Note Title 6/2/2009

TUESDAY, JUNE 2, AM

FUNDAMENTAL THEOREM OF ARITHMETIC

ANY NATURAL NUMBER CAN BE FACTORED INTO PRIMES IN A UNIQUE WAY.

#1 S.R. CHAPTER 2

$$500 = 5 \cdot 100 = 5 \cdot 10^2 = 5 \cdot 2^2 \cdot 5^2 = 2^2 \cdot 5^3$$

a	b	GCF(a,b)	
1	$2^2 \cdot 5^3$	1	
2	$2 \cdot 5^3$	2	
2^2	5^3	1	4×125
$2 \cdot 5$	$2 \cdot 5^2$	$2 \cdot 5 = 10$	
$2^2 \cdot 5$	5^2	5	
⋮	⋮	⋮	

LINEAR PROGRAMMING PROBLEM

INEQUALITIES (CONSTRAINTS)

$-x + y \le 2$ $\quad$ $-x+y=2$

$4x + 5y \le 20$

$4x+5y=20$

solve.

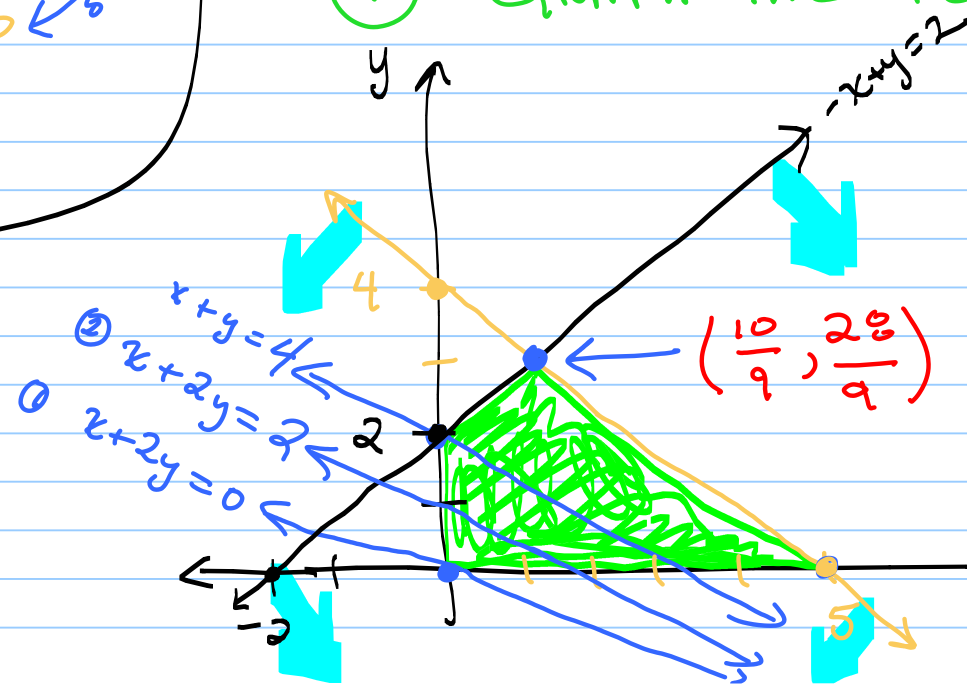
$x \ge 0$

$y \ge 0$

OBJECTIVE FUNCTION

$x + 2y \leftarrow$ maximize

① GRAPH THE FEASIBLE REGION.



$$4(-x+y=2)$$
$$+\ (4x+5y=20)$$
$$9y = 28$$
$$y = \frac{28}{9}$$
$$x = \frac{10}{9}$$

① $x+2y=0$ ← SLOPE $-\frac{1}{2}$

② $x+2y=2$

③ $x+2y=4$

$Ax+By=C$

$y=\frac{-A}{B}x+\frac{C}{B}$

C.P.	$x+2y$
$(0,0)$	0
$(0,2)$	4
$(5,0)$	5
$(\frac{10}{9},\frac{28}{9})$	$\frac{10}{9}+\frac{56}{9}=\frac{64}{9}\ (>7)$ ← MAX.

"$x+2y$" IS MAXIMIZED AND MEETS THE CONSTRAINTS WHEN $x=\frac{10}{9}$ AND $y=\frac{28}{9}$, AND THE MAXIMUM VALUE IS $64/9$.

