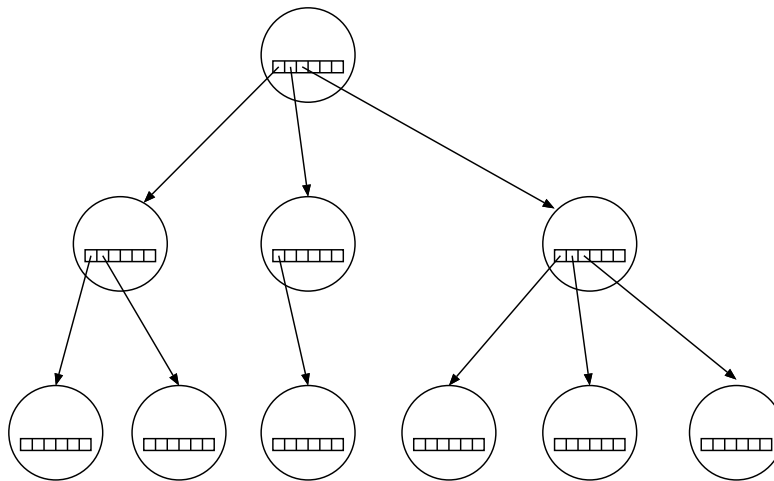
09-0: **Trees with > 2 children**

How can we implement trees with nodes that have > 2 children?



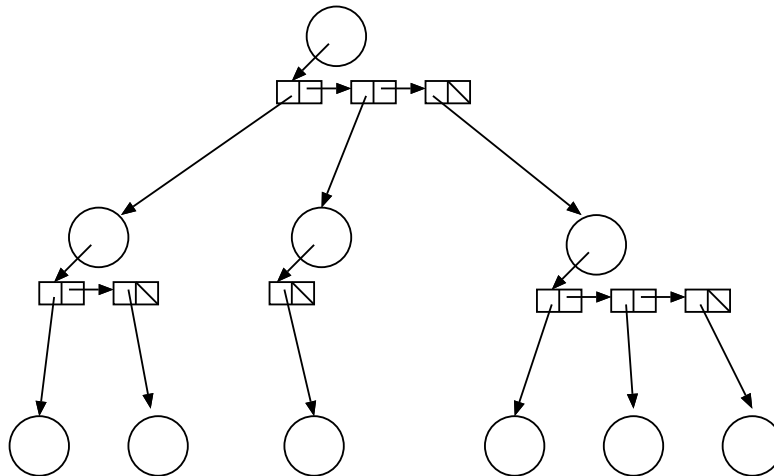
09-1: **Trees with > 2 children**

- Array of Children



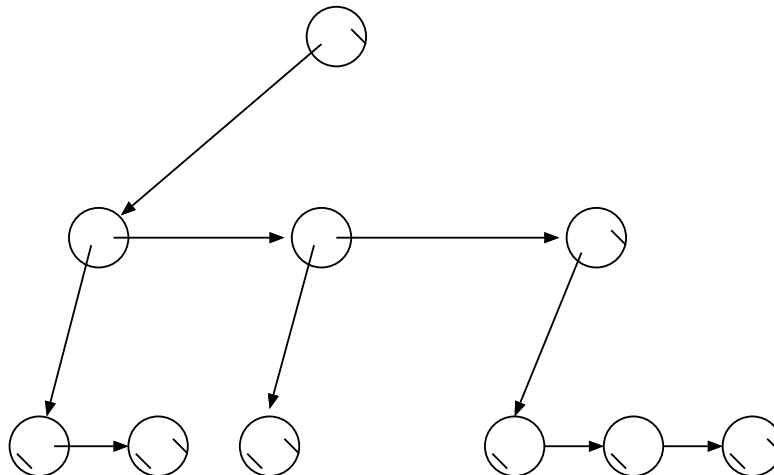
09-2: **Trees with > 2 children**

- Linked List of Children



09-3: **Left Child / Right Sibling**

- We can integrate the linked lists with the nodes themselves:



09-4: **Working with General Tree**

```

class Node {
    private Node leftchild_;
    private Node rightsib_;
    private Object element_;

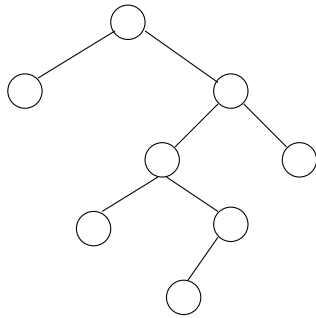
    Node leftchild() {
        return leftchild_;
    }
    void setLeftchild(Node leftchild) {
        leftchild_ = leftchild;
    }

    Node rightsib() {
        return rightsib_;
    }
    void setRightsib(Node leftchild) {
        rightsib_ = rightsib;
    }

    Node element() {
        return element_;
    }
    void setElement(Object element) {
        element_ = element;
    }
}
    
```

09-5: **General Trees – NumNodes**

- Returns the number of nodes in a tree



Number of Nodes = 8

Number of Nodes = 6

09-6: General Trees – NumNodes

```

int numnodes(Node tree) {
    int descendants = 0;
    Node tmp;

    if (tree == null)
        return 0;
    for (tmp = tree.leftchild(); tmp != null;
         tmp = tmp.rightsib())
        descendants = descendants + numnodes(tmp);

    return descendants + 1;
}
  
```

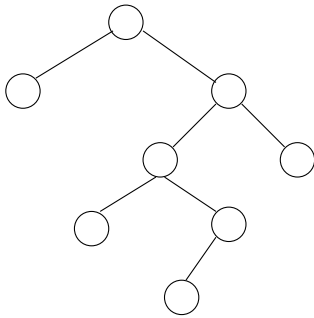
09-7: General Trees – NumNodes II

```

int numnodes(Node tree) {
    if (tree == null)
        return 0;
    return 1 + numnodes(tree.leftchild())
           + numnodes(tree.rightsib());
}
  
```

09-8: Tree Operations – Height

- Returns the height of the tree
 - (Length of the path to the deepest leaf) + 1



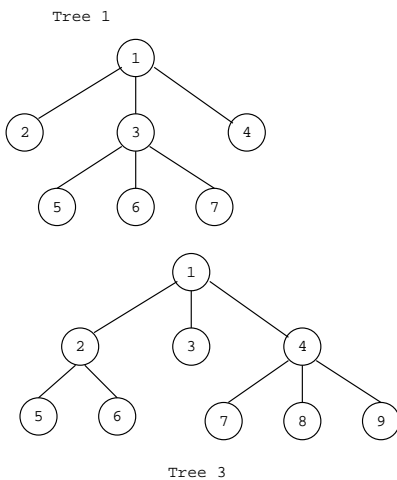
Height = 5

09-9: General Trees – Height

```
int height(Node tree) {
    if (tree == null)
        return 0;
    int childHeight = 0;
    for (Node tmp = tree.leftchild(); tmp != null;
         tmp=tmp.rightsib())
    {
        childHeight = MAX(childHeight, height(tmp));
    }
    return childHeight + 1;
}
```

09-10: General Trees – Height

```
int height(Node tree) {
    if (tree == null)
        return 0;
    return MAX((1 + height(tree.leftchild())),
               height(tree.rightsib()));
}
```

09-11: General Trees

Write numLeaves and print 09-12: **General Trees – numLeaves**

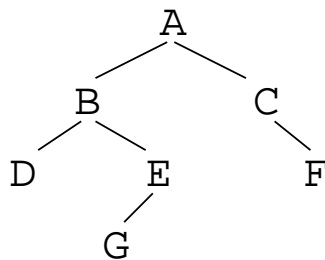
```
int numLeaves(Node tree) {
    if (tree == null)
        return 0;
    if (tree.leftchild() == null)
        return 1 + numLeaves(tree.rightsib());
    return numLeaves(tree.leftchild()) +
        numLeaves(tree.rightsib());
}
```

