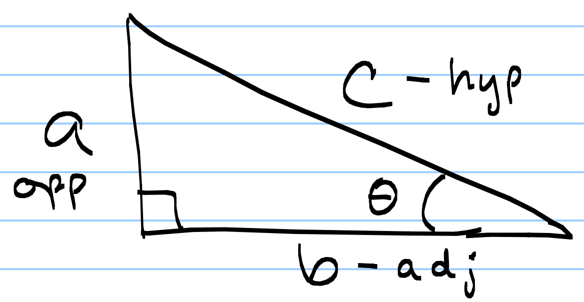
TRIGONOMETRY



$$a^2 + b^2 = c^2$$

SOHCAHTOA

S - ine
O - pposite
H - ypotenuse
C - osine
A - DJACENT
T - ANGENT

$$\sin \theta = \frac{opp}{hyp}$$

$$\cos \theta = \frac{adj}{hyp}$$

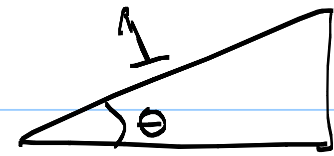
$$\tan \theta = \frac{opp}{adj} = \frac{\sin\theta}{\cos\theta}$$

cosecant — $\csc \theta = \frac{1}{\sin\theta} = \frac{hyp}{opp}$

secant — $\sec \theta = \frac{1}{\cos\theta} = \frac{hyp}{adj}$

cotangent — $\cot \theta = \frac{1}{\tan\theta} = \frac{adj}{opp} = \frac{\cos\theta}{\sin\theta}$

$$\sin^2\theta + \cos^2\theta = 1$$



$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\left(\mathrm{trig}^3\theta = (\mathrm{trig}\,\theta)^3 \right)$$

