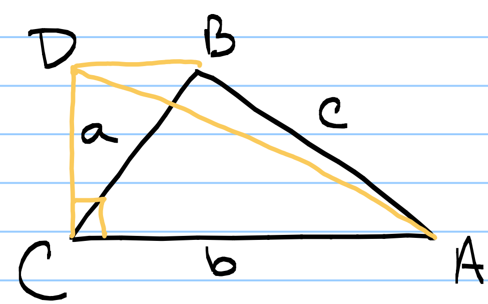
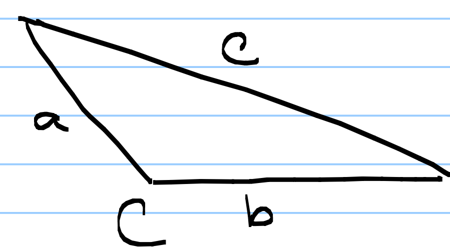
Wed, June 3, AM

Note Title 6/3/2009

Converse of Pytheg Thm.

$a^2+b^2=c^2 \implies$ TRIANGLE IS RIGHT





$a^2+b^2 = AD^2 = c^2$

$AD \neq c$

#11 $\cos x = 1-\sin x$ $\qquad$ $\left(\cos x = \sqrt{1-\sin^2 x} \right)$

$\sqrt{1-\sin^2 x} = 1-\sin x$

$$1-\sin^2 x = 1 - 2\sin x + \sin^2 x$$

$$0 = 2\sin^2 x - 2\sin x$$

$$0 = 2\sin x(\sin x - 1)$$

$\sin x = 0$ OR $\sin x - 1 = 0$

$\downarrow$ $\qquad$ $\sin x = 1$

$x = \cancel{0}, \cancel{\pi}, 2\pi$ ✓ $\qquad$ $x = \pi/2$ ✓

$$\{\pi/2, 2\pi\}$$

$\cos^2 x = \sin^2 x$

$1-\sin^2 x = \sin^2 x$

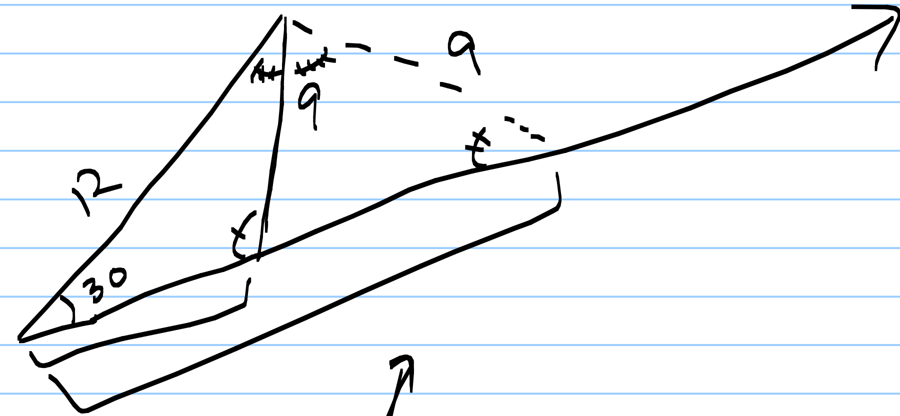
$1 = 2\sin^2 x$

$$\frac{1}{2} = \sin^2 x$$

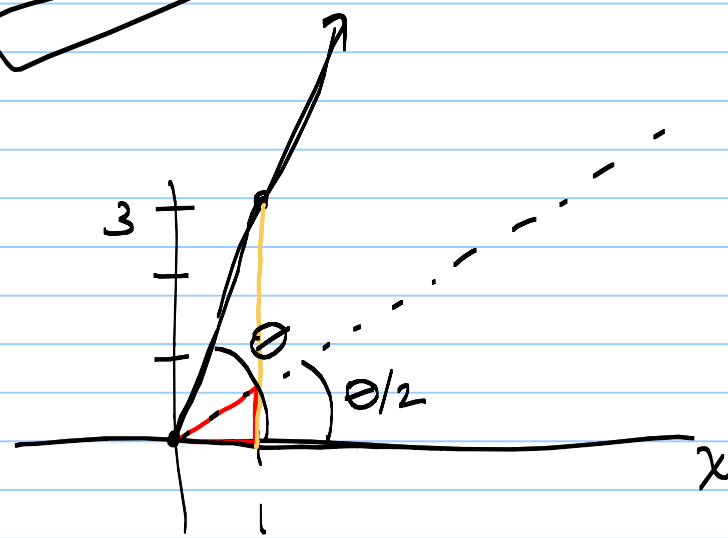
$$\pm\frac{\sqrt{2}}{2} = \sin x$$

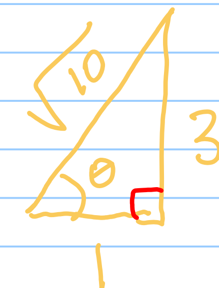
$$\{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$$

