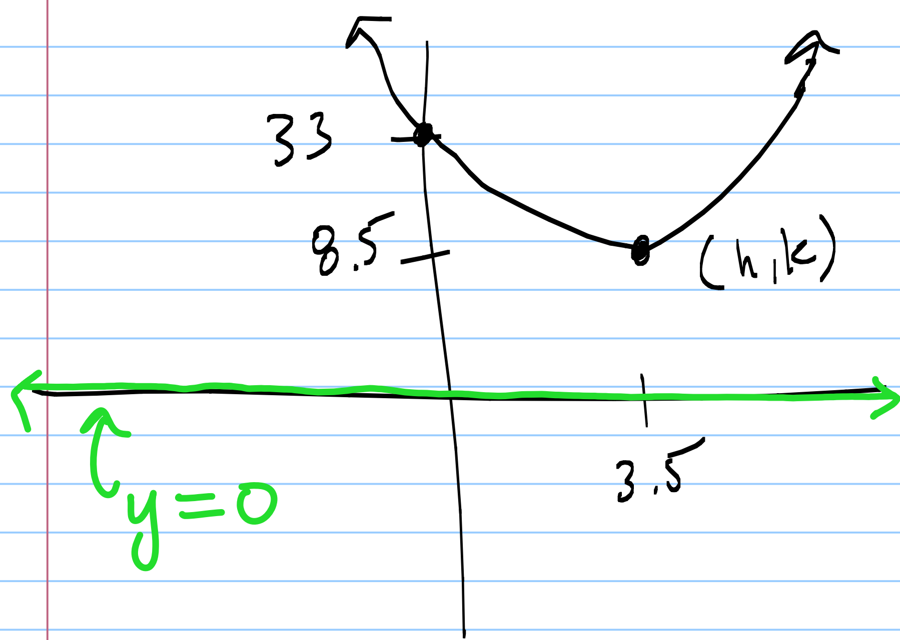
Note Title 6/8/2009

Monday, June 8

$$\underbrace{1+2+\dots+n}_{= \sum_{k=1}^{n} k} = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} 2k^2-1 = (2\cdot 1^2-1)+(2\cdot 2^2-1)+\dots+(2n^2-1)$$

#7 (PRACTICE 1)



