1. The questions below refer to the hash table that was obtained by hashing the following keys using the hash function $h(k) = k \mod 11$:
   5, 7, 16, 6, 15, 1, 12, 23

   (a) Show the resulting hash table assuming open hashing and that the collisions are handled with separate chaining method. (2 points)

   (b) Show the resulting hash table from the previous exercise assuming closed hashing and that the collisions are handled by linear probing approach. (2 points)

   (c) Show what will happen when the same keys are hashed using the quadratic probing. Be careful! (2 points)

2. Assume that Disjoint Sets ADT is implemented using an array of parent indices. Consider the following operations:
   • makeSets(10)
   • Union(1,2)
   • Union(2,3)
   • Union(3,4)
   • Union(4,5)
   • Union(5,6)
   • Union(7,8)
   • Union(8,9)
   • Union(1,8)

   (a) Show the resulting parent array after the operations above, assuming that no heuristics are used, and we always have the root of the tree represented by the first argument of Union point to the root of the tree pointed to by the second argument of Union. (1 point)

   (b) Show the resulting parent array after the operations above, assuming Union by Rank heuristic is used. (Please note that there is no path compression.) (2 points)

3. Consider a hash table of size 10 that uses linear probing, with $f(i) = i$. The elements 13, 12, 22, 52 are inserted into an empty table

   (a) (1 point) Show the contents of the table after these 4 elements are inserted

   (b) (2 points) Assume that the 5th element to be inserted has a 1/10 chance of hashing to 0, a 1/10 chance of hashing to 1, a 1/10 change of hashing to 2, and so on. For each of the 10 table locations, give the probability that the 5th element will be placed at that location.

   (c) (3 points) Repeat parts a and b, assuming that the table uses quadratic probing, with $f(i) = i \times i$. 