PROJECT: ENUMERATORS

ALGORITHMIC LEARNING THEORY, SUMMER 2014

1. Introduction

Imagine a learner that, instead of trying to learn a language (i.e., a set) is trying to learn a *function*. This is something that scientists often do when they are trying to find a model to describe the behavior of some system. Moreover, scientists often have some idea of what kind of model they're looking for (an ordinary differential equation, a linear system, etc.) and are, in a sense, just trying to choose the right one from a "list" of all possible models. Learners choosing from a list of possible answers like this are called *enumerators*, and in this project you'll investigate the ability of these learners to learn sets and functions.

2. Computability theory: Lemmas and exercises

Exercise 1. *Let* L_n $\{x \mid x \geq n\}$. *Find a computable function* $h : \mathbb{N} \to \mathbb{N}$ *such that* $h(n)$ *is an index for* L_n *(i.e.,* $W_{h(n)} = L_n$*).*

3. Learning Theory I: Identification, lemmas and exercises

Lemma 1. *Let* $L_n = \{x \mid x \geq n\}$ *and* $\mathcal{L} = \{L_n \mid n \in \mathbb{N}\}$. *Then* $\mathcal{L}$ *is identifiable. Moreover,* $\mathcal{L}$ *is recursively identifiable.*

Lemma 2. *If* $\varphi \in \mathcal{F}^{REC}$ *identifies* RE_{SVT} *and*

$$\sigma = \langle 0, x_0\rangle, \langle 1, x_1\rangle, \langle 2, x_2\rangle, \ldots, \langle n, x_n\rangle$$

then there are natural numbers j *and* k *so that if*

$$\tau = \sigma\hat{\ }\langle n+1, 0\rangle, \ldots, \langle n+j, 0\rangle$$
$$\tau' = \tau\hat{\ }\langle n+j+1\rangle, \ldots, \langle n+j+k, 1\rangle$$

then $\varphi(\tau) \neq \varphi(\tau')$.

4. Learning Theory II: Limitations

Definition 1. *A learning function* $\varphi \in \mathcal{F}$ *is called an* enumerator *if there is a total function* $f \in \mathcal{F}$ *such that for all* $\sigma \in SEQ$, $\varphi(\sigma) = f(i)$, *where* i *is the least natural number for which* $rng(\sigma) \subset W_{f(i)}$. *We call such an* f *an* enumerating function *for* φ.

Proposition 1. $RE_{SVT} \in \left[\mathcal{F}^{ENUM}\right]$.

Proposition 2. $\left[\mathcal{F}^{ENUM}\right] \subsetneq [\mathcal{F}]$.

Corollary 1. $\left[\mathcal{F}^{ENUM} \cap \mathcal{F}^{REC}\right] \subsetneq \left[\mathcal{F}^{REC}\right]$.

Proposition 3. *Let* $\mathcal{L} \in RE_{SVT}$ *be r.e. indexable. Then* $\mathcal{L} \in \left[\mathcal{F}^{ENUM} \cap \mathcal{F}^{REC}\right]$.