$$y = ax^2 + bx + c$$

$$y - k = a(x-h)^2$$

$$y - 8.5 = a(x-3.5)^2$$

$$33 - 8.5 = a(-3.5)^2$$

$$a = 2$$

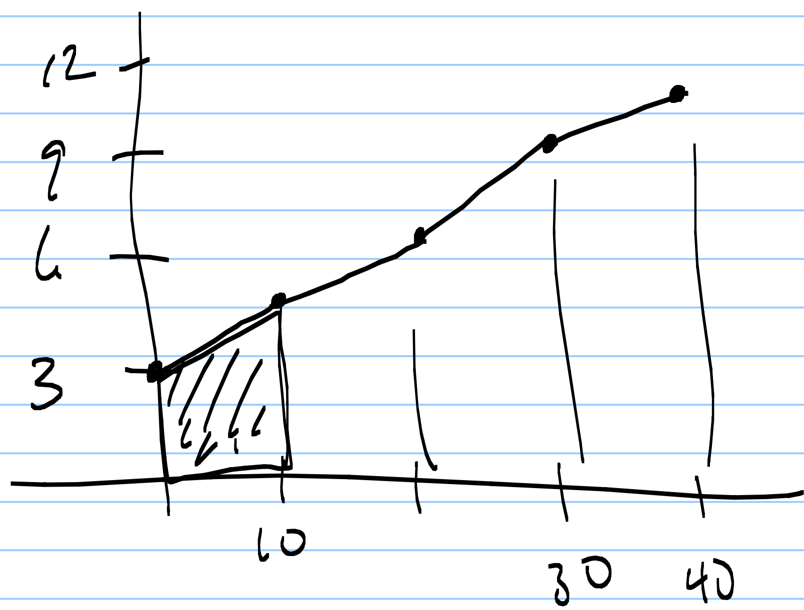
$$y - 8.5 = 2(x-3.5)^2$$

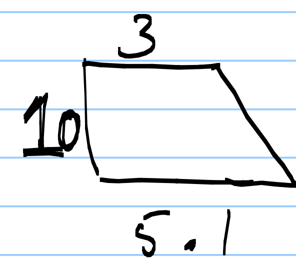
$$y = 2(x-3.5)^2 + 8.5 = 0$$

$$(x-3.5)^2 = -4.25 = \frac{-17}{4}$$

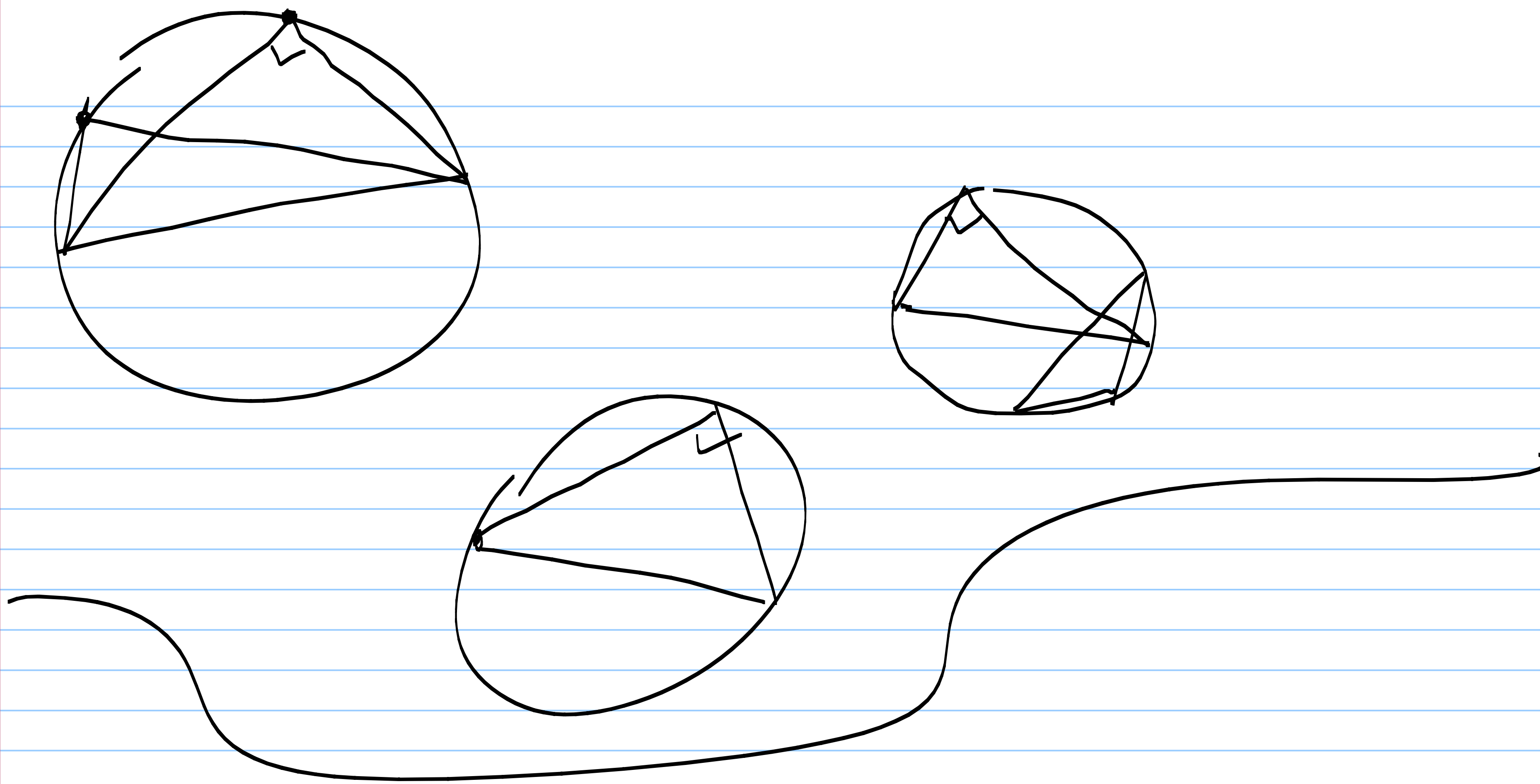
$$(x-3.5) = \pm\sqrt{\frac{-17}{4}} = \pm i\,\frac{\sqrt{17}}{2}$$