DIRECT PROOFS. (A.2)

#2 IF a IS DIVISIBLE BY 4, THEN a IS THE DIFFERENCE OF TWO PERFECT SQUARES.

PROOF: ASSUME a IS DIVISIBLE BY 4.

THEN $a = 4k$ FOR SOME k.

SO WE CAN WRITE

$a = 4k + k^2 - k^2 + 1 - 1$, AND REARRANGING

GIVES

$a = k^2 + 2k + 1 - k^2 + 2k - 1$

OR

$a = (k+1)^2 - (k-1)^2$,

AND WE SEE THAT a IS THE

DIFFERENCE OF SQUARES.

#3 IF a and b are real, THEN $a^2 + b^2 \geq 2ab$.

" Since $(a-b)^2$ IS THE SQUARE OF A REAL NUMBER, IT CANNOT BE NEGATIVE, HENCE $a^2 + b^2 \geq 2ba$.

#4 sum of rationals a, b is rational.

(THE INTEGERS ARE CLOSED UNDER + AND ·)

TRICK IS TO WRITE a, b AS RATIOS OF INTEGERS, FIND COMMON DENOMINATOR, MAKE THIS CLAIM.

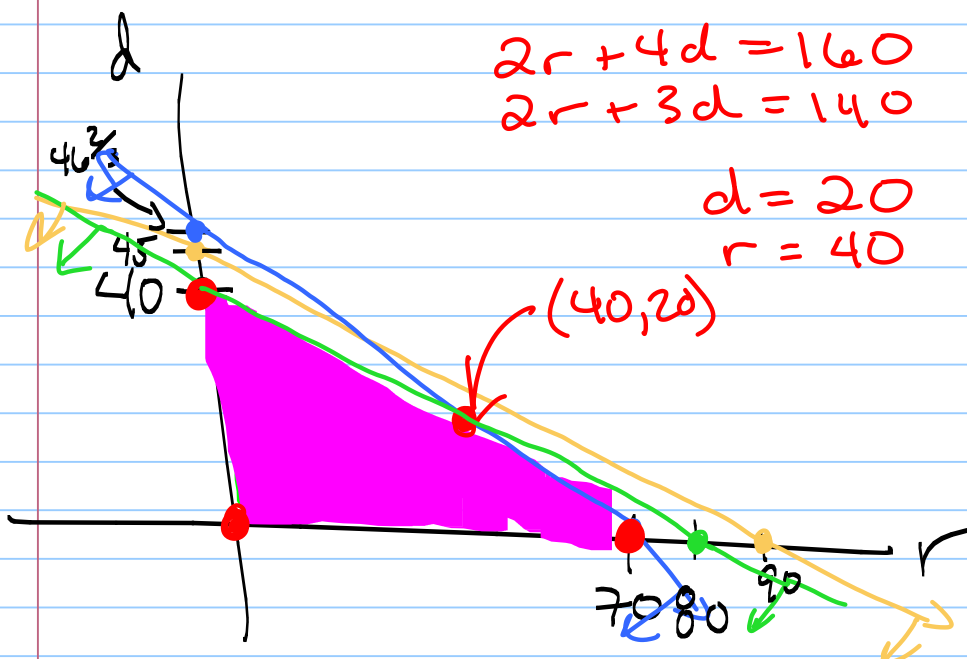
THE FLOWER PROBLEM.

		car	ros	tul	profit
r	Reg.	2	1	2	5.50
d	Del.	4	2	3	7.50
	AVAIL.	160	90	140	P

MAXIMIZE PROFIT SUBJECT TO CONSTRAINTS

PROFIT: $5.5r + 7.5d$ OBJECTIVE FCTN

$$\left.\begin{aligned} 2r + 4d &\le 160 \\ r + 2d &\le 90 \\ 2r + 3d &\le 140 \\ r, d &\ge 0 \end{aligned}\right\} \text{CONSTRAINTS}$$



