07-0: **Binary Tree Definition**

class Node {
    Node() { }
    Node(Comparable elem) {
        this.element = element;
    }
    Node(Object element, Node left, Node right) {
        this.element = element;
        this.left = left;
        this.right = right;
    }
    /* Access methods on next slide */
    private Node left;
    private Node right;
    private Comparable element;
}

07-1: **Binary Tree Access Methods**

Node left() {
    void setLeft(Node left) {
        return left;
        this.left = left;
    }
}

Node right() {
    void setRight(Node right) {
        return right;
        this.right = right;
    }
}

Comparable element() {
    return element;
}

void setElement(Comparable element) {
    this.element = element;
}

07-2: **Tree Operations – Height**

- Returns the height of the tree
  - (Length of the path to the deepest leaf) + 1

![Binary Tree Diagram]
07-3: **Tree Operations – Height**

```java
int height(Node tree) {
    if (tree == null)
        return 0;
    return 1 + MAX(height(tree.left()),
                    height(tree.right()));
}
```

07-4: **Tree Operations – NumNodes**

- Returns the number of nodes in a tree

Number of Nodes = 8 Number of Nodes = 6

07-5: **Tree Operations – NumNodes**

```java
int numNodes(Node tree) {
    if (tree == null)
        return 0;
    return 1 + numNodes(tree.left()) +
           numNodes(tree.right());
}
```

07-6: **Writing Tree Functions**

![Tree Diagrams]
Write `find`, `numLeaves`, `shallowestLeaf`

07-7: **Tree Operations – NumLeaves**

- Returns the number of leaves in a tree

```
Number of Leaves = 4
Number of Leaves = 1
```

```
int numLeaves(Node tree) {
    if (tree == null)
        return 0;
    if ((tree.left() == null) && (tree.right() == null))
        return 1;
    return numLeaves(tree.left()) +
            numLeaves(tree.right());
}
```

07-8: **Tree Traversals**

- **PREORDER Traversal**
  - Do operation on root of the tree
  - Traverse left subtree
  - Traverse right subtree

- **INORDER Traversal**
  - Traverse left subtree
  - Do operation on root of the tree
  - Traverse right subtree

- **POSTORDER Traversal**
  - Traverse left subtree
  - Traverse right subtree
  - Do operation on root of the tree
07-10: **PREORDER Traversal**

Printing out trees (Showing the shape of the tree in the printout)

07-11: **PREORDER Traversal**

Printing out trees (Showing the shape of the tree in the printout)

- First print the root at current indent level
- Print the left subtree with larger indentation
- Print the right subtree with larger indentation

07-12: **Printing Binary Trees**

```java
void print(Node tree, int indent) {
    if (tree != null) {
        for(int i=0; i<indent; i++) {
            System.out.print("\t");
        }
        System.out.println(tree.element().toString());
        print(tree.left(), indent + 1);
        print(tree.right(), indent + 1);
    }
}
```

07-13: **INORDER Traversal**

Printing all elements in a Binary Search Tree in order

- (Already covered in previous slides)

07-14: **POSTORDER Traversal**

Calculating the Value of an expression tree
07-15: **POSTORDER Traversal**
Calculating the Value of an expression tree

- **Base case:**
  - Return value stored at leaf
- **Recursive case:**
  - Calculate value of left subtree
  - Calculate value of right subtree
  - Calculate expression value

07-16: **Expression Tree Value**

```java
int value(Node tree) {
    if (tree.left() == null && tree.right() == null)
        return ((Integer) tree.element()).intValue();
    int left = value(tree.left());
    int right = value(tree.right());
    char op = ((Character) tree.element()).charValue();
    switch (op) {
        case '+':
            return left + right;
        case '*':
            return left * right;
        ...
    }
    ...
}
```