$$1 + \cot^2\theta = \csc^2\theta$$

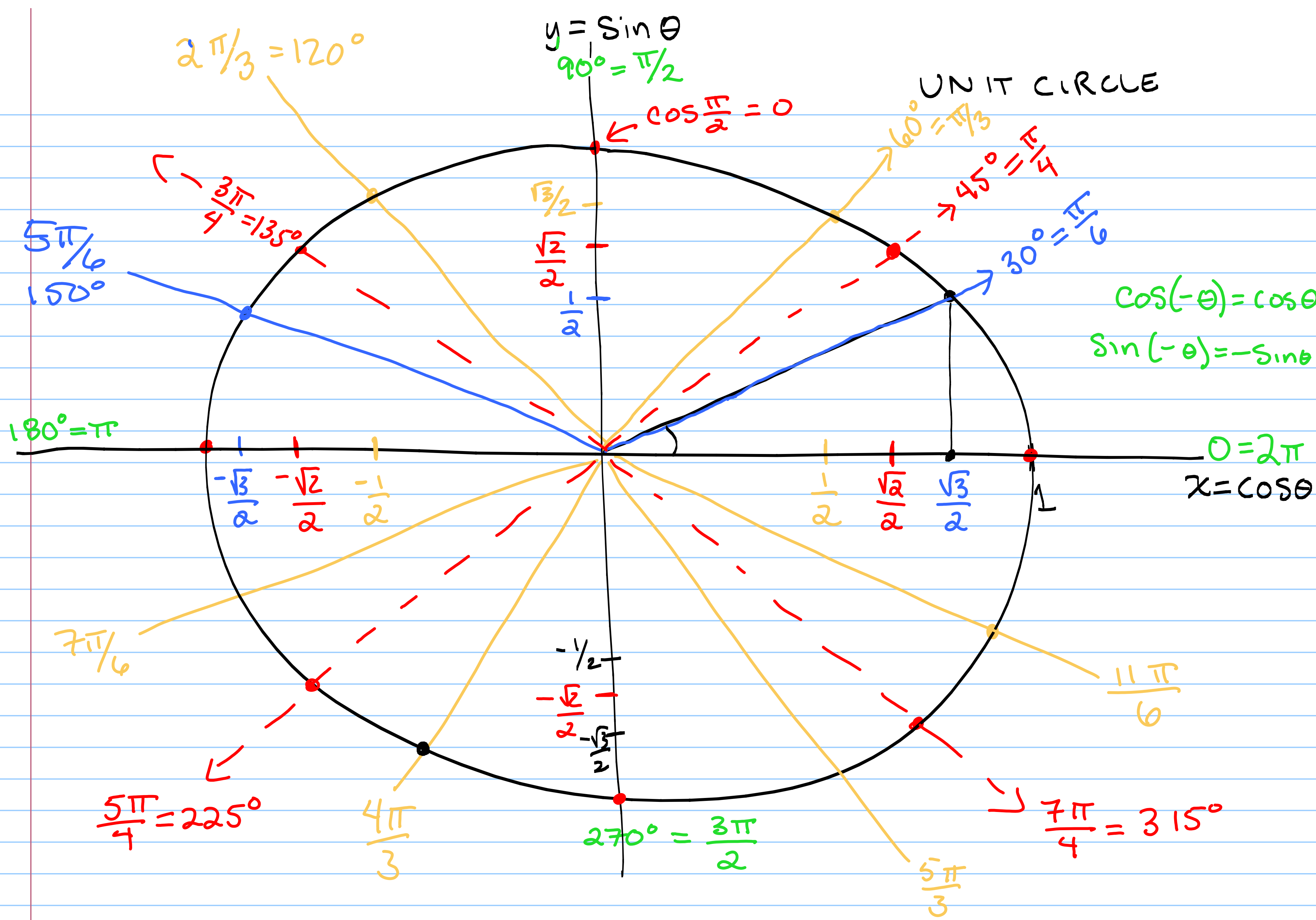
$$\tan^2\theta + 1 = \sec^2\theta$$

HALF ANGLE / SUM FORMULAS

DOUBLE ANGLE

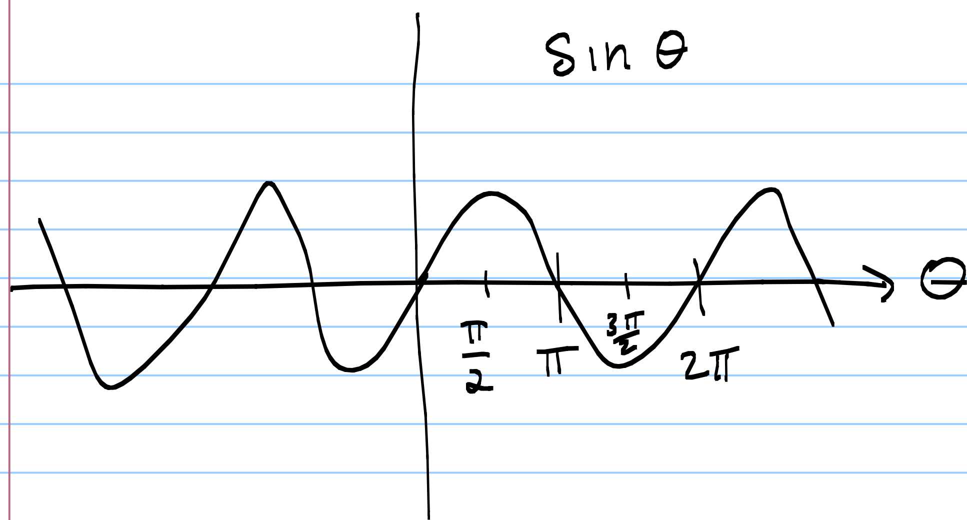
$$\sin(2\theta) = \sin(\theta+\theta) \xrightarrow{\text{SUM FORMULA}} = \sin\theta\cos\theta + \cos\theta\sin\theta$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