$2r + 4d = 160$
$2r + 3d = 140$

$d = 20$
$r = 40$

CP	$5.5r + 7.5d$
(0,0)	0
(70,0)	385 ← MAX
(0,40)	300
(40,20)	370

GEOMETRY

Congruent — EXACTLY THE SAME

Similar — SAME SHAPE, MAYBE DIFFERENT SIZES

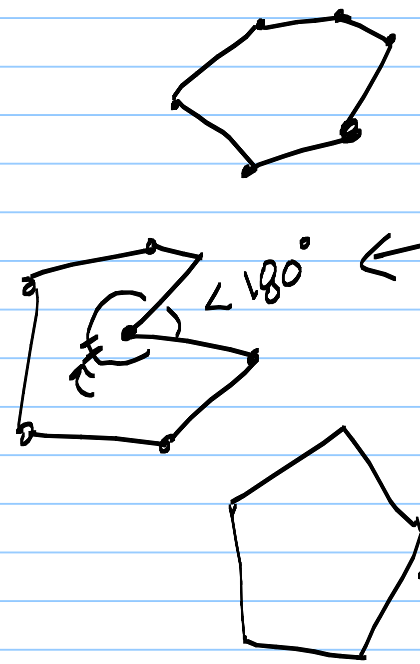
POLYGON

TRIANGLE

QUADRILATERAL

PENTAGON

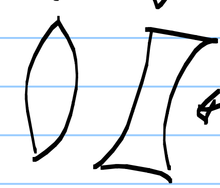
HEXAGON



CONVEX

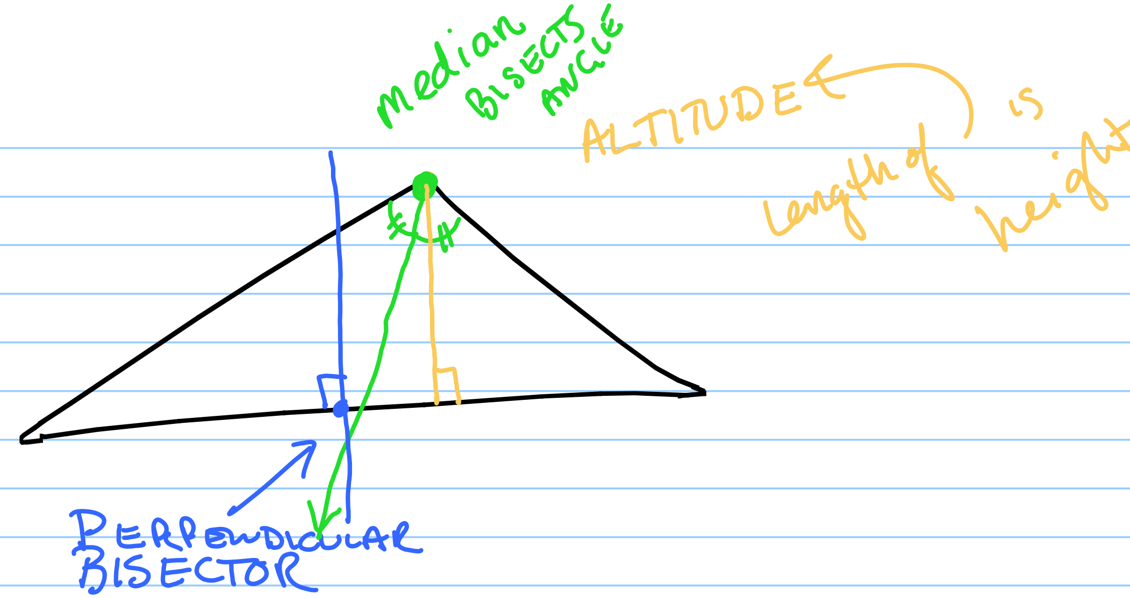
CONCAVE

REGULAR



n-GON → SUM OF INTERIOR ANGLES OF AN n-GON IS $(n-2)180°$

TRIANGLES



EQUILATERAL

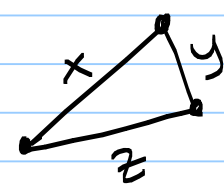
ISOSCELES

SCALENE – ALL INT. ANGLES $< 90°$

