Data Structures and Algorithms
CS245-2016S-06
Binary Search Trees

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Operations:

- Insert an element in the list
- Check if an element is in the list
- Remove an element from the list
- Print out the contents of the list, in order
Implementing Ordered List

Using an Ordered Array – Running times:

Check
Insert
Remove
Print
Implementing Ordered List

Using an Ordered Array – Running times:

- Check: $\Theta(\lg n)$
- Insert: $\Theta(n)$
- Remove: $\Theta(n)$
- Print: $\Theta(n)$
Using an *Unordered* Array – Running times:

Check
Insert
Remove
Print
Using an *Unordered* Array – Running times:

- Check: $\Theta(n)$
- Insert: $\Theta(1)$
- Remove: $\Theta(n)$ Need to find element first!
- Print: $\Theta(n \lg n)$
  (Given a fast sorting algorithm)
Using an Ordered Linked List – Running times:

Check
Insert
Remove
Print
Implementing Ordered List

Using an Ordered Linked List – Running times:

- Check $\Theta(n)$
- Insert $\Theta(n)$
- Remove $\Theta(n)$
- Print $\Theta(n)$
The Best of Both Worlds

- Linked Lists – Insert fast / Find slow
- Arrays – Find fast / Insert slow
- The only way to examine nth element in a linked list is to traverse (n-1) other elements

- If we could leap to the middle of the list ...
06-8: The Best of Both Worlds
Move the initial pointer to the middle of the list:

We’ve cut our search time in half! Have we changed the $\Theta()$ running time?
Move the initial pointer to the middle of the list:

We’ve cut our search time in half! Have we changed the $\Theta()$ running time?

Repeat the process!
06-11: The Best of Both Worlds

[Diagram of a linked list or tree structure]
The Best of Both Worlds

Grab the first element of the list:

4 → 8 → 12 → 15 → 22 → 25 → 28

Give it a good shake -
Binary Trees are Recursive Data Structures

- **Base Case:** Empty Tree
- **Recursive Case:** Node, consisting of:
  - Left Child (Tree)
  - Right Child (Tree)
  - Data
The following are all Binary Trees (Though not Binary Search Trees)
06-15: **Tree Terminology**

- Parent / Child
- Leaf node
- Root node
- Edge (between nodes)
- Path
- Ancestor / Descendant
- Depth of a node $n$
  - Length of path from root to $n$
- Height of a tree
  - (Depth of deepest node) + 1
Each node has 0 or 2 children

Full Binary Trees

Not Full Binary Trees
06-17: **Complete Binary Tree**

- Can be built by starting at the root, and filling the tree by levels from left to right
- **Complete Binary Trees**

  ![Complete Binary Trees Diagram]

- *Not* Complete Binary Trees

  ![Not Complete Binary Trees Diagram]
Binary Trees

For each node n, (value stored at node n) ≥ (value stored in left subtree)

For each node n, (value stored at node n) < (value stored in right subtree)
Example Binary Search Trees
Each Node in a BST is implemented as a class:

```java
public class Node {
    public Comparable data;
    public Node left;
    public Node right;
}
```
public class Node {

    public Node(Comparable data, Node left, Node right) {
        this.data = data;
        this.left = left;
        this.right = right;
    }

    public Node left() {
        return left;
    }
    public Node setLeft(Node newLeft) {
        left = newLeft
    }

    ... (etc)

    private Comparable data;
    private Node left;
    private Node right;
}

06-21: Implementing BSTs
Binary Search Trees are recursive data structures, so most operations on them will be recursive as well.

Recall how to write a recursive algorithm ...
Writing a Recursive Algorithm

- Determine a small version of the problem, which can be solved immediately. This is the base case.
- Determine how to make the problem smaller.
- Once the problem has been made smaller, we can assume that the function that we are writing will work correctly on the smaller problem (Recursive Leap of Faith).
  - Determine how to use the solution to the smaller problem to solve the larger problem.
First, the Base Case – when is it easy to determine if an element is stored in a Binary Search Tree?
06-25: Finding an Element in a BST

- First, the Base Case – when is it easy to determine if an element is stored in a Binary Search Tree?
  - If the tree is empty, then the element can’t be there
  - If the element is stored at the root, then the element is there
Next, the Recursive Case – how do we make the problem smaller?
Next, the Recursive Case – how do we make the problem smaller?

