Distributed Software Development

Auctions

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21-0: Auctions

6 An auction is a negotiation mechanism where:
   △ The mechanism is well-specified (it runs according to explicit rules)
   △ The negotiation is mediated by an intermediary
   △ Exchanges are market/currency-based

6 Agents place bids on items or collections of items.

6 An auctioneer determines how goods are allocated.

6 Desiderata: the auction should be fair, efficient, easy to use, and computationally efficient.

6 We’ll need to trade these against each other.
Private-value auctions are easier to think about at first. In this case, the value agent A places on a job has nothing to do with the value that agent B places on the object.

- For example, an hour of computing time.

In common-value auctions, the value an agent places on an item depends on how much others value it.

- Example: art, collectibles, precious metals.
An English (or first-price) auction is the kind we’re most familiar with.

Bids start low and rise. All agents see all bids.

May be a reserve price involved.

Dominant strategy: bid $\epsilon$ more than the highest price, until your threshold is reached.

Problems: requires multiple rounds, not efficient for the seller, requires agents to reveal their valuations to each other.

There may be technical problems to solve with making sure all agents see all bids within a limited period of time.
21-3: *First-price sealed-bid auction*

- Each agent submits a single sealed bid. Highest wins and pays what they bid.
  - This is how you buy a house.
- Single round of bidding. All preferences remain private.
- Problems: No Nash equilibrium - agents need to counterspeculate. Item may not go to the agent who valued it most. (inefficient).
Dutch auction

- Prices start high and decline.
- First agent to bid wins.
- Strategically equivalent to first-price sealed-bid.
- In practice, closes quickly.
21-5: Continuous double auction

- This is how the stock exchange works.
- Agents can submit 'bids' and 'asks' for a good.
- Pairs are made continuously
  - Lowest ask with highest bid.
  - No closing time.
21-6: Vickrey auction

- The Vickrey, or second-price, auction, has a number of appealing aspects from a computational point of view.
- Single round of bidding.
- Efficient allocation of goods.
- Truth-telling is the dominant strategy.
- Rule: each agent bids. Highest bid wins, but pays the second price.
  - (the example we used earlier is isomorphic to the Vickrey auction).
Angel, Buffy and Cordelia are bidding on a sandwich.

- Angel is willing to pay $5, Buffy $3, and Cordelia $2.

Each participant bids the amount they’re willing to pay.

Angel gets the sandwich and pays $3.
Let’s prove that truth-telling is a dominant strategy.

Angel:
- If he overbids, he still pays $3. No advantage.
- If he bids between $3 and $5, he still pays $3. No advantage.
- If he bids less than $3, then he doesn’t get the sandwich - but he was willing to pay $5, so this is a loss.
Buffy (the same reasoning will hold for Cordelia)

- If she bids less than $3, she still doesn’t get the sandwich. (notice that we assume she doesn’t care how much Angel pays.)
- If she bids between $3 and $5, she still doesn’t get the sandwich. No benefit.
- If she bids more than $5, she gets the sandwich and pays $5. But she was only willing to pay $3, so this is a loss.
21-10: *Vickrey Auctions in real life*

Because of these properties, Vickrey auctions have been adopted for:
- Allocation of computer resources
- Distribution of electrical power
- Bandwidth allocation
- Scheduling problems.

Interestingly, they are not widely used in human auctions.
- Perhaps people are not rational ...
Selfish routing revisited:
- Assume that we want to find the shortest path through a graph.
- Each edge is associated with an agent.
- Each edge has a privately known transmission cost.
- How can we find the shortest path?
Rule:

- Accept each agent’s bid.
- If they are not on the shortest path, they get 0.
- If they are on the shortest path, they get:
  - Cost of next shortest path - (cost of shortest path without their contribution).
Assume each agent bids truthfully.

Agents A, B, and C are each paid $8 - (6 - 2) = 4$

△ This is their contribution to the 'best solution'

Other agents are paid nothing.
Why is truth-telling a dominant strategy?