$$x = 3.5 \pm i\,\frac{\sqrt{17}}{2} = \frac{7 \pm i\sqrt{17}}{2}$$





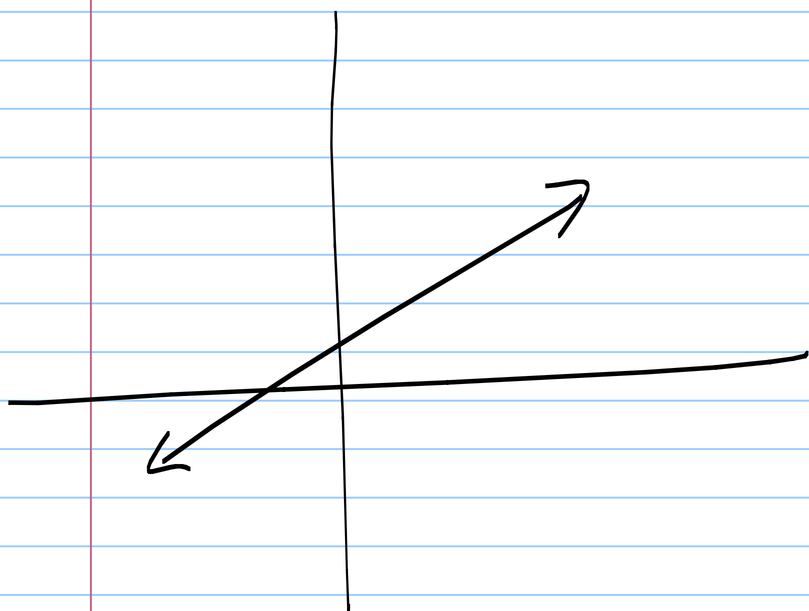
$$A = \left(\frac{3+5.1}{2}\right)\cdot 10$$

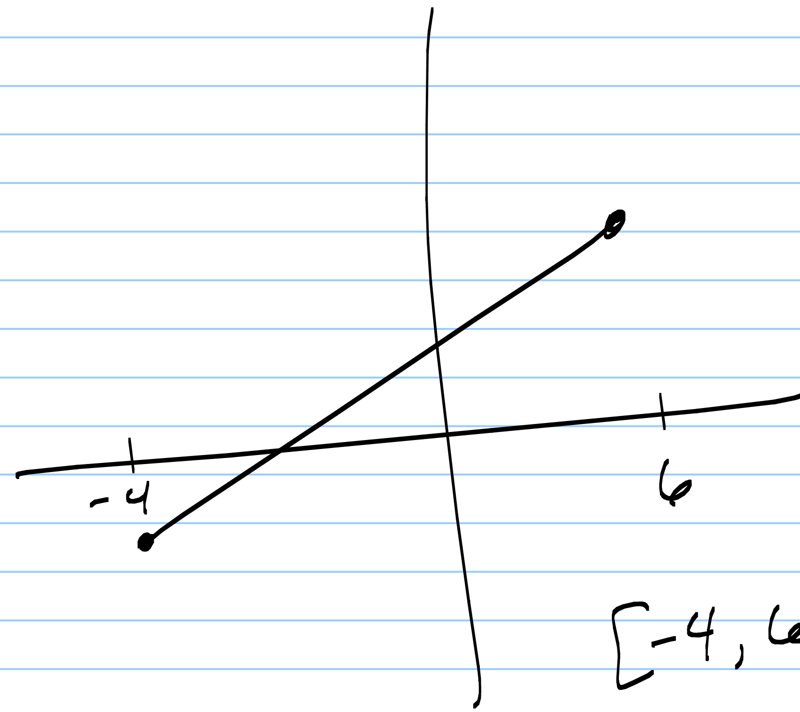


A RELATION IS A RULE OF ASSIGNMENT

<u>ex</u> $= \; = \{(0,0),(1,1),(2,2)\ldots\} \subseteq \mathbb{N}\times\mathbb{N}$

$< \; = \{(1,2),(1,3),(1,4),(2,7),(2,13)\ldots\} \subseteq \mathbb{N}^2$





[-4, 6]

\# 28

$$g(f(x)) = g\left(\frac{2x+6}{x+2}\right) = \frac{2x+6}{x+2} + 2$$

#50

A IS A SUBSET OF B

"every element of A is an element of B."

$A \subseteq B$, $A \subset B$

TRANSITIVE.

