21-0: Auctions

- An auction is a negotiation mechanism where:
  - The mechanism is well-specified (it runs according to explicit rules)
  - The negotiation is mediated by an intermediary
  - Exchanges are market/currency-based
- Agents place bids on items or collections of items.
- An auctioneer determines how goods are allocated.
- Desiderata: the auction should be fair, efficient, easy to use, and computationally efficient.
- We'll need to trade these against each other.

21-1: Auctions

- Private-value auctions are easier to think about at first.
- In this case, the value agent A places on a job has nothing to do with the value that agent B places on the object.
  - For example, an hour of computing time.
- In common-value auctions, the value an agent places on an item depends on how much others value it.
  - Example: art, collectibles, precious metals.

21-2: English auctions

- An English (or first-price) auction is the kind we're most familiar with.
- Bids start low and rise. All agents see all bids.
- May be a reserve price involved.
- Dominant strategy: bid \( \epsilon \) more than the highest price, until your threshold is reached.
- Problems: requires multiple rounds, not efficient for the seller, requires agents to reveal their valuations to each other.
- There may be technical problems to solve with making sure all agents see all bids within a limited period of time.

21-3: First-price sealed-bid auction

- Each agent submits a single sealed bid. Highest wins and pays what they bid.
  - This is how you buy a house.
- Single round of bidding. All preferences remain private.
- Problems: No Nash equilibrium - agents need to counterspeculate. Item may not go to the agent who valued it most. (inefficient).

21-4: Dutch auction

- Prices start high and decline.
- First agent to bid wins.
- Strategically equivalent to first-price sealed-bid.
- In practice, closes quickly.
21-5: **Continuous double auction**

- This is how the stock exchange works.
- Agents can submit 'bids' and 'asks' for a good.
- Pairs are made continuously
  - Lowest ask with highest bid.
  - No closing time.

21-6: **Vickrey auction**

- The Vickrey, or second-price, auction, has a number of appealing aspects from a computational point of view.
- Single round of bidding.
- Efficient allocation of goods.
- Truth-telling is the dominant strategy.
- Rule: each agent bids. Highest bid wins, but pays the second price.
  - (the example we used earlier is isomorphic to the Vickrey auction).

21-7: **Example**

- Angel, Buffy and Cordelia are bidding on a sandwich.
  - Angel is willing to pay $5, Buffy $3, and Cordelia $2.
- Each participant bids the amount they’re willing to pay.
- Angel gets the sandwich and pays $3.

21-8: **Proof**

- Let’s prove that truth-telling is a dominant strategy.
- Angel:
  - If he overbids, he still pays $3. No advantage.
  - If he bids between $3 and $5, he still pays $3. No advantage.
  - If he bids less than $3, then he doesn’t get the sandwich - but he was willing to pay $5, so this is a loss.

21-9: **Proof**

- Buffy (the same reasoning will hold for Cordelia)
  - If she bids less than $3, she still doesn’t get the sandwich. (notice that we assume she doesn’t care how much Angel pays.)
  - If she bids between $3 and $5, she still doesn’t get the sandwich. No benefit.
  - If she bids more than $5, she gets the sandwich and pays $5. But she was only willing to pay $3, so this is a loss.

21-10: **Vickrey Auctions in real life**

- Because of these properties, Vickrey auctions have been adopted for:
  - Allocation of computer resources
  - Distribution of electrical power
  - Bandwidth allocation
  - Scheduling problems.
- Interestingly, they are not widely used in human auctions.
  - Perhaps people are not rational ...
Selfish routing revisited:
- Assume that we want to find the shortest path through a graph.
- Each edge is associated with an agent.
- Each edge has a privately known transmission cost.
- How can we find the shortest path?

Rule:
- Accept each agent’s bid.
- If they are not on the shortest path, they get 0.
- If they are on the shortest path, they get:
  - Cost of next shortest path - (cost of shortest path without their contribution).

Assume each agent bids truthfully.

Agents A, B, and C are each paid 8 - (6 - 2) = 4
- This is their contribution to the ‘best solution’

Other agents are paid nothing.

