June 8, PM

Note Title 6/8/2009

#21 $a(t) = -5 \quad m/s^2$

Find $v(t)$, given that at $t=0$, $v=2$.

$$v(t) = \int -5\,dt = -5\int dt = -5t + C.$$

$$v(0) = 2 = c$$

$$v(t) = -5t + 2.$$

FIND $s(t)$ GIVEN THAT $s(0) = 0$.

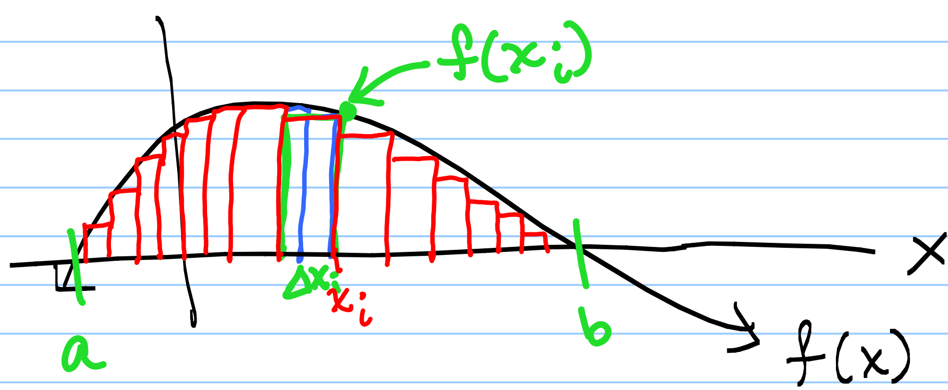
$$s(t) = \int v(t)\,dt = \int -5t + 2\,dt = \frac{-5t^2}{2} + 2t + c$$

$s(0) = 0 = c$, so $s(t) = \dfrac{-5t^2}{2} + 2t$

$$s(3) = \frac{-5 \cdot 3^2}{2} + 2 \cdot 3 = \frac{-45}{2} + 6.$$

19c $\quad \frac{(4x^2-1)^{19}}{19} \cdot \frac{1}{8} + C$

Riemann Sums



$$\sum_{i=1}^{\overbrace{2000}^{n}} f(x_i)\,\Delta x_i$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x_i$$

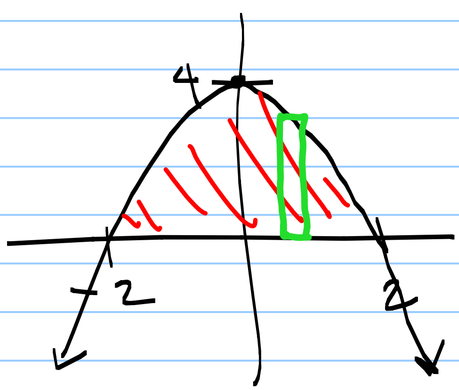
↑ AND ALSO $\max(\Delta x_i)$ should be $\to 0$.

FUNDAMENTAL THEOREM OF CALCULUS

$$A = \int_a^b f(x)\,dx = F(b) - F(a)$$

WHERE $F(x)$ IS <u>ANY</u> ANTIDERIVATIVE of $f(x)$.

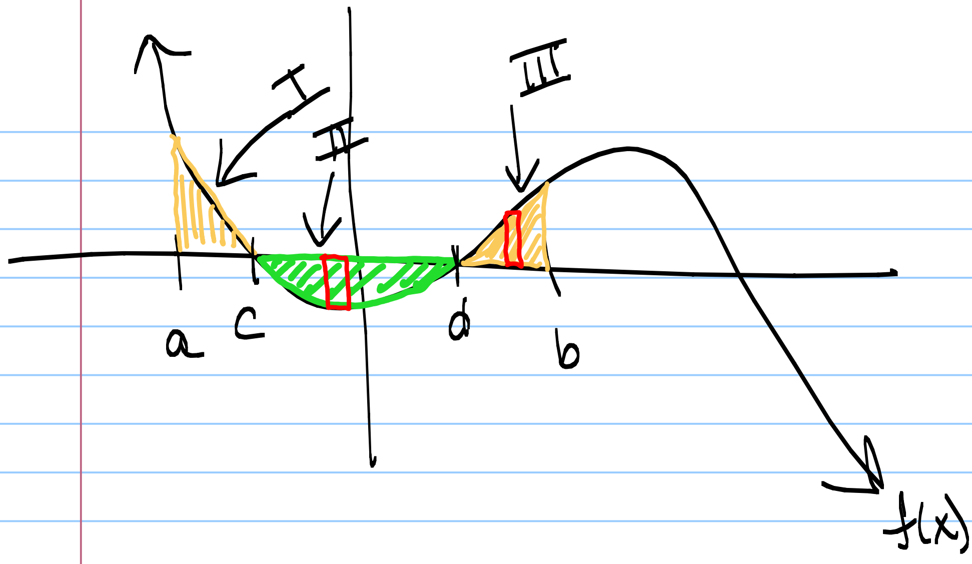
FIND THE AREA BOUND BY THE X-AXIS AND $y = -x^2 + 4$