Both the left and right subtrees are smaller versions of the problem. Which one do we use?
Next, the Recursive Case – how do we make the problem smaller?

- Both the left and right subtrees are smaller versions of the problem. Which one do we use?
- If the element we are trying to find is $<$ the element stored at the root, use the left subtree. Otherwise, use the right subtree.
06-29: Finding an Element in a BST

- Next, the Recursive Case – how do we make the problem smaller?
  - Both the left and right subtrees are smaller versions of the problem. Which one do we use?
  - If the element we are trying to find is $<$ the element stored at the root, use the left subtree. Otherwise, use the right subtree.

- How do we use the solution to the subproblem to solve the original problem?
Next, the Recursive Case – how do we make the problem smaller?

- Both the left and right subtrees are smaller versions of the problem. Which one do we use?
  - If the element we are trying to find is \(<\) the element stored at the root, use the left subtree. Otherwise, use the right subtree.

How do we use the solution to the subproblem to solve the original problem?

- The solution to the subproblem \(\text{is}\) the solution to the original problem (this is not always the case in recursive algorithms)
To find an element $e$ in a Binary Search Tree $T$:

- If $T$ is empty, then $e$ is not in $T$
- If the root of $T$ contains $e$, then $e$ is in $T$
- If $e <$ the element stored in the root of $T$:
  - Look for $e$ in the left subtree of $T$
  - Otherwise
    - Look for $e$ in the right subtree of $T$
boolean find(Node tree, Comparable elem) {
    if (tree == null)
        return false;
    if (elem.compareTo(tree.element()) == 0)
        return true;
    if (elem.compareTo(tree) < 0)
        return find(tree.left(), elem);
    else
        return find(tree.right(), elem);
}
06-33: Printing out a BST

To print out all element in a BST:

- Print all elements in the left subtree, in order
- Print out the element at the root of the tree
- Print all elements in the right subtree, in order
To print out all element in a BST:

- Print all elements in the left subtree, in order
- Print out the element at the root of the tree
- Print all elements in the right subtree, in order
  - Each subproblem is a smaller version of the original problem – we can assume that a recursive call will work!
What is the base case for printing out a Binary Search Tree – what is an easy tree to print out?
06-36: **Printing out a BST**

- What is the base case for printing out a Binary Search Tree – what is an easy tree to print out?
- An empty tree is extremely easy to print out – do nothing!
- Code for printing a BST ...
```java
void print(Node tree) {
    if (tree != null) {
        print(tree.left());
        System.out.println(tree.element());
        print(tree.right());
    }
}
```
06-38: Printing out a BST

Examples
06-39: Tree Traversals

- **PREORDER Traversal**
  - Do operation on root of the tree
  - Traverse left subtree
  - Traverse right subtree

- **INORDER Traversal**
  - Traverse left subtree
  - Do operation on root of the tree
  - Traverse right subtree

- **POSTORDER Traversal**
  - Traverse left subtree
  - Traverse right subtree
  - Do operation on root of the tree
06-41: POSTORDER Examples
INORDER Examples
To find the minimal element in a BST:

- **Base Case**: When is it easy to find the smallest element in a BST?
- **Recursive Case**: How can we make the problem smaller?

