From stories to models…

In order to apply a mathematical theory to answer questions about real world situations, we need to construct a 'mathematical model' which captures essential features of the problem, and which omits aspects of the situation which are unimportant or irrelevant. Our problem 'model' needs to be framed in language which is suited to the theory we intend to apply.

In particular, if we have developed a general theory for solving linear systems, then we can use it to investigate a real world problem provided we can somehow turn that problem into one which has the format of a linear system. Stories about quasi-realistic situations provide practice in building mathematical models.

Consider the following story problem:

> *The Busy Bee Bakery sells two special cakes: Chocolate Dream Cake, which requires 20 minutes of the baker's time to prepare and costs $8 for ingredients, and Coconut Surprise Cake, which requires 15 minutes to prepare and costs $5 for ingredients.*
>
> *How many cakes of each kind can the bakery turn out each day if the baker is on duty for 8 hours per day and has $180 to pay for ingredients?*

We can readily translate this story situation into mathematical language (i.e., into algebraic equations) by following a three-step methodology.

Step 1: Identify the unknown quantities we are being asked to determine, and use letters to represent these 'unknowns' as algebraic variables.

Let x = number of chocolate cakes the bakery can turn out daily
and y = number of coconut cakes the bakery can turn out daily

Step 2: Arrange the problem data in the form of a convenient summary-table.

	time	cost
chocolate cake	20 minutes	8 dollars
coconut cake	15 minutes	5 dollars
totals	8 hours	$180

Step 3: Translate the story relationships into algebraic equations involving the unknown quantities.

preparation-time (in minutes): $20x + 15y = 4800$
cost-of-ingredients (in dollars): $8x + 5y = 180$

Thus, in three steps, this story problem has been transformed into an algebraic model to which we can now apply general methods for finding solutions.

Notice that we had to make a choice here, between expressing time in minutes or in hours.
Our choice was somewhat arbitrary, and we could have made it either way. Less work was required to convert 8 hours into 4800 minutes, and we avoided introducing fractions. But we could have expressed times in hours, and thus arrived at the alternative equation:

preparation-time (in hours): $(1/3)x + (1/4)y = 8$

Similarly, although it might be silly to do so, we could have expressed costs in cents instead of in dollars:

cost-of-ingredients (in cents): $800x + 500y = 18000$

So long as we are consistent in our choice of units for measuring time or cost, we will get equations that are correct and useful. The mistake to avoid would be to mix two different ways of measuring time or cost within the same equation!

The homework exercises are intended to provide additional practice in the art of translating story situations into mathematical models.