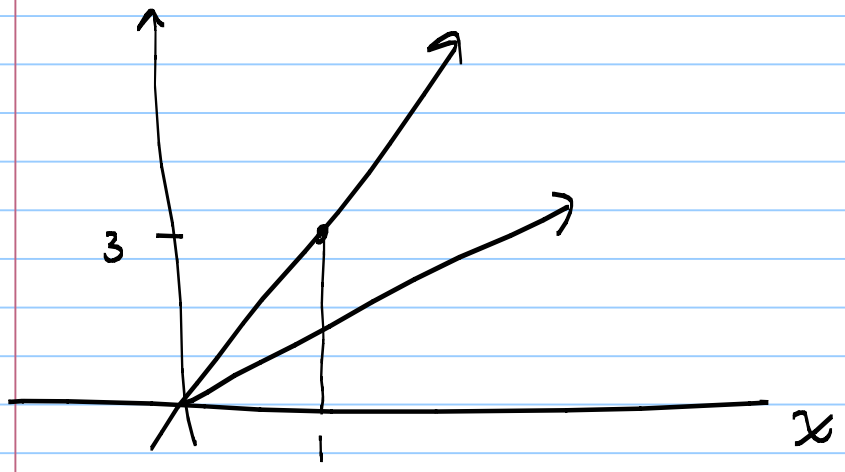
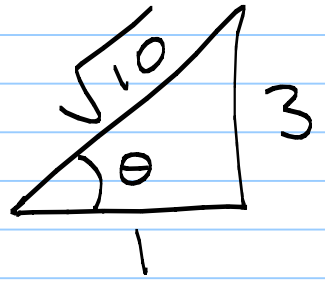
Wed, June 3, 2009

Note Title 6/3/2009





$$\cos\theta = \frac{1}{\sqrt{10}}$$

$$\sin\theta = \frac{3}{\sqrt{10}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$$

WHAT IS TAN $\frac{\theta}{2}$

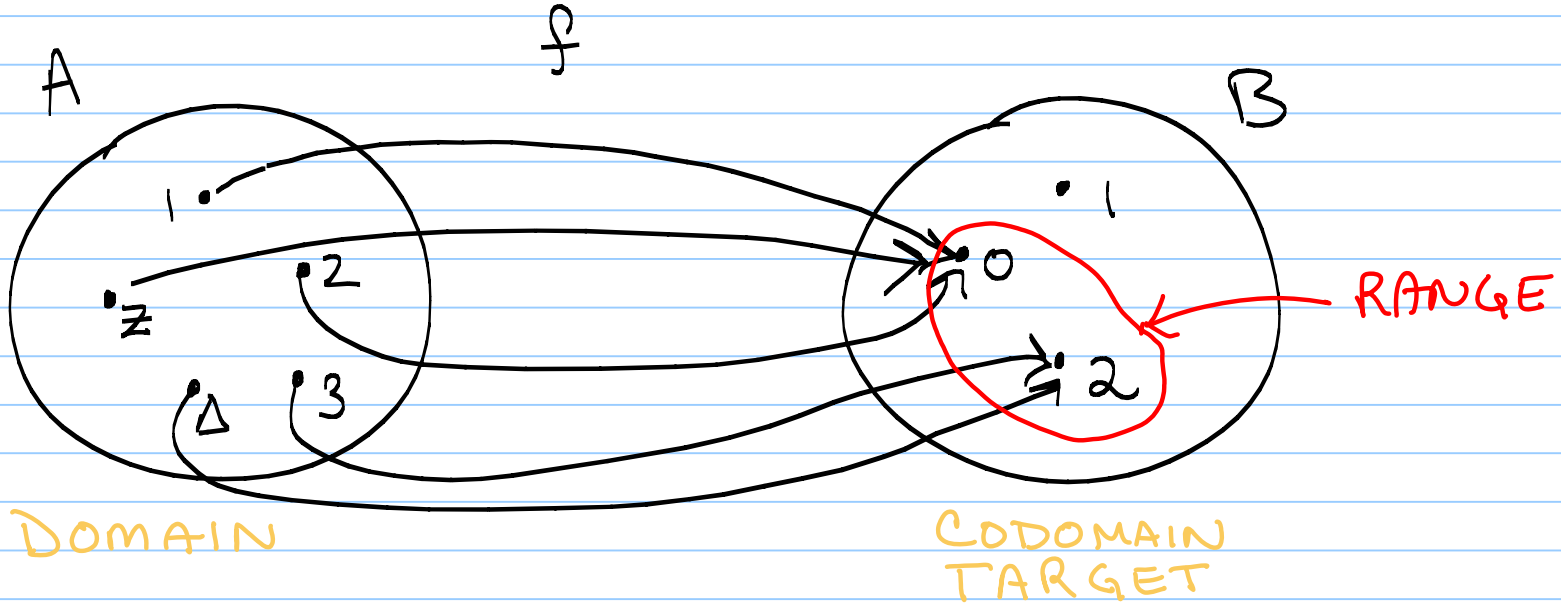
$$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}} = \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}} = \frac{1-\cos\theta}{\sin\theta} = \frac{1-1/\sqrt{10}}{3/\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}-1}{3}$$

FUNCTIONS

Defn

A FUNCTION IS RULE THAT ASSIGNS [redacted] ELEMENT OF ONE SET [redacted] [redacted] ELEMENT FROM ANOTHER SET.



$$f(3) = 2$$

$$f = \{ (1,0), (2,0), (\Delta, 2), (3,2), (2,0) \}$$

$$f(x) = x^2 + 2$$

"WHAT IS THE NATURAL DOMAIN OF f?"

↳ THE LARGEST SET OF REALS THAT CAN BE LEGALLY PLUGGED INTO f.