OBTUSE – ONE ANGLE $> 90°$

RIGHT TRIANGLE – ONE ANG $= 90°$

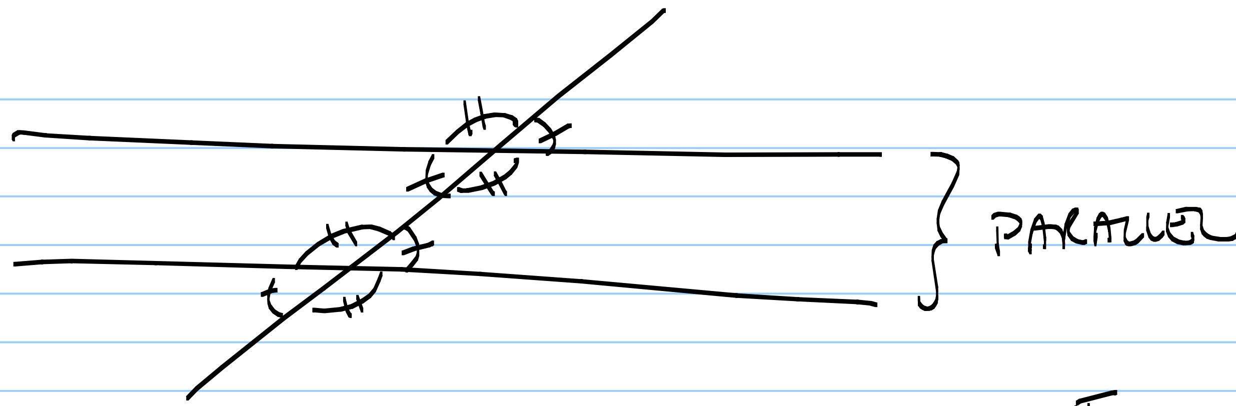
TRIANGLE INEQUALITY



$x + y > z$

SSS, SAS, ASA, AAS ← CONGRUENCE

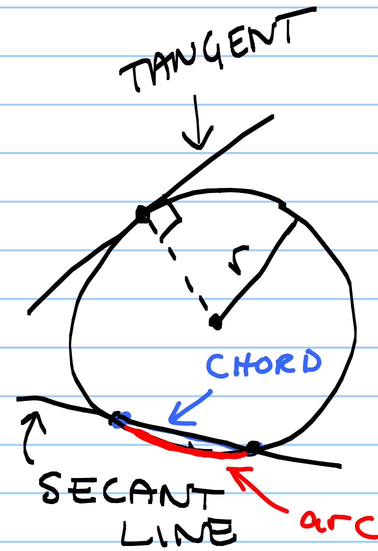
AAA ← SIMILAR



CIRCLES

DISTANCE

ALL POINTS EQUIDISTANT FROM A CHOSEN POINT.



DISTANCE FORMULA

$$d(P_1, P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$P_1 = (x_1, y_1)$

$P_2 = (x_2, y_2)$

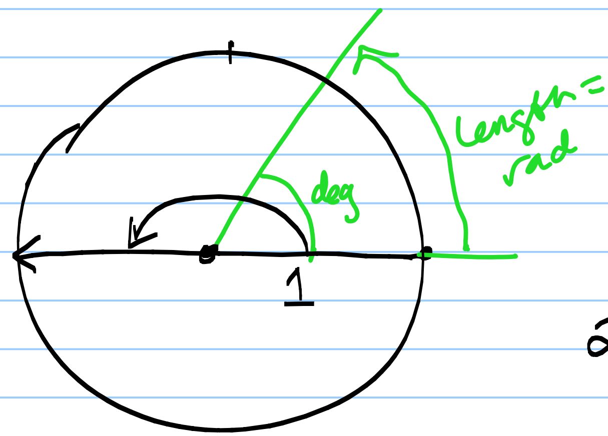
$P_2 = (h, k)$

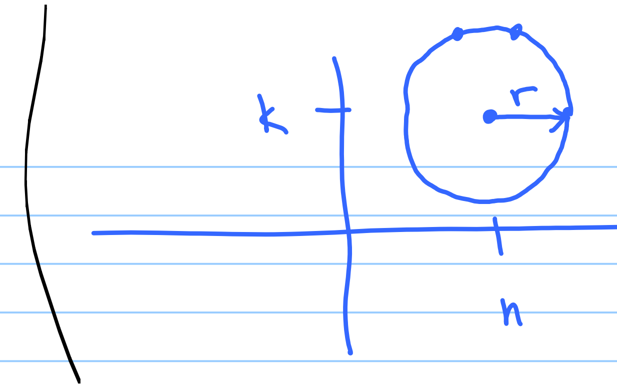
COLLECTION OF ALL POINTS DISTANCE r FROM P_1.