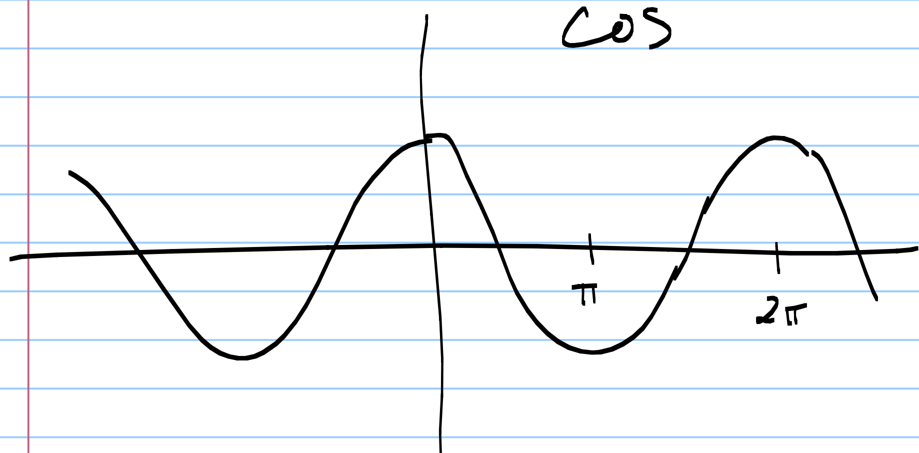


UNIT CIRCLE
$y = \sin\theta$
$90° = \pi/2$
$\cos\frac{\pi}{2} = 0$
$2\pi/3 = 120°$
$60° = \pi/3$
$45° = \frac{\pi}{4}$
$30° = \frac{\pi}{6}$
$\frac{3\pi}{4} = 135°$
$5\pi/6$
$150°$
$\cos(-\theta) = \cos\theta$
$\sin(-\theta) = -\sin\theta$
$180° = \pi$
$0 = 2\pi$
$x = \cos\theta$
1
$\sqrt{3}/2$
$\frac{\sqrt{2}}{2}$
$\frac{1}{2}$
$-\frac{\sqrt{3}}{2}$
$-\frac{\sqrt{2}}{2}$
$-\frac{1}{2}$
$\frac{1}{2}$
$\frac{\sqrt{2}}{2}$
$\frac{\sqrt{3}}{2}$
$-1/2$
$-\frac{\sqrt{2}}{2}$
$-\frac{\sqrt{3}}{2}$
$7\pi/6$
$\frac{11\pi}{6}$
$\frac{5\pi}{4} = 225°$
$\frac{4\pi}{3}$
$270° = \frac{3\pi}{2}$
$\frac{5\pi}{3}$
$\frac{7\pi}{4} = 315°$