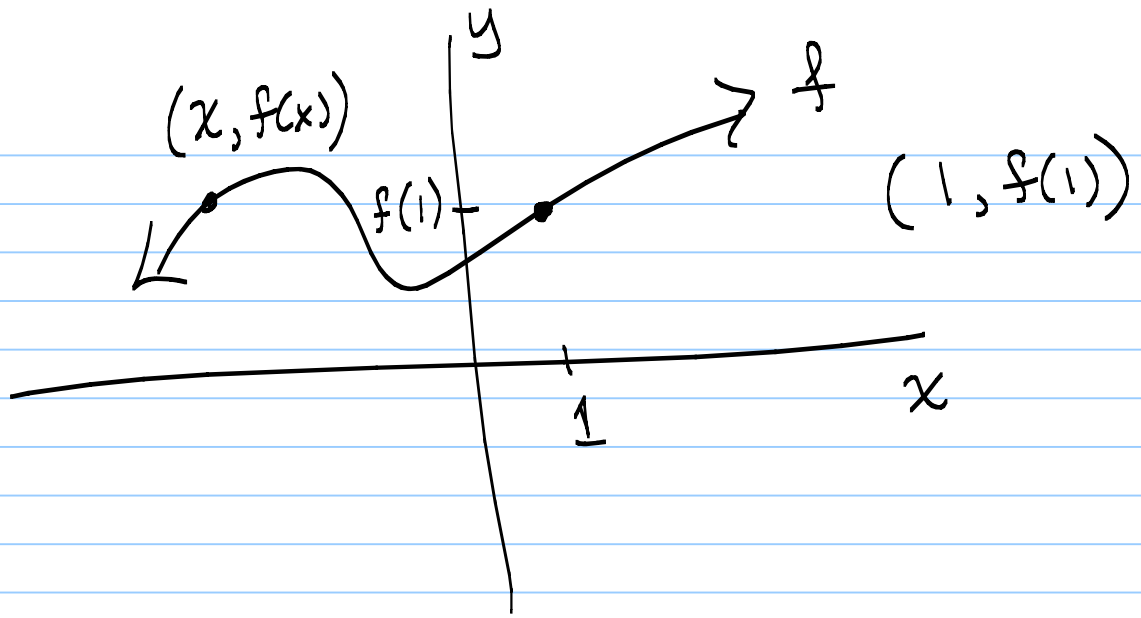
$$f(x) = \frac{1}{\sqrt{x-1}}$$

$x \neq 1$

$x \not< 1$

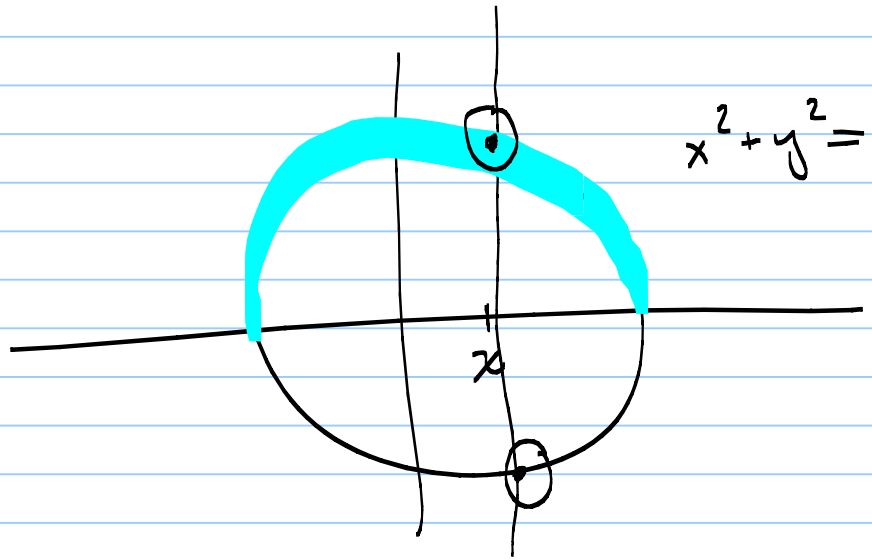
$\text{Dom}(f) = (1, \infty)$

GRAPHICAL REPRESENTATION



$y = f(x)$ — x: independent

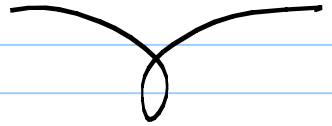
y: DEPENDENT



$x^2 + y^2 = r^2$

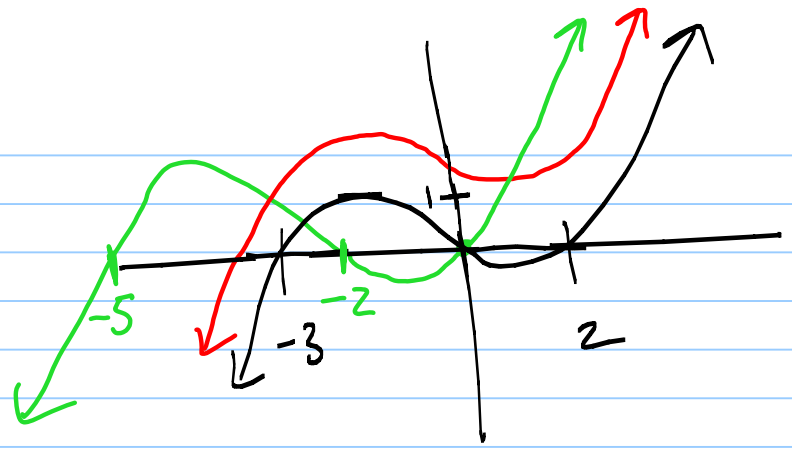
$y = \pm\sqrt{r^2 - x^2}$

$y = \sqrt{r^2 - x^2}$



$y = f(x)$

$y = f(x) + k$



$y = f(x+k)$

$y = f(x+2)$

$g(x) = f(x+2)$

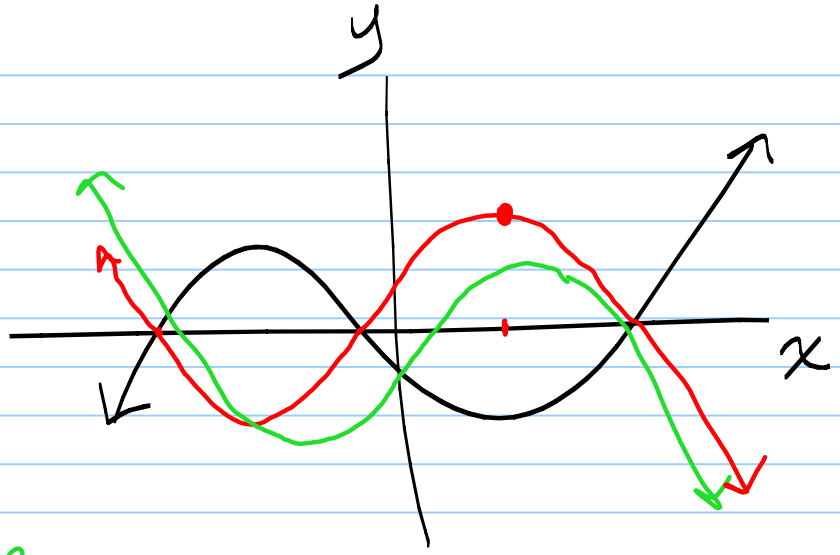
x	$g(x) = f(x+2)$
0	$g(0) = f(0+2) = f(2) = 0$
-2	$g(-2) = f(-2+2) = f(0) = 0$

Reflections

$y = f(x)$

Refl. across x-axis.