How can we use the solution to the smaller problem to solve the original problem?
To find the minimal element in a BST:

Base Case:

• When is it easy to find the smallest element in a BST?
To find the minimal element in a BST:

Base Case:

- When is it easy to find the smallest element in a BST?
  - When the left subtree is empty, then the element stored at the root is the smallest element in the tree.
To find the minimal element in a BST:

Recursive Case:

- How can we make the problem smaller?
To find the minimal element in a BST:
Recursive Case:

- How can we make the problem smaller?
  - Both the left and right subtrees are smaller versions of the same problem
- How can we use the solution to a smaller problem to solve the original problem?
BST Minimal Element

To find the minimal element in a BST:

Recursive Case:

- How can we make the problem smaller?
  - Both the left and right subtrees are smaller versions of the same problem
- How can we use the solution to a smaller problem to solve the original problem?
  - The smallest element in the left subtree is the smallest element in the tree
Comparable minimum(Node tree) {
    if (tree == null) {
        return null;
    }
    if (tree.left() == null) {
        return tree.element();
    } else {
        return minimum(tree.left());
    }
}
Iterative Version

Comparable minimum(Node tree) {
    if (tree == null)
        return null;
    while (tree.left() != null)
        tree = tree.left();
    return tree.element();
}
What is the base case – an easy tree to insert an element into?
06-52: **Inserting** $e$ **into BST** $T$

- What is the base case – an easy tree to insert an element into?
  - An empty tree
  - Create a new tree, containing the element $e$
Recursive Case: How do we make the problem smaller?
Recursive Case: How do we make the problem smaller?
- The left and right subtrees are smaller versions of the same problem.
- How do we use these smaller versions of the problem?
Inserting $e$ into BST $T$

- Recursive Case: How do we make the problem smaller?
  - The left and right subtrees are smaller versions of the same problem
  - Insert the element into the left subtree if $e \leq$ value stored at the root, and insert the element into the right subtree if $e >$ value stored at the root
06-56: **Inserting** $e$ **into** BST $T$

- **Base case** – $T$ is empty:
  - Create a new tree, containing the element $e$

- **Recursive Case**:
  - If $e$ is less than the element at the root of $T$, insert $e$ into left subtree
  - If $e$ is greater than the element at the root of $T$, insert $e$ into the right subtree
Tree manipulation functions return trees

Insert method takes as input the old tree and the element to insert, and returns the new tree, with the element inserted
  - Old value (pre-insertion) of tree will be destroyed

To insert an element $e$ into a tree $T$:
  - $T = \text{insert}(T, e)$;
Node insert(Node tree, Comparable elem) {
  if (tree == null) {
    return new Node(elem);
  }
  if (elem.compareTo(tree.element() <= 0)) {
    tree.setLeft(insert(tree.left(), elem));
    return tree;
  } else {
    tree.setRight(insert(tree.right(), elem));
    return tree;
  }
}
06-59: Deleting From a BST

• Removing a leaf:
06-60: Deleting From a BST

- Removing a leaf:
  - Remove element immediately
06-61: Deleting From a BST

- Removing a leaf:
  - Remove element immediately
- Removing a node with one child:
06-62: Deleting From a BST

- Removing a leaf:
  - Remove element immediately
- Removing a node with one child:
  - Just like removing from a linked list
  - Make parent point to child
Removing a leaf:
  • Remove element immediately
Removing a node with one child:
  • Just like removing from a linked list
  • Make parent point to child
Removing a node with two children:
Deleting From a BST

- Removing a leaf:
  - Remove element immediately
- Removing a node with one child:
  - Just like removing from a linked list
  - Make parent point to child
- Removing a node with two children:
  - Replace node with largest element in left subtree, or the smallest element in the right subtree
We have been storing “Comparable” elements in BSTs

Alternately, could use a “key()” method – elements stored in BSTs must implement a key() method, which returns an integer.

We can combine the two methods

- Each element stored in the tree has a key() method
- key() method returns Comparable class
BST Implementation Details

- Use BSTs to implement Ordered List ADT
- Operations
  - Insert
  - Find
  - Remove
  - Print in Order
- The specification (interface) should not specify an implementation
  - Allow several different implementations of the same interface
BST functions require the root of the tree be sent in as a parameter.

Ordered list functions should *not* contain implementation details!

What should we do?
BST Implementation Details

- BST functions require the root of the tree be sent in as a parameter.
- Ordered list functions should not contain implementation details!
- What should we do?
  - Private variable, holds root of the tree
  - Private recursive methods, require root as an argument
  - Public methods call private methods, passing in private root