Why is truth-telling a dominant strategy?
- What if a underbids?
  - A bids 1: paid 8 - (5 - 1) = 4. No benefit.
- What if A overbids?
  - A bids 3: paid 8 - (7 - 3) = 4. No benefit.
  - A bids 5. No longer on the shortest path, so A gets 0.
- What if d underbids?
  - D bids 3: no change.
  - D bids 1: paid 6 - (5 - 1) = 2. But his cost is 4.
- D overbids: no change.

Advantages and disadvantages

- Advantages of the Vickrey auction/Clarke tax:
  - Truth-telling as a dominant strategy
    - Easy for participants, no need for multiple rounds of bidding.
    - Most efficient solution is always discovered.
  - Disadvantages:
    - Leaves money ‘on the table’ (payments are more than cost of job)
    - Payments are a function of the quality of the second-best solution.
    - Not intuitive for humans.

Everything we’ve said so far applies only to private value auctions.

Common or correlated-value auctions are much less predictable.

In particular, common-value auctions are subject to the winner’s curse
- As soon as you win a common-value auction, you know you’ve paid too much.
21-17: Winner’s curse

- Example: Oil drilling
  - Suppose that four firms are bidding on drilling rights. Each has an estimate of how much oil is available in that plot.
    - A thinks $5M, B thinks $10M, C thinks $12M, and D thinks $20M.
    - Let’s say it’s really $10M, but the firms don’t know this.
  - In an English auction, D will win for $12M (plus 1 dollar)
  - They lose $2M on this deal.
  - Problem: The winner is the firm who tended to overestimate by the most.
  - (Assumption: all firms have access to the same information.)

21-18: Winner’s curse

- This also explains why sports free agents seem to underperform their contracts.
  - They’re not underperforming, they’re overpaid.
- How to avoid the winner’s curse:
  - Better information gathering
  - Caution in bidding

21-19: Combinatorial auctions

- Often, goods that are being sold in an auction have complementarities.
  - Owning one good makes a second good more valuable.
- For example, let’s say supercomputer access is sold in 1-hour increments.
- Lab 1 needs three hours before 5 pm - less time is worthless.
- Lab 2 needs two hours before noon.
- How to approach this:
  - 1. Separate auctions for each hour.
    - Complicated rules for backing out and reallocating needed.
  - 2. Auction combinations (or bundles) of goods.

21-20: Winner-determination problem

- Finding the winner for a single-item Vickrey auction is easy.
- Finding the winner for a combinatorial auction is (computationally) hard.
- Formulation:
  - Given: n bidders, m items
  - Let a bundle S be a subset of the m items.
  - A bid b is a pair (v,s), where v is the amount an agent will pay for s.
  - An allocation $x_i(S)$ is described by a mapping from (i,s) into {0,1}. ($x_i(S)$ = 0 if i does not get s, and 1 if he does.)

21-21: Winner-determination problem

- We can then write the winner-determination problem as an optimization problem:
  - Find the set of allocations that maximizes: $\sum_{i \in N} (v_i, s) x_i(s)$
  - This program can be solved in a number of ways; integer linear programming or backtracking search are the most common.

21-22: Winner-determination problem

- Problem: The size of the WDP is exponential in the number of items that can be sold.
  - Every possible bundle must be considered.
- Formulating the problem as ILP helps some
  - This problem has been studied since the 50s, so good heuristic techniques exist.
21-23: Winner-determination problem

- Other solutions:
  - Limiting the sorts of bundles allowed.
  - OR bids and XOR bids.
  - This transforms the problem into the knapsack problem.
  - Still NP-hard, but good heuristics exist
- Limiting size of bundles.
- Approximation algorithms

21-24: Current research issues

- Auctions are a particularly hot area of research.

- Topics include:
  - Information revelation - how can we preserve the truth-telling strategy of Vickrey without agents revealing their preferences to each other?
  - Winner determination.
  - Languages for expressing more complex constraints.
  - Preventing collusion and false-name bids.
  - 'online' auctions
    - Not “on the Internet” - meaning agents continuously arrive and leave.

21-25: Summary

- Vickrey auctions are particularly appealing from a computational standpoint.
  - Easy for participants to decide how to act.
  - Hard to manipulate.
- Resources always allocated to the agent that values them the most.
- Challenges:
  - Dealing with imperfect information
  - Combinatorial auctions run us up against NP-completeness (again).