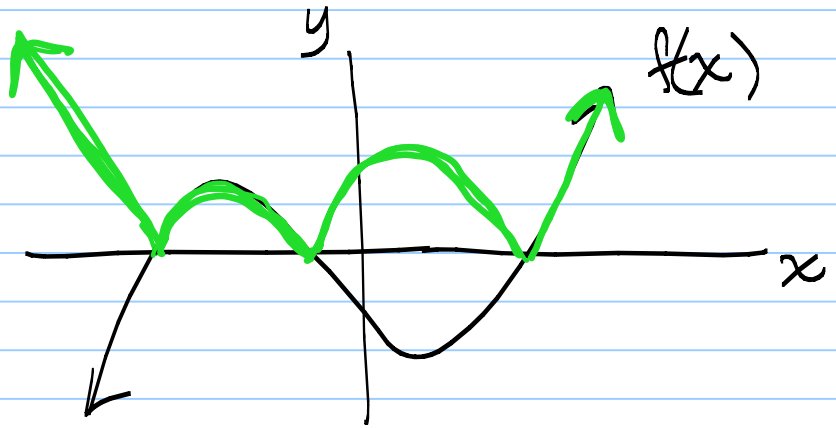
$y = -f(x)$



$y = f(-x)$ — REFLECTION ACROSS y-AXIS

$y = f(x)$

$y = |f(x)|$

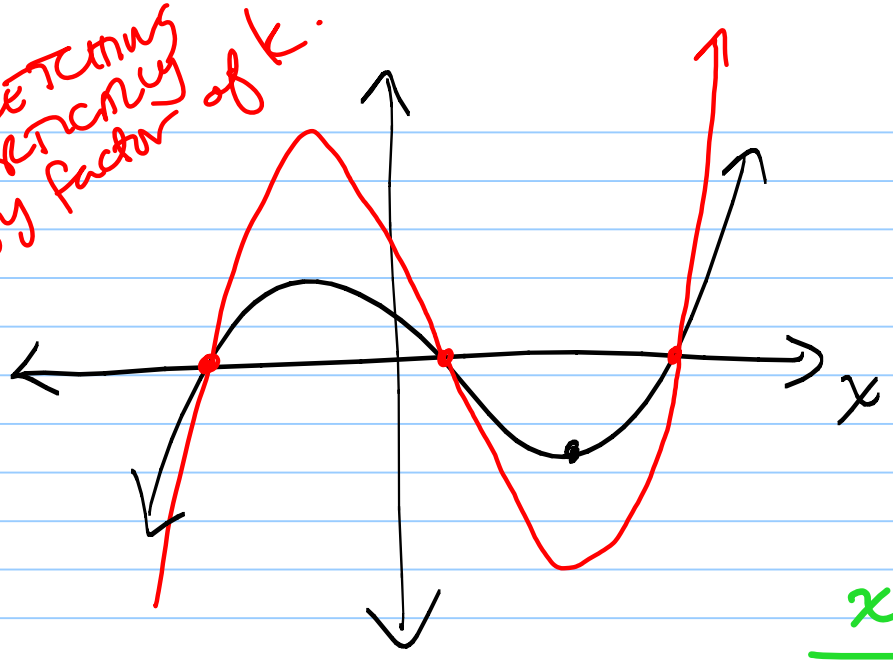


$y = f(x)$

$y = k \cdot f(x)$ ← STRETCHING VERTICALLY BY FACTOR of k.

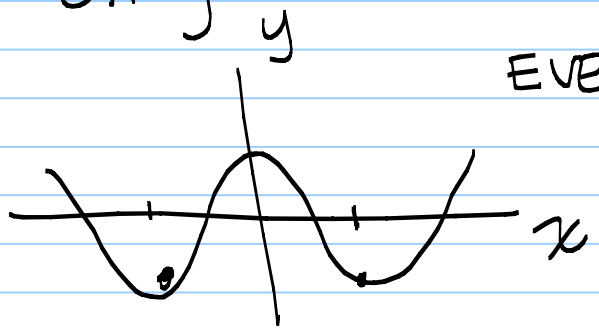
$y = f(kx) = g(x)$

↑ SHRINKS HORIZ. BY FACTOR OF k.



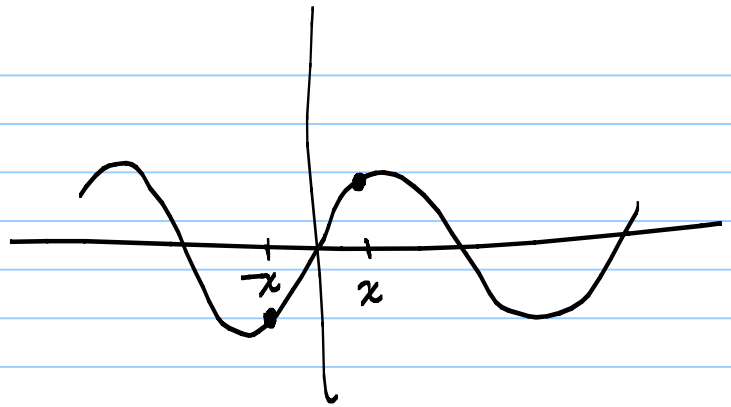
x	$g(x) = f(xk)$
1	$g(1) = f(k)$

SYMMETRY



EVEN-SYMMETRY @ y-axis

$f(x) = f(-x)$

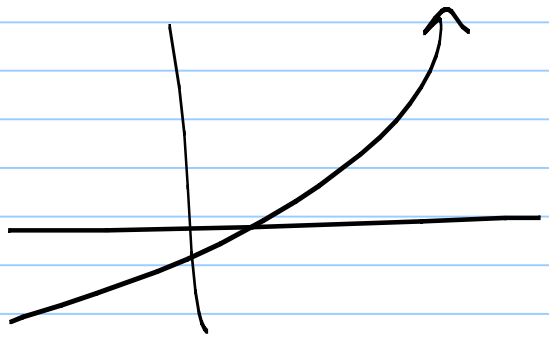


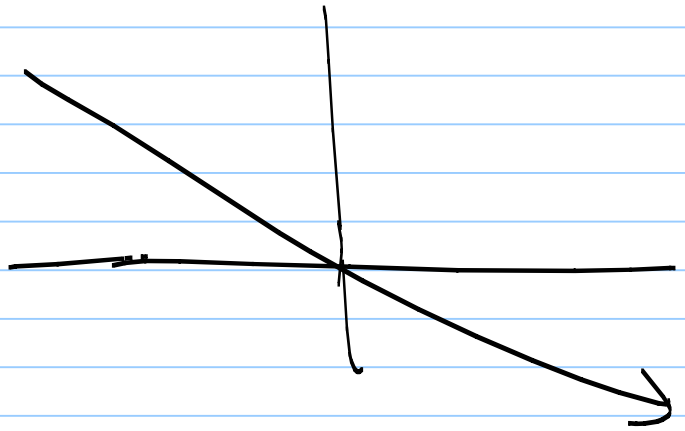
ODD - SYMMETRY @ ORIGIN

$f(-x) = -f(x)$

MONOTONE — ALWAYS CLIMBING (INCREASING)