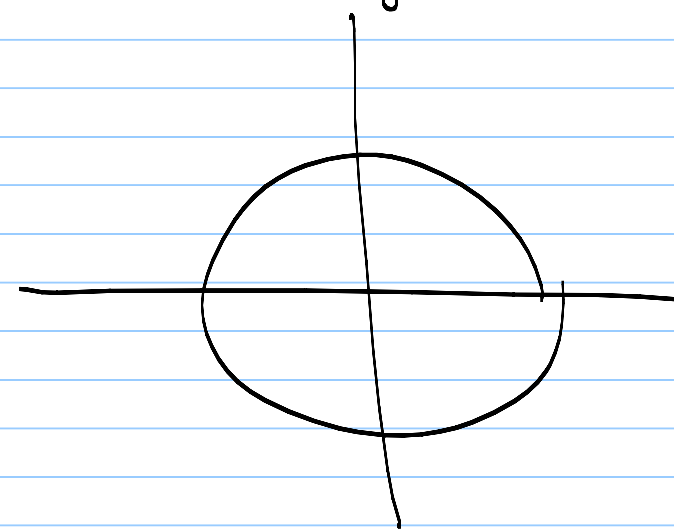
$\sin\theta$



$\cos$



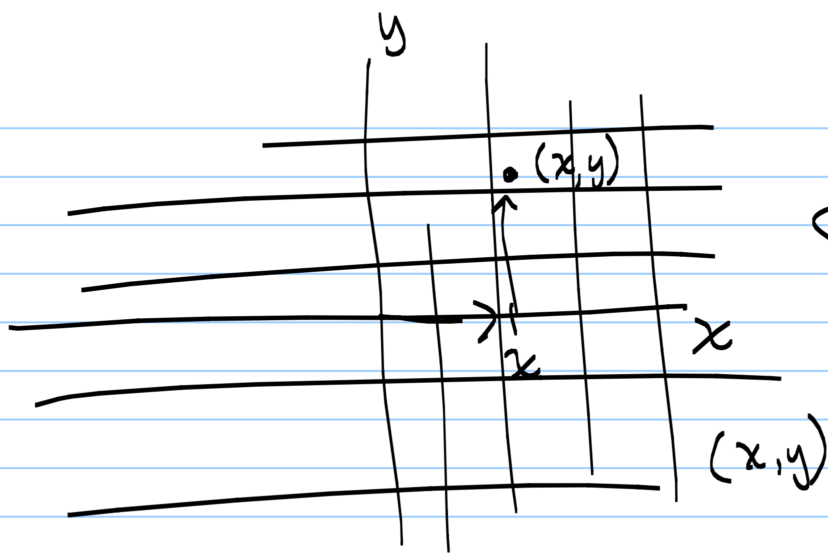
$y = \sin\theta$

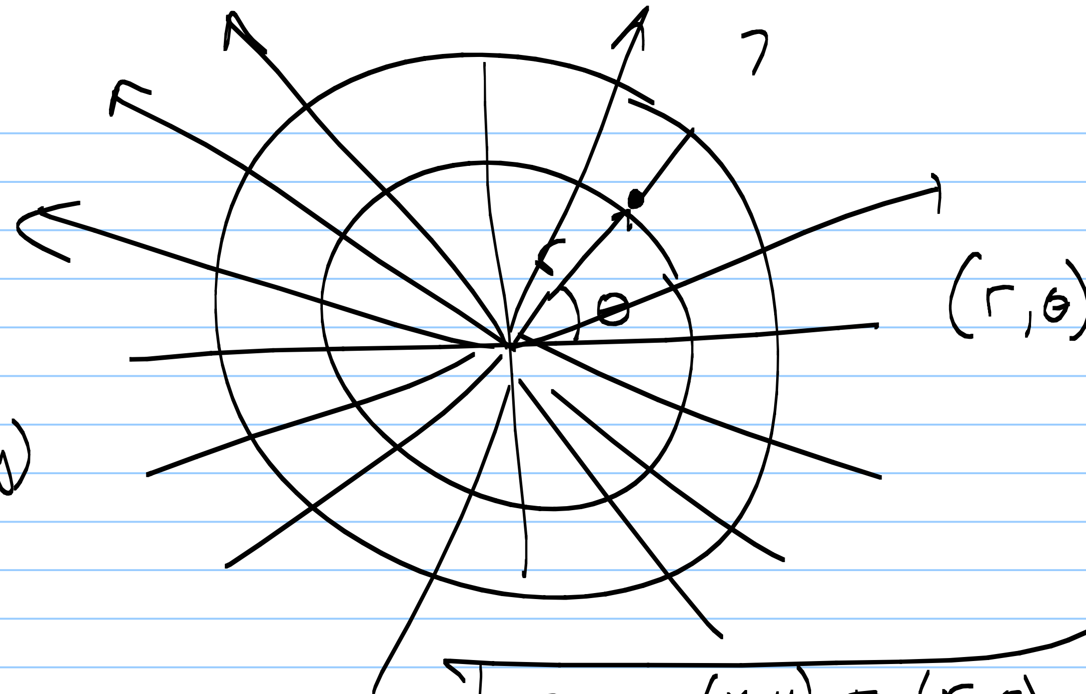


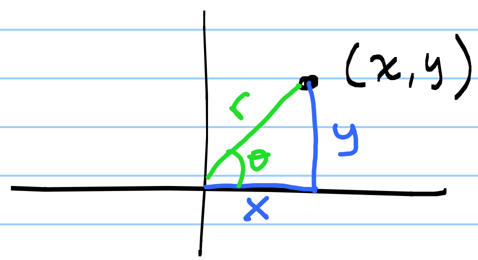
$$\frac{1}{2} = \sin\theta$$

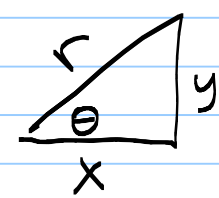
$$\theta = \frac{\pi}{6} + 2\pi n, \text{ or}$$

$$\theta = \frac{5\pi}{6} + 2\pi n.$$









From (x,y) to (r,θ)

$r = \sqrt{x^2+y^2}$

$\tan\theta = \frac{y}{x}$

FROM (r,θ) TO (x,y)