$A \subseteq B$, $B \subseteq C$

Then

$A \subseteq C$

#3

a) $f(x) + f(-x)$ IS AN EVEN FUNCTION.

proof. Let $g(x) = f(x) + f(-x)$. (W.T.S. $g(x) = g(-x)$)

$$g(-x) = f(-x) + f(--x)$$
$$= f(-x) + f(x) = g(x)$$
.

b) $f(x) - f(-x)$ IS AN ODD FUNCTION

proof. Let $h(x) = f(x) - f(-x)$. (W.T.S. $h(-x) = -h(x)$)

$$h(-x) = f(-x) - f(--x) = f(-x) - f(x)$$
$$= -(-f(-x) + f(x)) = -(f(x) - f(-x))$$
$$= -h(x).$$

c) SHOW EVERY FUNCTION $f(x)$ IS THE SUM OF AN EVEN & ODD FUNCTION

proof Note that $f(x) = \frac{g(x)}{2} + \frac{h(x)}{2} = \frac{(f(x) + \cancel{f(-x)}) + (f(x) - \cancel{f(-x)})}{2}$

$$= \frac{2f(x)}{2}.$$

Since $g(x)$ is even, so is $\frac{g(x)}{2}$, and since $h(x)$ is odd, so is $\frac{h(x)}{2}$. Thus

$$f(x) = \frac{g(x)}{2} + \frac{h(x)}{2}$$

is a decomposition of f as the sum of an even function and an odd function

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#4

½m → next 1 mi → last ½ m

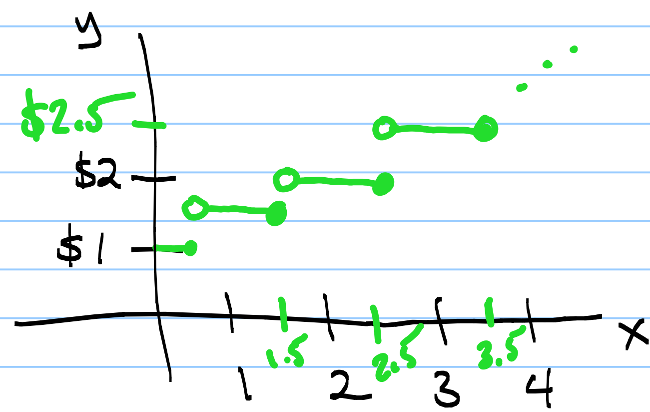
a) (i) $1 + .50 + 0.50 = \$2$

(ii) $1 + .50 = \$1.50$

b) $y = (\text{FIRST } 1/2 \text{ mi charge}) + .50\left(\lceil x - 1/2 \rceil\right)$

$y = 1 + 0.50\left(\lceil x - 1/2 \rceil\right)$

c)

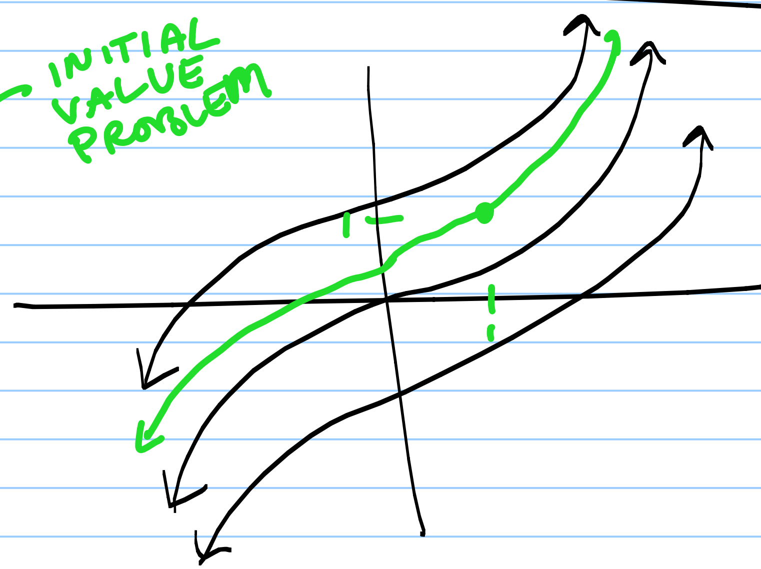


ANTI DERIVATIVES

$f'(x) = x^2$, $\boxed{f(1) = 1}$ INITIAL VALUE PROBLEM

$f(x) = \frac{x^3}{3} + C$

$f(1) = \frac{1^3}{3} + C = \underline{1}$



$C = \frac{2}{3}$

$f(x) = \frac{x^3}{3} + \frac{2}{3}$

INDEFINITE INTEGRAL $\equiv$ MOST GENERAL ANTIDERIVATIVE.