09-13: **General Trees – numLeaves**

```
void print(Node tree, int offset) {
    if (tree != null)
    {
        for (int i = 0; i < offset; i++)
            System.out.print("\t");
        System.out.println(tree.element());
        print(tree.leftchild(), offset+1);
        print(tree.rightsib(), offset);
    }
}
```

09-14: **Serializing Binary Trees**

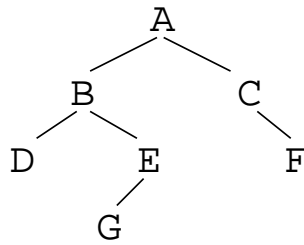
- Print a tree to a file, saving structure information
- First Try: Print out nodes, in order that they would appear in a PREORDER traversal.
 - Why doesn't this work?



ABDEGCF

09-15: **Serializing Binary Trees**

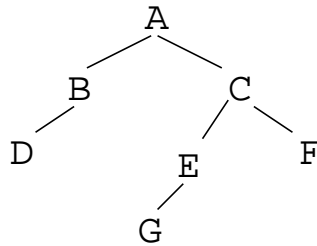
- Printing out nodes, in order that they would appear in a PREORDER traversal does not work, because we don't know when we've hit a null pointer
- Store null pointers, too!



ABD//EG///C/F//

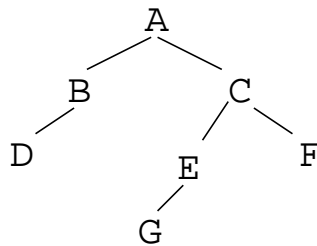
09-16: Serializing Binary Trees

- Printing out nodes, in order that they would appear in a PREORDER traversal does not work, because we don't know when we've hit a null pointer
- Store null pointers, too!



09-17: Serializing Binary Trees

- Printing out nodes, in order that they would appear in a PREORDER traversal does not work, because we don't know when we've hit a null pointer
- Store null pointers, too!



ABD///CEG///F//

09-18: Serializing Binary Trees

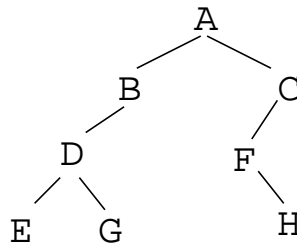
- Printing out nodes, in order that they would appear in a PREORDER traversal does not work, because we don't know when we've hit a null pointer
- Store null pointers, too!

ABDE//G///CF/H///

09-19: Serializing Binary Trees

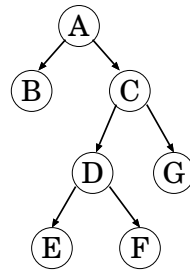
- Printing out nodes, in order that they would appear in a PREORDER traversal does not work, because we don't know when we've hit a null pointer
- Store null pointers, too!

ABDE//G///CF/H///



09-20: Serializing Binary Trees

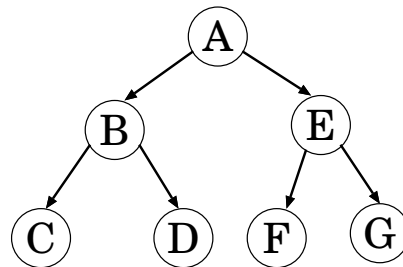
- If we are serializing a full binary tree (each node contains exactly 0 or 2 children), we can store a single extra bit for each node 0 for an internal node, 1 for a leaf:



A₀B₁C₀D₀E₁F₁G₁

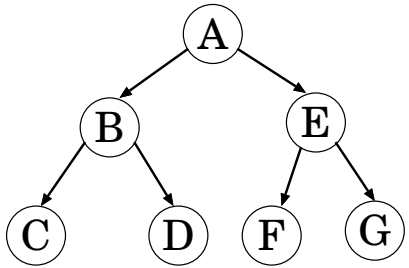
09-21: Serializing Binary Trees

- If we are serializing a full binary tree (each node contains exactly 0 or 2 children), we can store a single extra bit for each node 0 for an internal node, 1 for a leaf:



09-22: Serializing Binary Trees

- If we are serializing a full binary tree (each node contains exactly 0 or 2 children), we can store a single extra bit for each node 0 for an internal node, 1 for a leaf:



$A_0B_0C_1D_1E_0F_1G_1$

09-23: **Serializing Binary Trees**

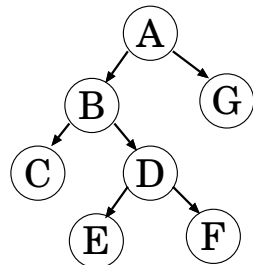
- If we are serializing a full binary tree (each node contains exactly 0 or 2 children), we can store a single extra bit for each node 0 for an internal node, 1 for a leaf:

$A_0B_0C_1D_0E_1F_1G_1$

09-24: **Serializing Binary Trees**

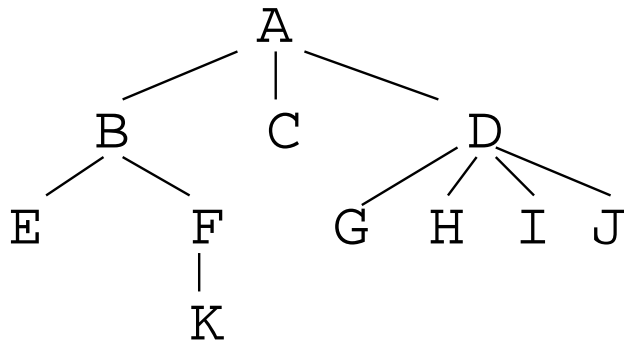
- If we are serializing a full binary tree (each node contains exactly 0 or 2 children), we can store a single extra bit for each node 0 for an internal node, 1 for a leaf:

$A_0B_0C_1D_0E_1F_1G_1$



09-25: **Serializing General Trees**

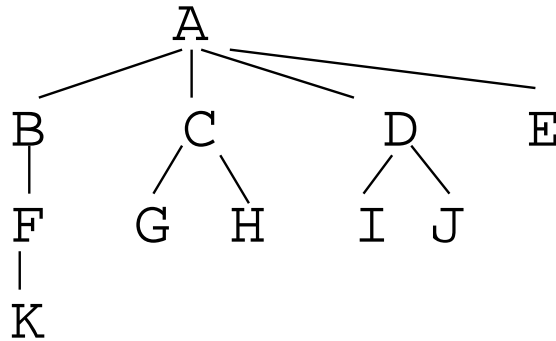
- Store an “end of children” marker



$ABE)FK)))C)DG)H)I)J)))$

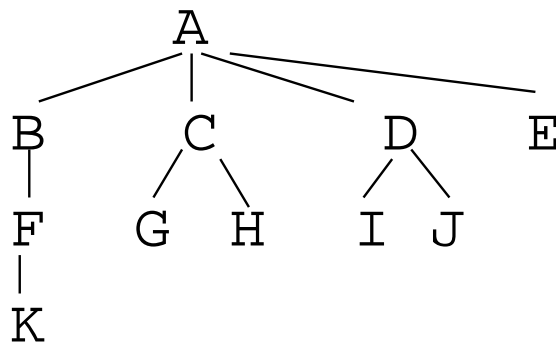
09-26: **Serializing General Trees**

- Store an “end of children” marker



09-27: **Serializing General Trees**

- Store an “end of children” marker



ABFK)))CG)H))DI)J))E))

09-28: **Serializing General Trees**

- Store an “end of children” marker

ABDK)))CE)F)GI)J))H)))

09-29: **Serializing General Trees**

- Store an “end of children” marker

ABDK)))CE)F)GI)J))H)))