MC #13

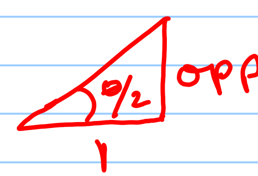


MC #14

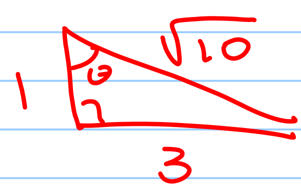




$$\tan\theta = \frac{3}{1} = 3$$



$$\tan\frac{\theta}{2} = \frac{\sqrt{\frac{1-\cos\theta}{2}}}{\sqrt{\frac{1+\cos\theta}{2}}}$$



$$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$\tan\frac{\theta}{2} = \sqrt{\frac{1-1/\sqrt{10}}{1+1/\sqrt{10}}}$$

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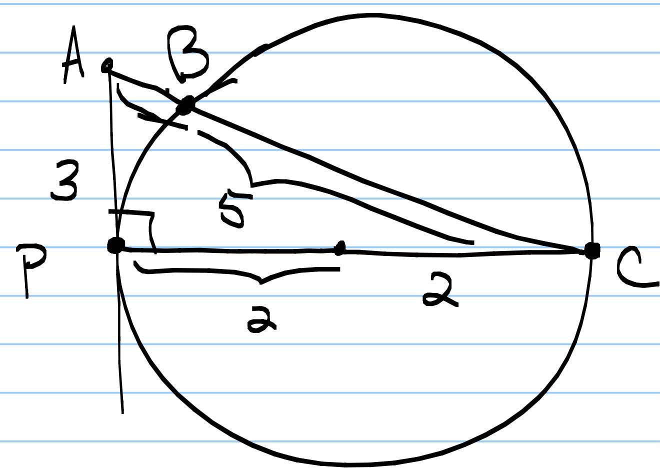