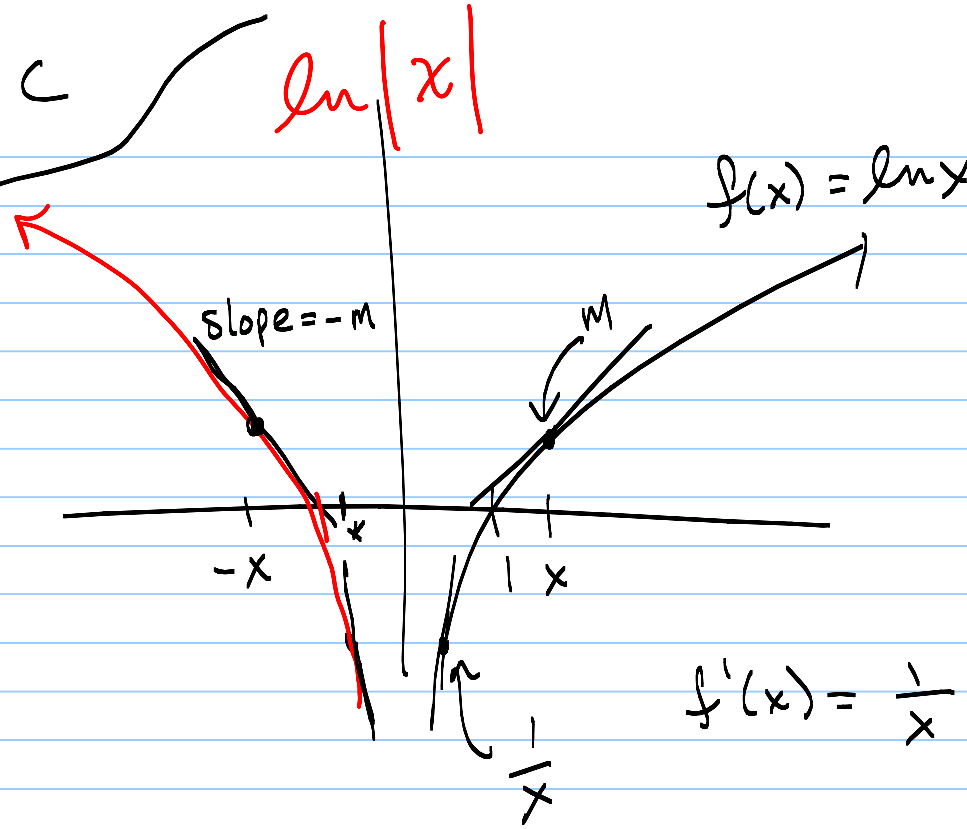
$$\int f(x)\,dx = F(x) + C$$

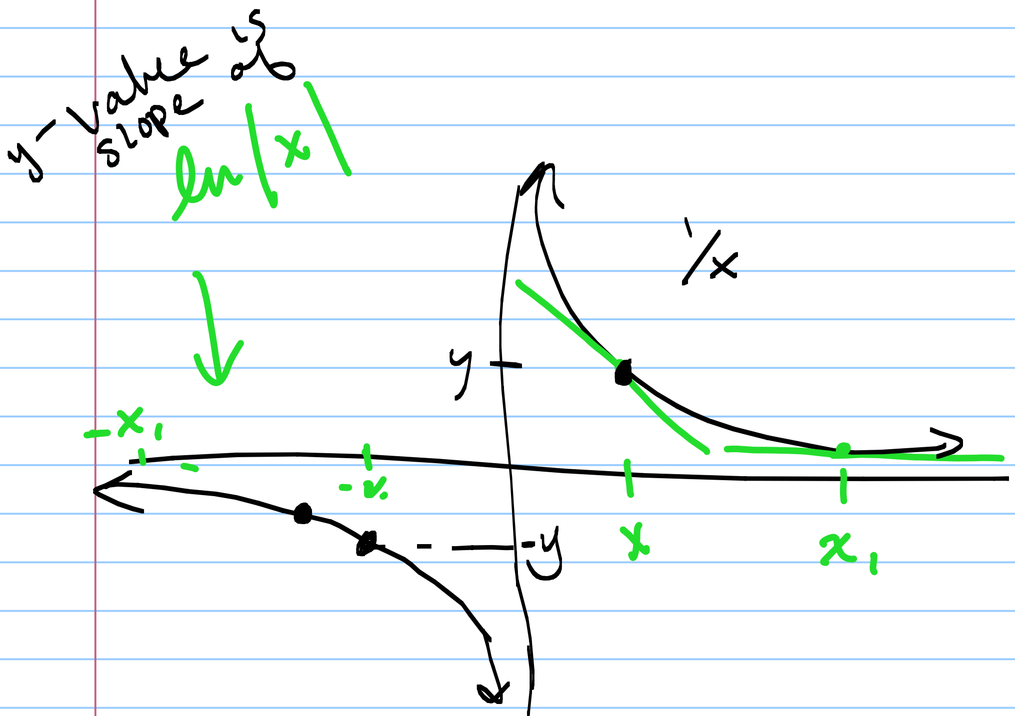
NOTATION SAYS "INTEGRATE THE STUFF INBETWEEN."