ALWAYS FALLING. (DECREASING)

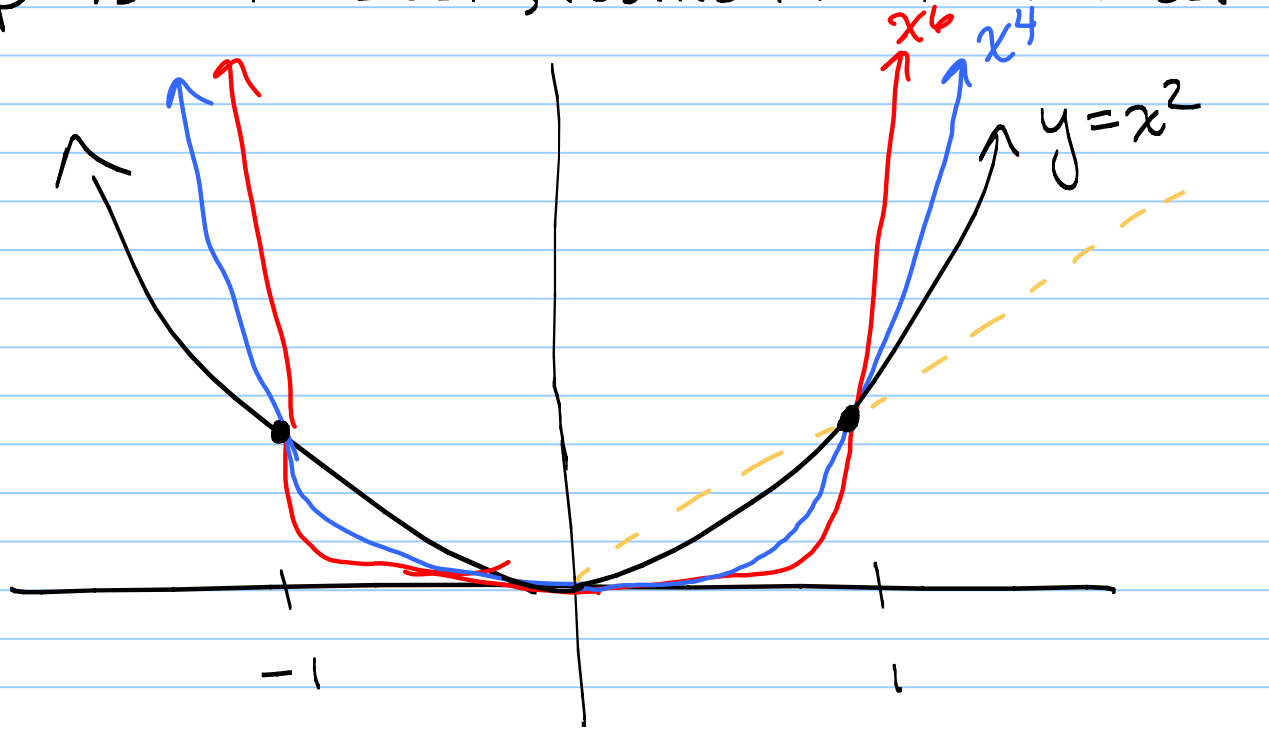




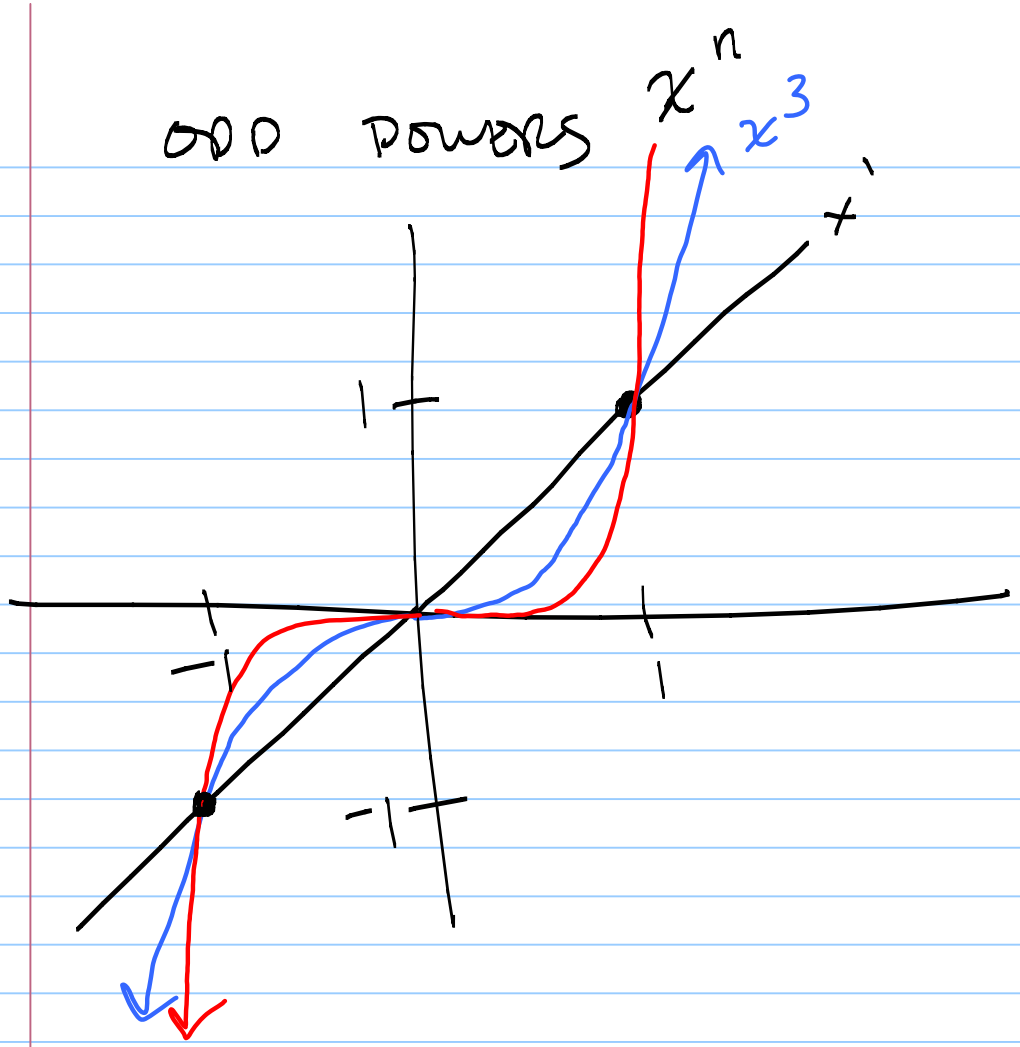
POWER FUNCTIONS

$$f(x) = x^p$$

p IS AN EVEN, POSITIVE NATURAL NUMBER

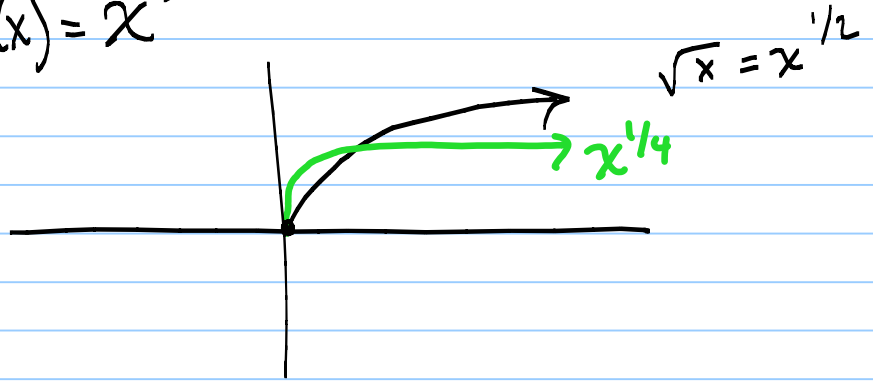


ODD POWERS x^n

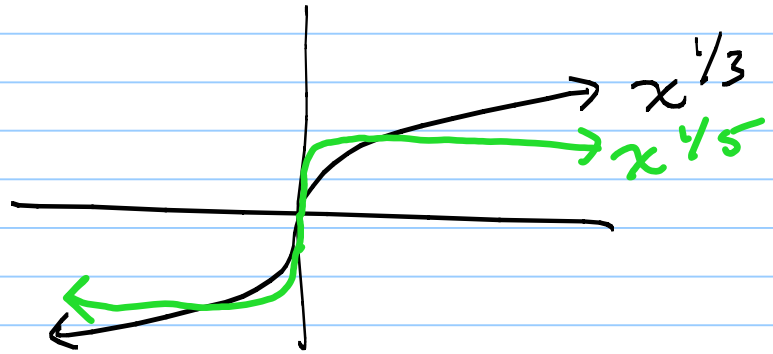


ROOT FUNCTIONS

$f(x) = x^{1/n}$ ← EVEN n



$f(x) = x^{1/n}$ ← ODD n



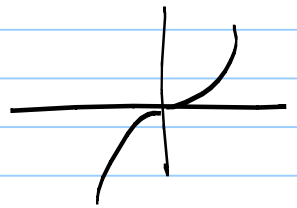
$y = f(x)$ HAS AN INVERSE IF ITS GRAPH IS NOT INTERSECTED BY ANY HORIZONTAL LINES IN MORE THAN ONE PLACE.

(ie, $f(x)$ IS ONE-TO-ONE OR INJECTIVE.)