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

$$r^2 = (x-h)^2 + (y-k)^2$$

UNIT CIRCLE





$2\pi \sim 360°$ IS THE CIRCUMF. OF UNIT CIRCLE.

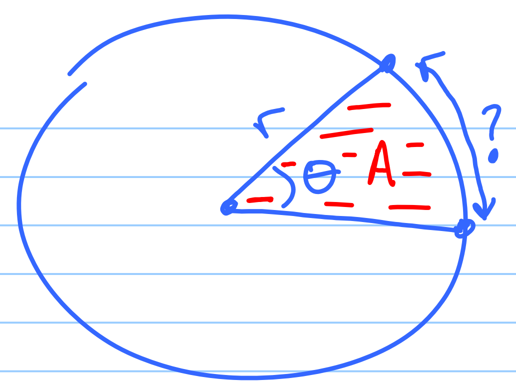
$\pi \sim 180°$

$\frac{\pi}{2} \sim 90°$

$\frac{deg}{360°} = \frac{rad}{2\pi}$

$\frac{45°}{360°} = \frac{?}{2\pi}$

$\frac{1}{8} = \frac{?}{2\pi} \rightarrow \frac{2\pi}{8} = ? = \frac{\pi}{4}$



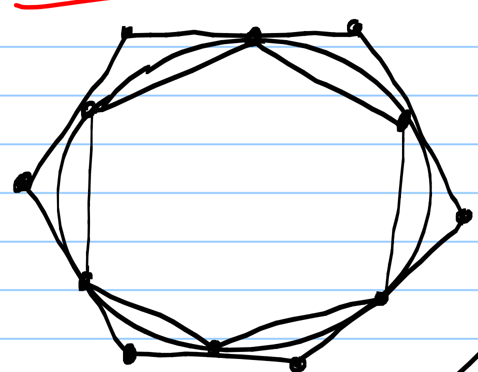
$$\frac{?}{2\pi r} = \frac{\theta}{2\pi}$$

$$? = \theta r$$ ← ARC LENGTH.

θ IN RADIANS!

$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}$$

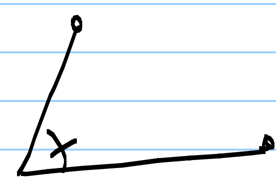
$$A = \frac{\theta r^2}{2}$$ ← θ IN RADIANS