$r\cos\theta = x$

$r\sin\theta = y$

TRY: $(3, 45°)$

$(x,y) = \left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$

$P = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$(x, y) \rightsquigarrow (r,\theta)$

$$r = \sqrt{x^2+y^2} = \sqrt{\left(-\tfrac{1}{2}\right)^2 + \left(-\tfrac{\sqrt{3}}{2}\right)^2} = \sqrt{\tfrac{1}{4}+\tfrac{3}{4}} = \underline{1}$$

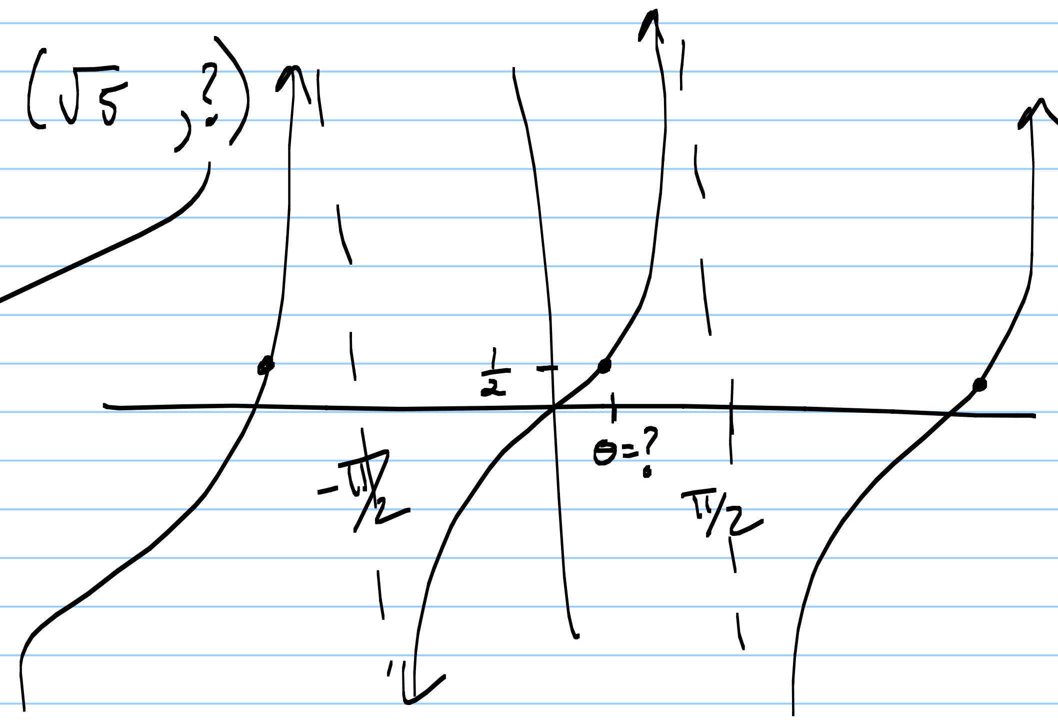
$$\tan\theta = \frac{\;\frac{-\sqrt{3}}{2}\;}{\frac{-1}{2}} = \sqrt{3} \rightsquigarrow \theta = \frac{4\pi}{3}$$

$$(x,y) = (2,1) \rightsquigarrow (r,\theta) = (\sqrt{5}, ?)$$

$$r = \sqrt{2^2+1^2} = \sqrt{5}$$

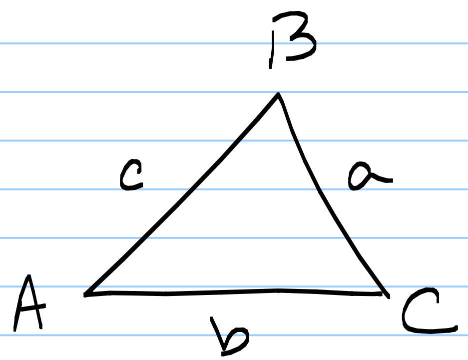
$$\tan\theta = \frac{1}{2}$$

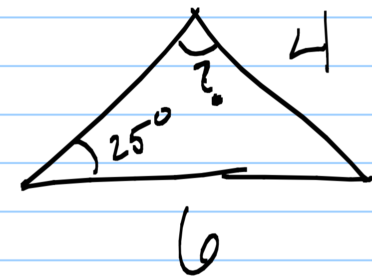
$$\theta = \tan^{-1}\left(\tfrac{1}{2}\right)$$



Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$





$$\frac{\sin 25^\circ}{4} = \frac{\sin \theta}{6}$$

$\sin\theta =$

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

PROOF BY CONTRADICTION

<u>THEOREM</u> IF $\underset{P}{\text{conditions}}$ THEN $\overset{Q}{\text{consequence}}$

(DIRECT PROOF : SUPPOSE P IS TRUE.)