RULES FOR INTEGRATING

$$\int x^p\,dx = \frac{x^{p+1}}{p+1} + C \quad , p \neq -1$$

$$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C$$





Sum / Diff / constant multiple rule

$$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

ex

$$\int x^2 + \sqrt{x}\,dx = \frac{x^3}{3} + \frac{2x^{3/2}}{3} + C$$

$$\int k\, f(x)\,dx = k \int f(x)\,dx$$

($\int \ln x \, dx$ dont know)

$$\int e^x\,dx = e^x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \frac{x^2 - 3x + \sqrt{x}}{x^2}\,dx$$

$$= \int 1 - \frac{3}{x} + x^{-3/2}\,dx$$

$$= x - 3\ln|x| - 2x^{-1/2} + C$$

U-SUBSITITUTION (undoes the chain rule)

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

$$(x^2-2x)^4 \xrightarrow{\frac{d}{dx}} 4(x^2-2x)^3(2x-2)$$

$$\int f'(g(x))\, g'(x)\, dx = f(g(x)) + C$$

$$\int \frac{4}{4} \underbrace{x^3}_{?} (x^4-2)^8\, dx = \frac{1}{4}\int \underbrace{4x^3}_{g'(x)} \underbrace{(x^4-2)^8}_{f'(g(x))}\, dx = \frac{1}{4}\cdot\frac{(x^4-2)^9}{9} + C$$

$$f'(x) = x^8 \rightarrow f(x) = \frac{x^9}{9}$$

IN PRACTICE

1. CHOOSE u.
2. FIND du.

GUIDELINES FOR CHOOSING u.

⓪ NEVER CHOOSE $u = x$.

① IF $(\sim)^r$, PROBABLY $u = \sim$

② IF $e^{\sim}$, PROBABLY $u = \sim$

3. MAKE SUBSTITUTION.

4. INTEGRATE
(IF YOU CAN. IF you can't, GOTO 1.)

③ IF $\ln(\sim)$, PROBABLY $u = \ln(\sim)$

weak → ④ IF $\text{trig}(\sim)$, PROB. $u = (\sim)$

5. BACK SUBSTITUTE

ex $-\frac{1}{2}\int -2x e^{-x^2}\,dx = -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + C$

$u = -x^2$

$du = -2x\,dx$

$= -\frac{1}{2}e^{-x^2} + C$

ex $\int \frac{1}{x\ln x}\,dx = \int \frac{1}{x}\cdot\frac{1}{\ln x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$= \ln|\ln x| + C$

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ex $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int -\sin x\,(\cos x)^{-1}\,dx = -\int u^{-1}\,du$

$u = \cos x$

$du = -\sin x\,dx$

$= -\ln|u| + C$

$$= -\ln|\cos x| + C$$

Integration By Parts

→ undo the product rule

Hopefully, gets simpler when you diff. it.

$$\int u\,dv = uv - \int v\,du$$

- something you can integrate
- includes dx

$$\frac{d}{dx} f\cdot g = f'g + g'f$$

$$f\cdot g = \int f'g + g'f\,dx$$

$$f\cdot g = \int f'g\,dx + \int g'f\,dx$$

$$u\cdot v - \int v\,du = \int u\,dv$$

$$f\cdot g - \int f'g\,dx = \int g'f\,dx$$

$u = f(x)$ $\quad v = g(x)$

$du = f'(x)\,dx$ $\quad dv = g'(x)\,dx$

$$\int x\sin x\,dx = -x\cos x + \int \cos x\,dx$$

$$\boxed{\begin{array}{ll} u = x & v = -\cos x \\ du = dx & dv = \sin x\,dx \end{array}}$$

$$= -x\cos x + \sin x + C$$

$$\int \ln x \, dx = x\ln x - \int x \cdot \frac{1}{x} dx = x\ln x - \int 1 dx$$

$u = \ln x \qquad v = x$

$$= x\ln x - x + C$$

$du = \frac{1}{x} dx \qquad dv = dx$

#17, 19, 21 (p. 27, 28)