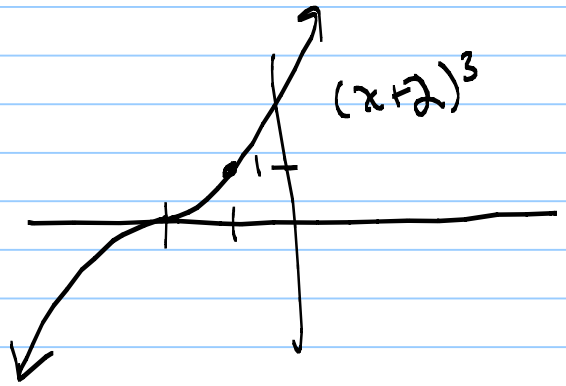
Power functions

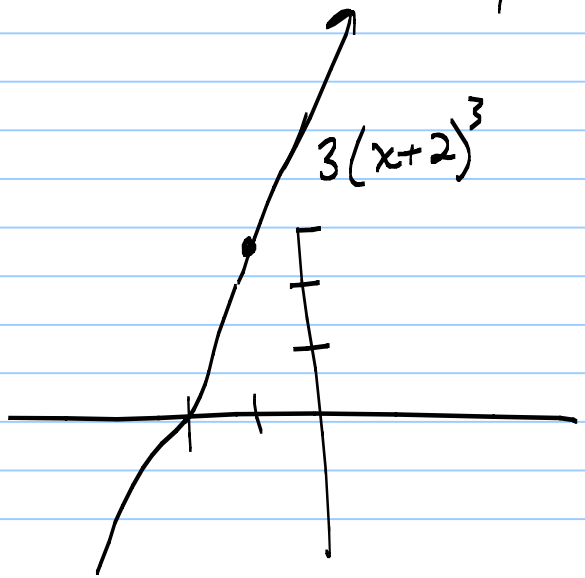
$f(x) = x^p$

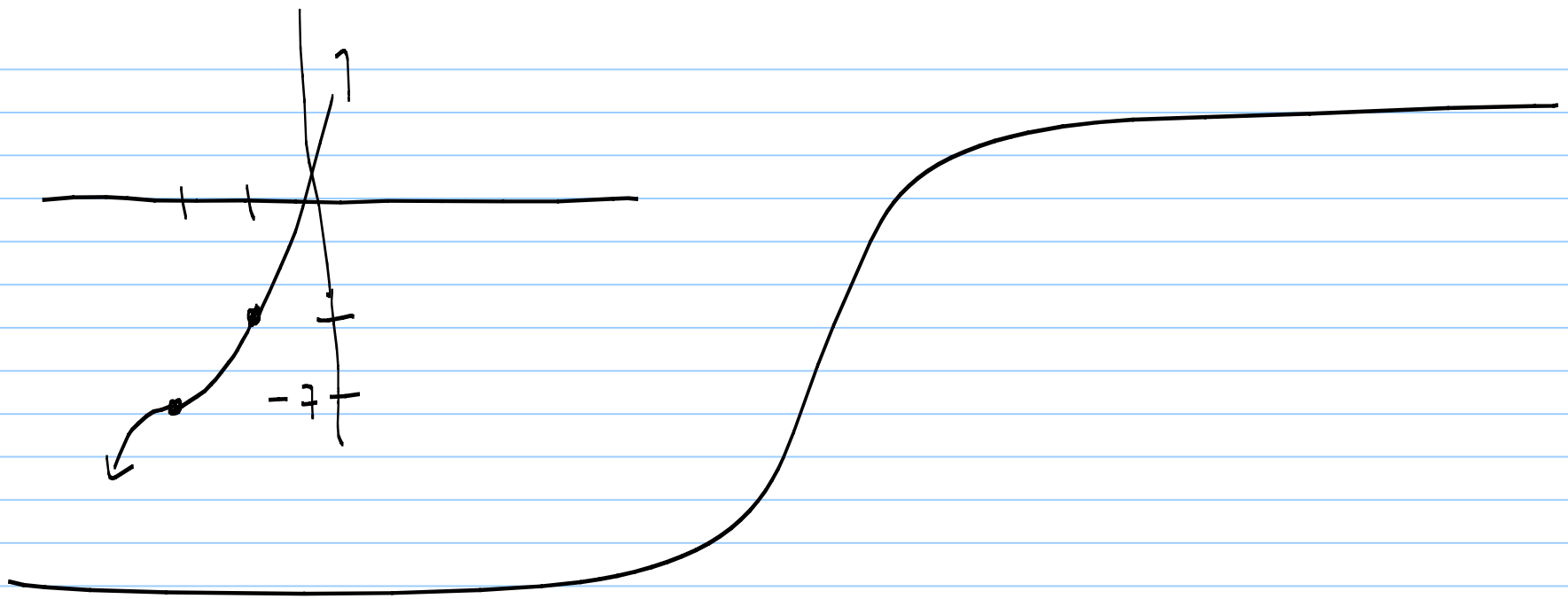
$f(x) = x^3$



$g(x) = 3(x+2)^3 - 7$







POLYNOMIALS

$$y = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

$$y = 3x^4 + 2x^3 - 3x^2 + x - 1$$

a_4 a_3 a_2 a_1 a_0

deg is ONE MORE THAN MAX # of turns allowed