PROOF BY CONTRADICTION.
(REDUTIO AD ABSURDUM)

SUPPOSE P AND NOT Q.

$\vdots$

CRAZINESS. $\rightarrow\leftarrow$

THERE, THE THM MUST HOLD.

__THM__ $\sqrt{2}$ IS NOT RATIONAL.

(IF $x = \sqrt{2}$ THEN x IS NOT RATIONAL.)

__PROOF__. Let $x = \sqrt{2}$ AND x IS RATIONAL.

THEN $X = \frac{a}{b}$ WHERE $a, b \in \mathbb{Z}$

AND $\frac{a}{b}$ IS REDUCED. So a & b HAVE

NO COMMON FACTORS.

NOW $x = \sqrt{2} = \frac{a}{b}$ SO $2 = \frac{a^2}{b^2}$.

So $2b^2 = a^2$, AND THUS a^2 AND HENCE a

ARE EVEN.

Since a is even, $a = 2 \cdot k$. So, $a^2 = 4k^2$

So $2b^2 = 4k^2$, AND $b^2 = 2k^2$. So,

b must BE EVEN AS WELL. BUT a AND b

HAVE NO COMMON PRIME FACTORS! $\rightarrow\leftarrow$

THIS CONTRADICTS OUR ASSUM'S & THUS THE THM MUST HOLD.

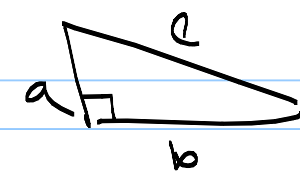
IF P THEN Q. (IMPLICATION)

$\not\equiv$

IF Q THEN P. (CONVERSE)

$\equiv$

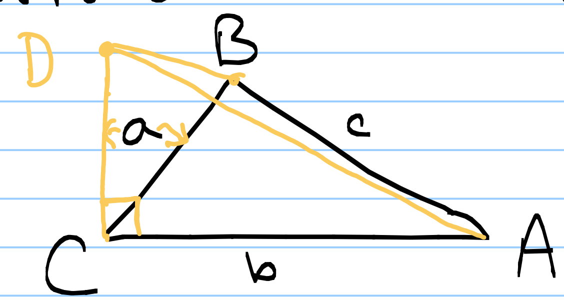
IF NOT Q THEN NOT P. (CONTRAPOSITIVE)

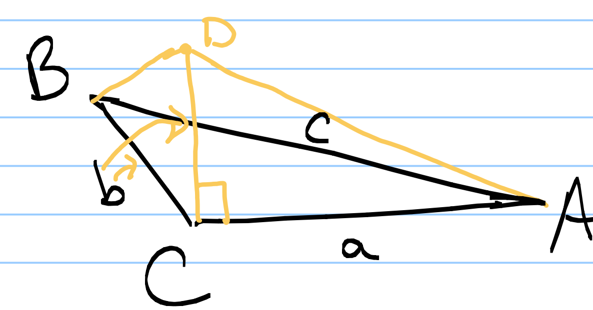
CONVERSE OF PYTHAGOREAN THM  T

IF $a^2+b^2=c^2$ THEN T IS A RIGHT Δ.

Pf SUPPOSE THIS ISN'T TRUE. ie,

SUPPOSE $a^2+b^2=c^2$, BUT $C \neq 90°$.





$C < 90°$

$C > 90°$.

$a^2+b^2=(AD)^2=c^2$

By P.T.

By ASSUMPTION

$a^2+b^2=(AD)^2=c^2$

$$AD = c.$$

BUT THIS CONTRADICTS THE APPARENT FACT THAT IF $C \neq 90^\circ$ THEN $AD \neq c$.