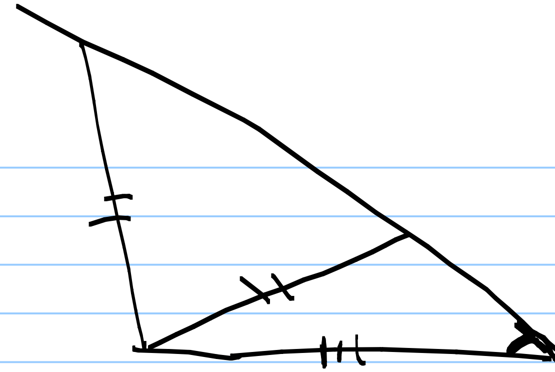


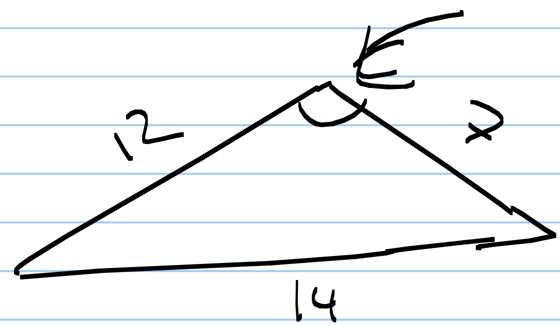
INSCRIBED / CIRCUMSCRIBED. HEXAGONS

PROBLEMS

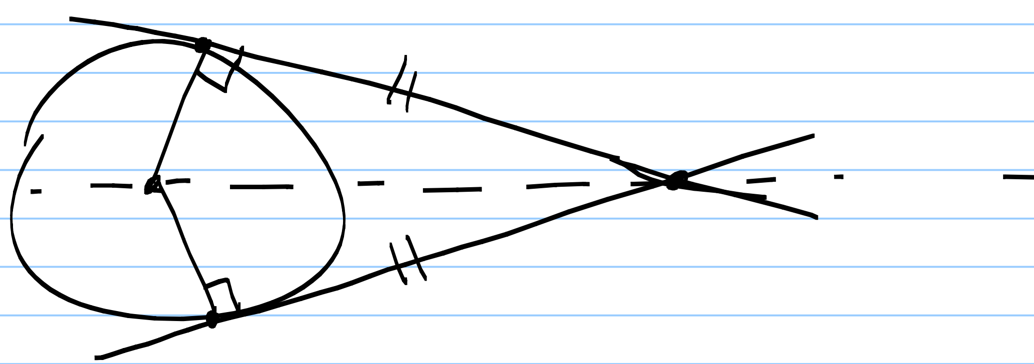
EX	1–9
MC	1–7
SR	1, 3, 4

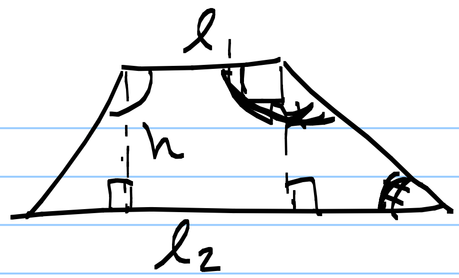




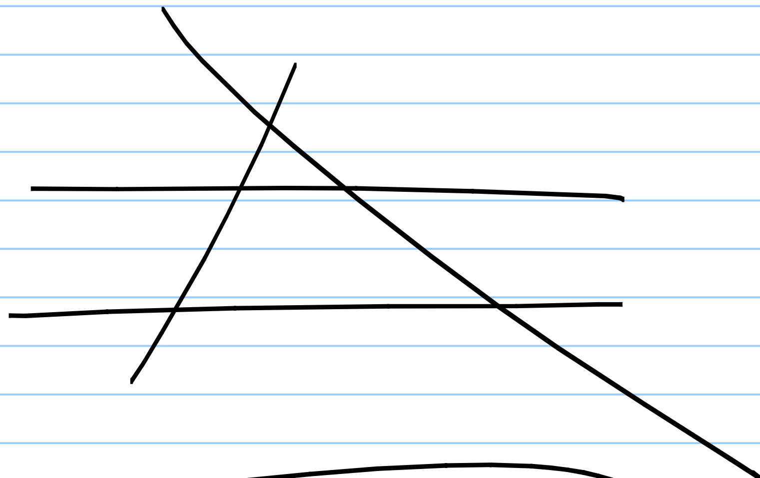


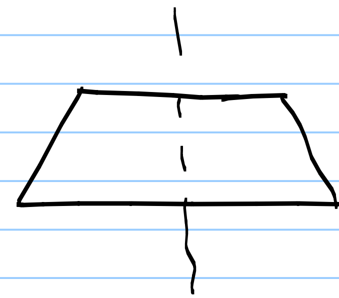
LAW OF
SINES
OR
COSINES

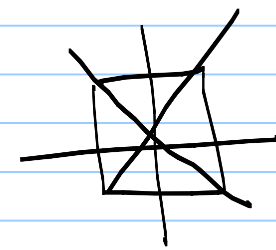




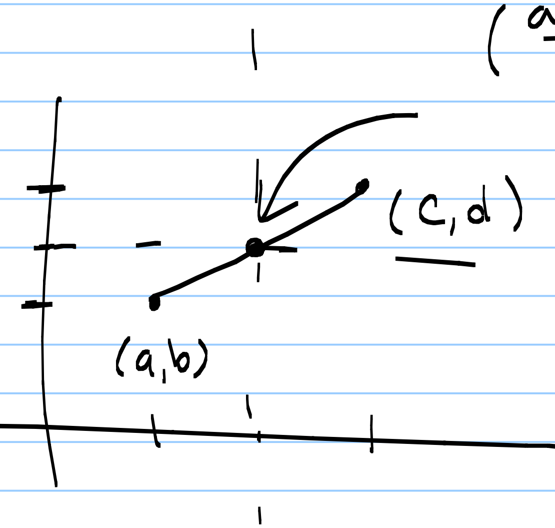
$$A = h \cdot \frac{(l_1 + l_2)}{2}$$



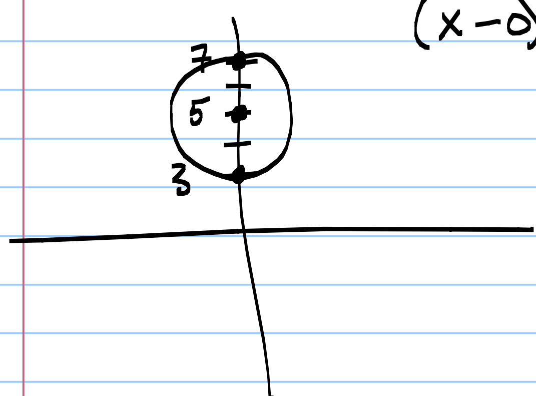




$$\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$$