EVEN DEGREE

← leading coeff is > 0

L.C. < 0

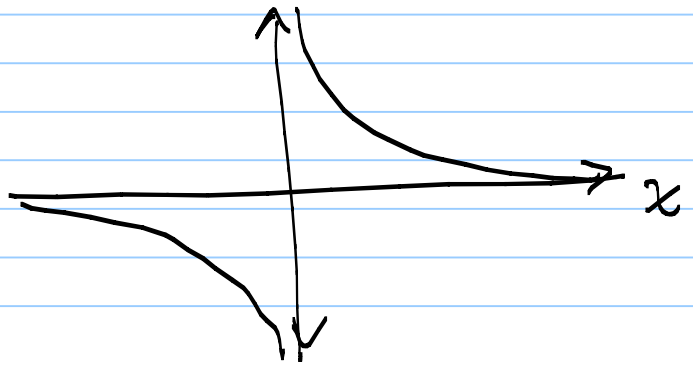
ODD DEGREE

$x^5 - 3x^4 + 1$?

∞, $-\infty$

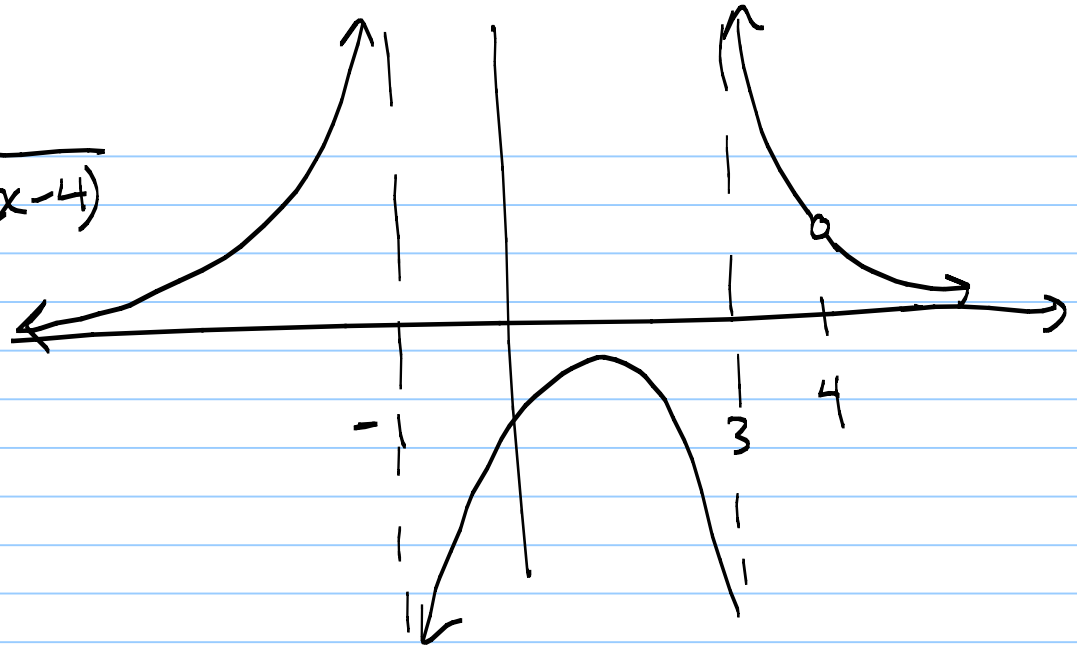
RATIONAL FUNCTIONS → RATIOS OF POLYNOMIALS

$f(x) = \frac{1}{x}$

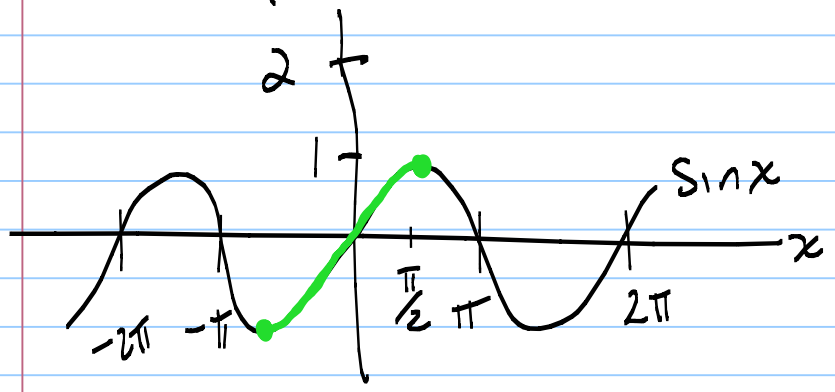


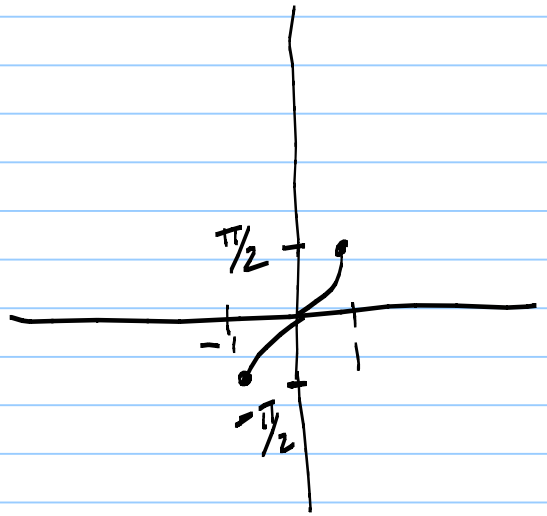
$$f(x) = \frac{(x+2)(x-4)}{(x-3)(x+1)(x-4)}$$