- What if a underbids?
  - A bids 1: paid 8 - (5 - 1) = 4. No benefit.

- What if A overbids?
  - A bids 3: paid 8 - (7 - 3) = 4. No benefit.
  - A bids 5. No longer on the shortest path, so A gets 0.

- What if d underbids?
  - D bids 3: no change.
  - D bids 1: paid 6 - (5 - 1) = 2. But his cost is 4.

- D overbids: no change.
Advantages and disadvantages

Advantages of the Vickrey auction/Clarke tax:
- Truth-telling as a dominant strategy
  - Easy for participants, no need for multiple rounds of bidding.
  - Most efficient solution is always discovered.
- Disadvantages:
  - Leaves money 'on the table' (payments are more than cost of job)
  - Payments are a function of the quality of the second-best solution.
  - Not intuitive for humans.
Everything we’ve said so far applies only to private value auctions.

Common or correlated-value auctions are much less predictable.

In particular, common-value auctions are subject to the winner’s curse
- As soon as you win a common-value auction, you know you’ve paid too much.
Example: Oil drilling

- Suppose that four firms are bidding on drilling rights. Each has an estimate of how much oil is available in that plot.
  - A thinks $5M, B thinks $10M, C thinks $12M, and D thinks $20M.
  - Let’s say it’s really $10 M, but the firms don’t know this.
- In an English auction, D will win for $12M (plus 1 dollar)
- They lose $2M on this deal.
- Problem: The winner is the firm who tended to overestimate by the most.
- (Assumption: all firms have access to the same information.)
This also explains why sports free agents seem to underperform their contracts.
  △ They’re not underperforming, they’re overpaid.

How to avoid the winner’s curse:
  △ Better information gathering
  △ Caution in bidding
Often, goods that are being sold in an auction have complementarities.

- Owning one good makes a second good more valuable.

For example, let’s say supercomputer access is sold in 1-hour increments.

Lab 1 needs three hours before 5 pm - less time is worthless.

Lab 2 needs two hours before noon.

How to approach this:

- 1. Separate auctions for each hour.
  - Complicated rules for backing out and reallocating needed.

- 2. Auction combinations (or bundles) of goods.
Finding the winner for a single-item Vickrey auction is easy.

Finding the winner for a combinatorial auction is (computationally) hard.

Formulation:
- Given: n bidders, m items
- Let a bundle S be a subset of the m items.
- A bid b is a pair (v,s), where v is the amount an agent will pay for s.
- An allocation $x_i(S)$ is described by a mapping from (i,s) into {0,1}. ($x_i(s) = 0$ if i does not get s, and 1 if he does.)
We can then write the winner-determination problem as an optimization problem:

- Find the set of allocations that maximizes:
  \[ \sum_{i \in N}(v, s)x_i(s) \]
- This problem can be solved in a number of ways; integer linear programming or backtracking search are the most common.
21-22: Winner-determination problem

- Problem: The size of the WDP is exponential in the number of items that can be sold.
  - Every possible bundle must be considered.

- Formulating the problem as ILP helps some
  - This problem has been studied since the 50s, so good heuristic techniques exist.
21-23: Winner-determination problem

- Other solutions:
  - Limiting the sorts of bundles allowed.
  - OR bids and XOR bids.
    - This transforms the problem into the knapsack problem.
    - Still NP-hard, but good heuristics exist
- Limiting size of bundles.
- Approximation algorithms
Auctions are a particularly hot area of research.

Topics include:

- Information revelation - how can we preserve the truth-telling strategy of Vickrey without agents revealing their preferences to each other?
- Winner determination.
- Languages for expressing more complex constraints.
- Preventing collusion and false-name bids.
- ‘online’ auctions
  - Not “on the Internet” - meaning agents continuously arrive and leave.
Vickrey auctions are particularly appealing from a computational standpoint.
- Easy for participants to decide how to act.
- Hard to manipulate.

Resources always allocated to the agent that values them the most.

Challenges:
- Dealing with imperfect information
- Combinatorial auctions run us up against NP-completeness (again).