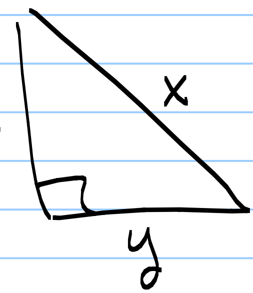


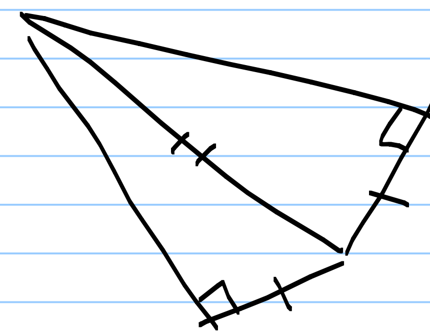
$$(x-0)^2 + (y-5)^2 = 4$$



Continuing a little ...

Ex #2





MC #6

$$\frac{(n-2)\cdot 180}{10} = 144^\circ \qquad n = 10$$

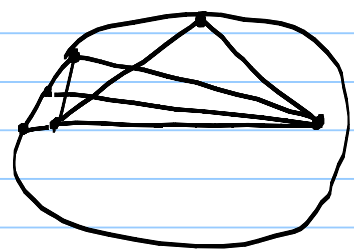
recall

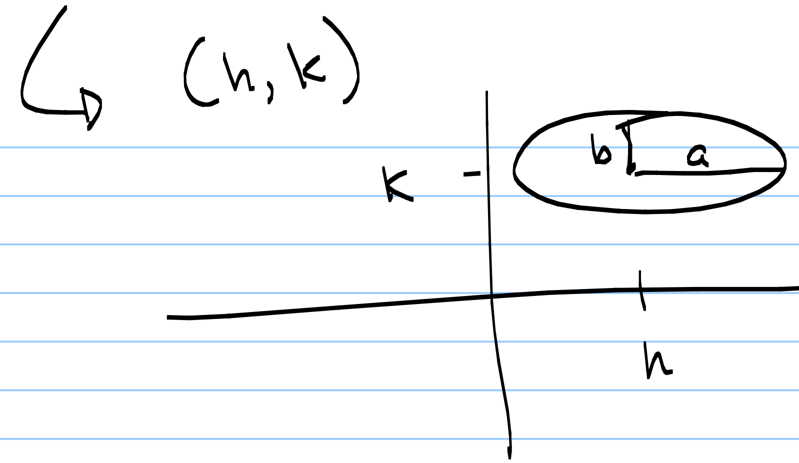
$$(x-h)^2 + (y-k)^2 = r^2$$

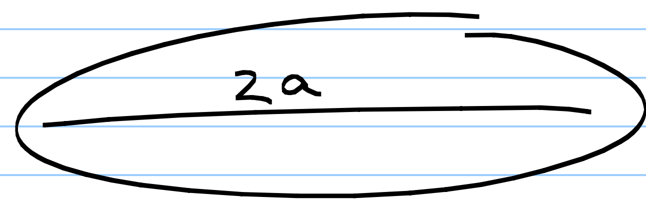
$$\frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = 1$$

ELLIPSE

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$




(h, k)
k
h
b
a


2a


2b