$$f(x) = \frac{x+2}{(x-3)(x+1)}, \quad x \neq 4$$



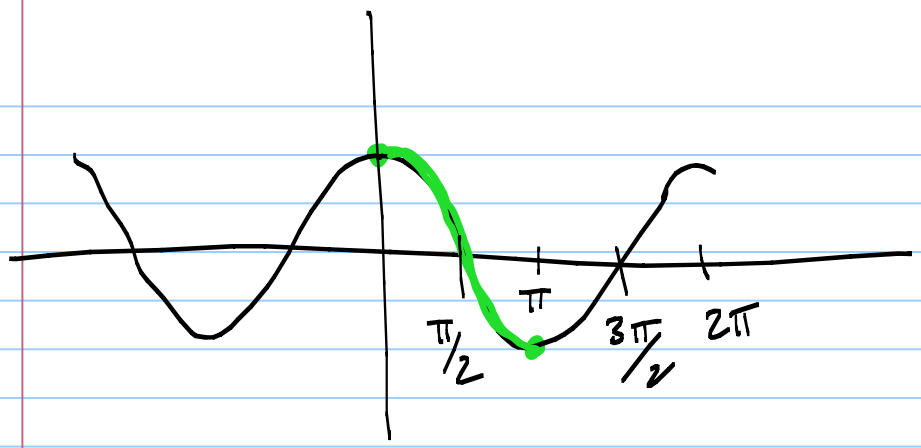
TRIG FCTNS

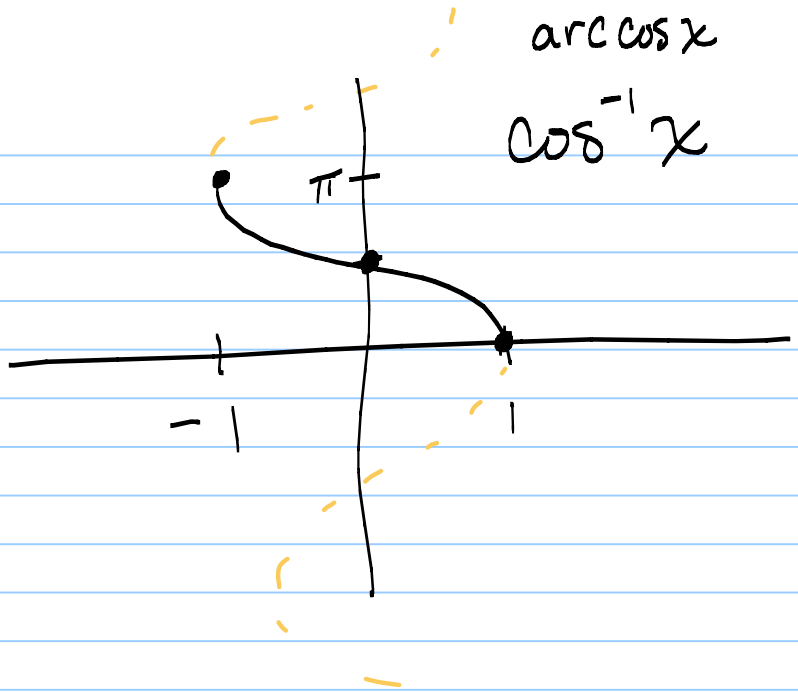


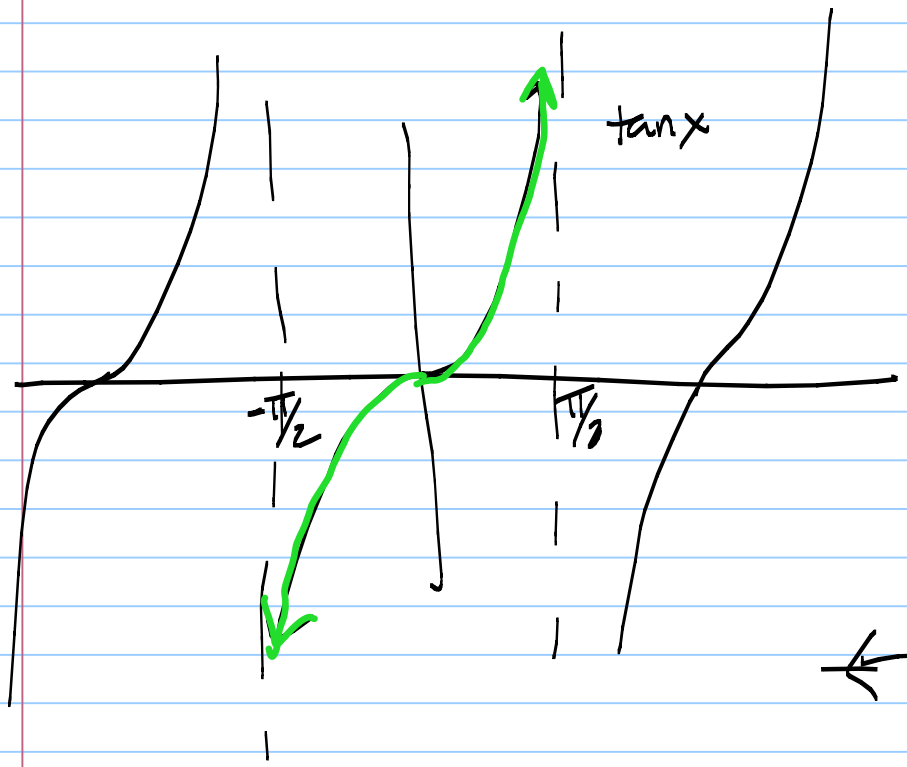


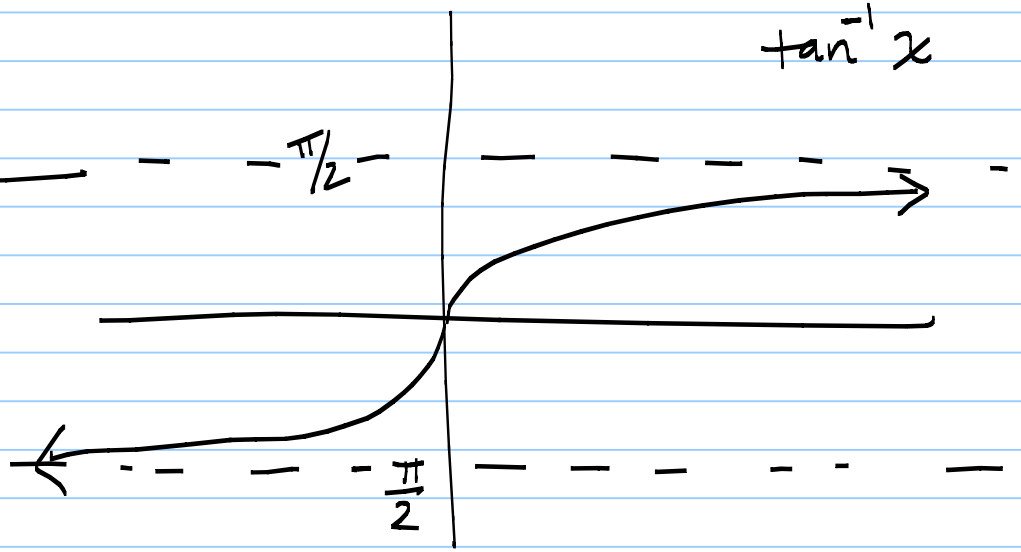
arc sin x

$\sin^{-1} x$


π/2
π
3π/2
2π


arc cos x
cos⁻¹ x
π
-1
1


tan x
-π/2
π/2

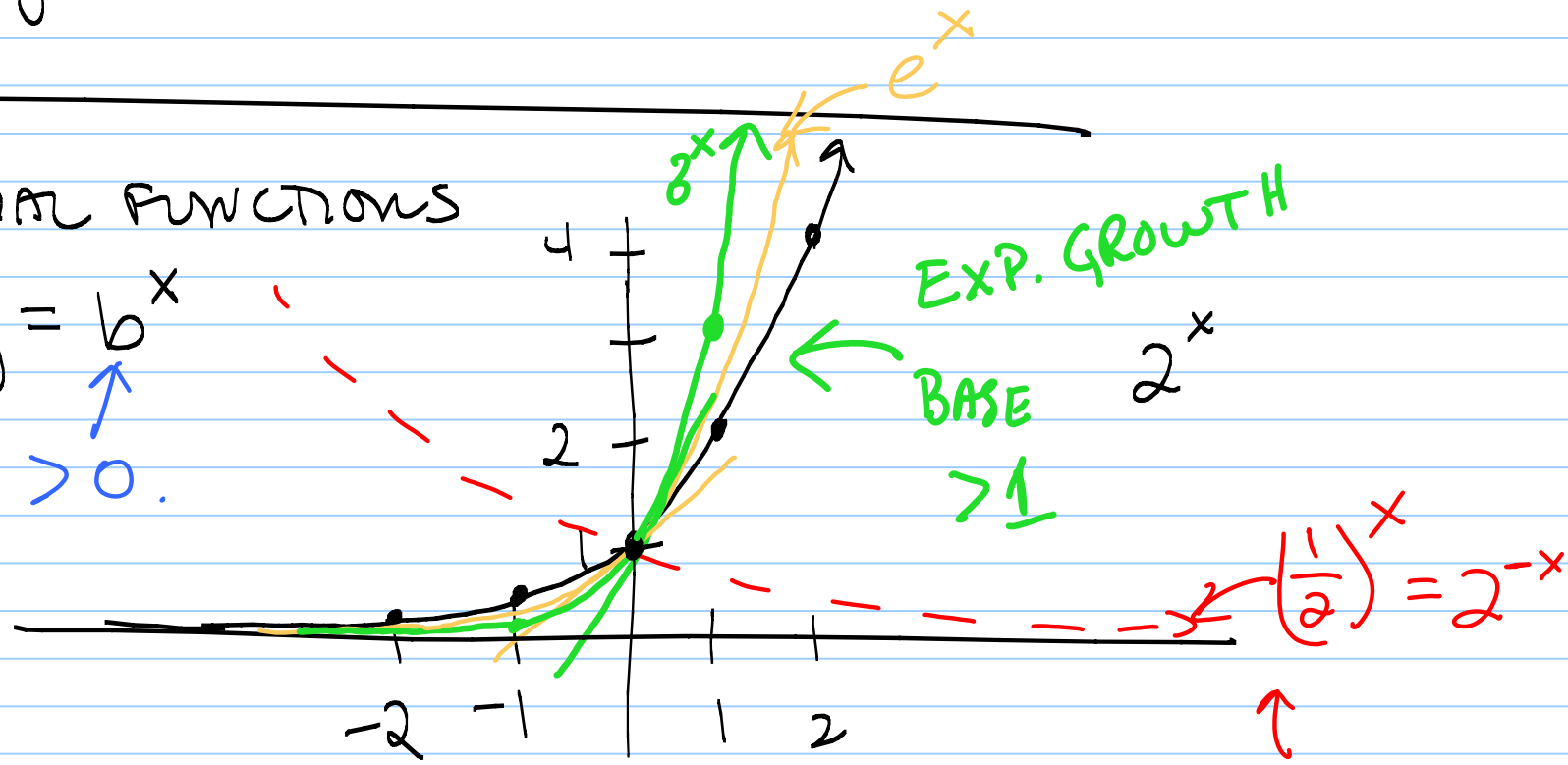

tan⁻¹ x
-π/2
π/2

Recall $y = x^p$ ← Power functions