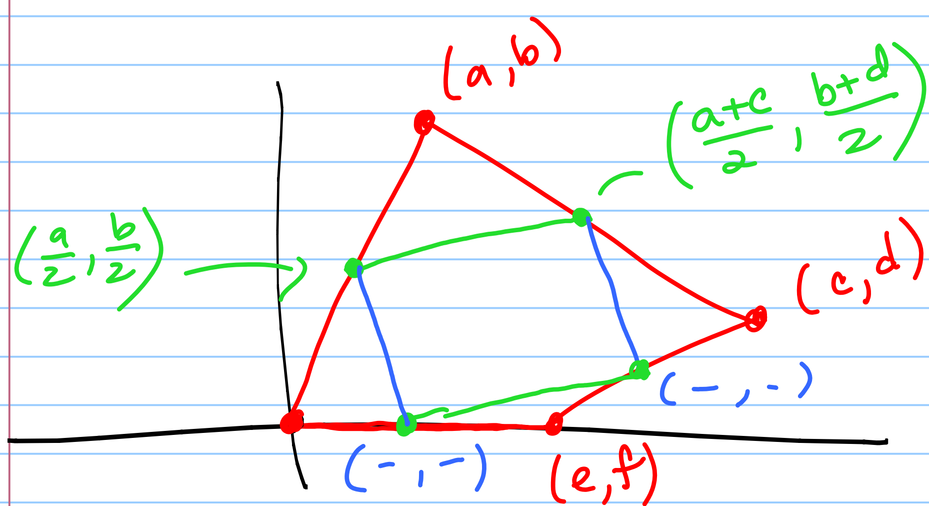
$\cot x + \tan x = \csc x \sec x$ IFF

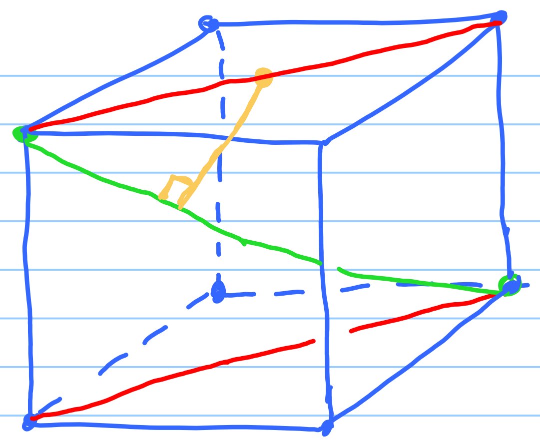
$\dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x} = \dfrac{1}{\sin x} \cdot \dfrac{1}{\cos x}$ IFF

$\dfrac{\overbrace{\cos^2 x + \sin^2 x}^{=1}}{\sin x \cos x} = \dfrac{1}{\sin x \cos x}$, SO THIS MUST HOLD

SINCE $\sin^2 x + \cos^2 x = 1$. THUS $\cot x + \tan x = \csc x \sec x$

QED $\square$





a)

b)

ERDÖS

EX (PROOF BY CONTRADICTION)

THM THERE ARE NO RATIONAL ROOTS OF $x^3+x+1=0$.

PROOF SUPPOSE THAT $\frac{p}{q}$ IS A RATIONAL SOLN OF

$x^3+x+1=0$, AND THAT $\frac{p}{q}$ IS REDUCED.

SO $\left(\frac{p}{q}\right)^3 + \frac{p}{q} + 1 = 0$.

MULTIPLYING OUT GIVES

$\frac{p^3}{q^3} + \frac{p}{q} + 1 = 0$, AND COMBINING TERMS

GIVES

$$\frac{p^3 + pq^2 + q^3}{q^3} = 0.$$

So $p^3 + pq^2 + q^3 = 0$

CASE 1 : p AND q ARE BOTH EVEN : $p = 2k$, $q = 2j$

CAN'T HAPPEN — $\frac{p}{q}$ IS REDUCED!

CASE 2 : p EVEN & q ODD : $p = 2k$ $q = 2j+1$

SUB IN ... $2(\quad) + 1 = 0$, WHICH IS A CONTRADICTION

SINCE 0 IS EVEN! (AND LHS IS NECESSARILY ODD.)

CASE 3: p odd & q even: SAME THING

CASE 4: BOTH ODD.: SAME THING.

THM $x^2 - y^2 = 10$ HAS NO INTEGER SOLUTIONS.

proof $(x-y)(x+y) = 10$

X 1	10	$\rightarrow$	$x-y=1$, $x+y=10$	$2x=11$, $x=\frac{11}{2}$
X 2	5		$x-y=2$, $x+y=5$	$2x=7$, $x=\frac{7}{2}$
5	2			
10	1			

HINT FOR 4

proof $\ni$ a, $a+b$ are rational AND b is NOT.

$$a = \frac{p}{q} \qquad a+b = \frac{m}{n}$$

HINT: $b = (a+b) - a$

$$a+b = a+b$$

$$b = (a+b) - (a)$$

PROOF BY CONTRAPOSITIVE

① $P \longrightarrow Q$ IMPLICATION

② $Q \longrightarrow P$ CONVERSE of

(3) not P $\longrightarrow$ NOT Q   INVERSE

(4) not Q $\longrightarrow$ NOT P   CONTRAPOSITIVE

(1) $\equiv$ (4)   (2) $\equiv$ (3)

<u>THM</u> IF (conditions hold) THEN (conclusion holds)

<u>proof</u> SUPPOSE THAT (CONCLUSION) DOESN'T HOLD.

$\vdots$

THEN (CONDITIONS) FAIL.

cond.

THM IF x AND y ARE INTEGER WHERE $x+y$ IS EVEN, THEN x & y HAVE THE SAME PARITY.

PROOF SUPPOSE THAT x & y HAVE DIFFERENT PARITY.

WITHOUT LOSS OF GENERALITY, ASSUME x IS EVEN & y IS ODD.

SO, $x = 2k$ $y = 2j+1$, FOR SOME k, j.

AND THEN $x+y = 2k+2j+1 = 2(k+j)+1$ IS ODD. THUS, IT MUST BE THAT IF $x+y$ IS EVEN, x & y HAVE THE SAME PARITY.