EXPONENTIAL FUNCTIONS

$y = b^x$

$b > 0$.

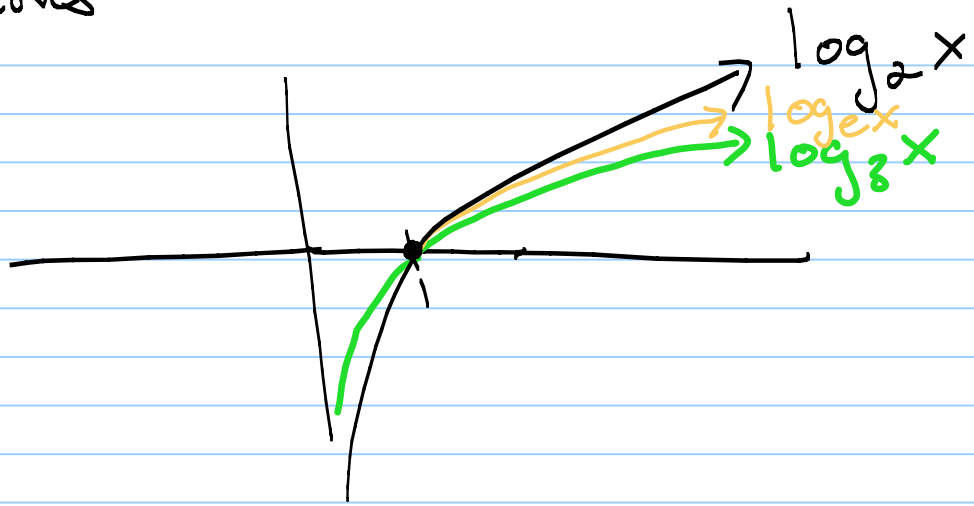


$$y = \left(\frac{1}{2}\right)^x = \frac{1}{2^x} = 2^{-x}$$

Logarithmic Functions

$y = \log_b x$

$\log_e x = \ln x$



$y = \log_b x$

MEANS

$x = b^y$

Log Rules

$\log_b (x \cdot y) = \log_b x + \log_b y$

$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$

$\log_b (x^y) = y \log_b x$

$$\log_b x = \frac{\ln x}{\ln b}$$