$$\int_{-2}^{2} -x^2+4\,dx = \left(-\frac{x^3}{3} + 4x\right|_{-2}^{2}$$

$$= \left(-\frac{(2)^3}{3} + 4\cdot 2\right) - \left(\frac{-(-2)^3}{3} + 4(-2)\right)$$

$$= \frac{-8}{3} + 8 - \frac{8}{3} + 8 = 16 - \frac{16}{3}$$



$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$= \text{I} + \text{III} - \text{II}$$

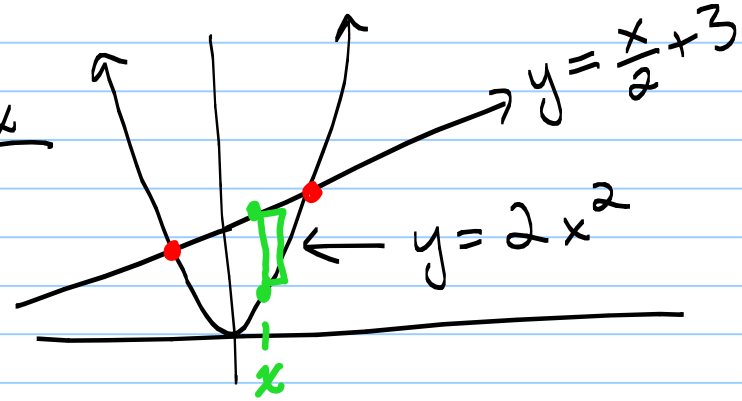
WHOLE AREA IS THE SUM OF

$$\text{I} = \int_a^c f(x)\,dx$$

$$\text{III} = \int_d^b f(x)\,dx$$

$$\text{II} = \left| \int_c^d f(x)\,dx \right|$$

ex



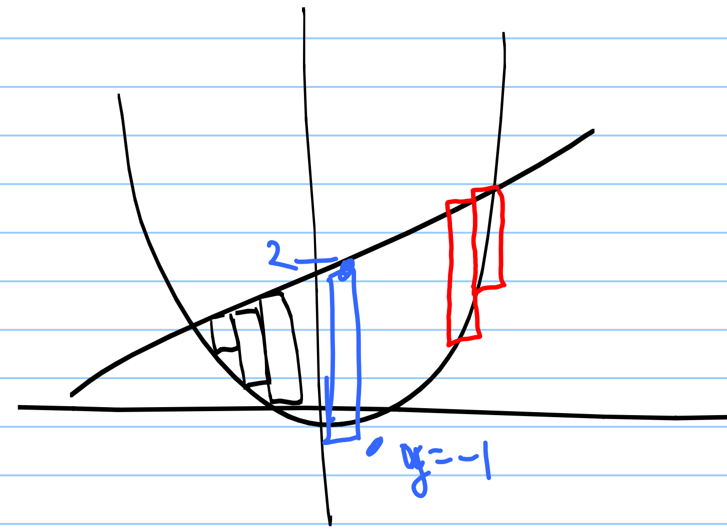
FIND THE AREA BOUND BY THE CURVES.

$$\int_{1-\sqrt{97}/8}^{1+\sqrt{97}/8} \left(\frac{x}{2}+3\right) - \left(2x^2\right)\,dx$$

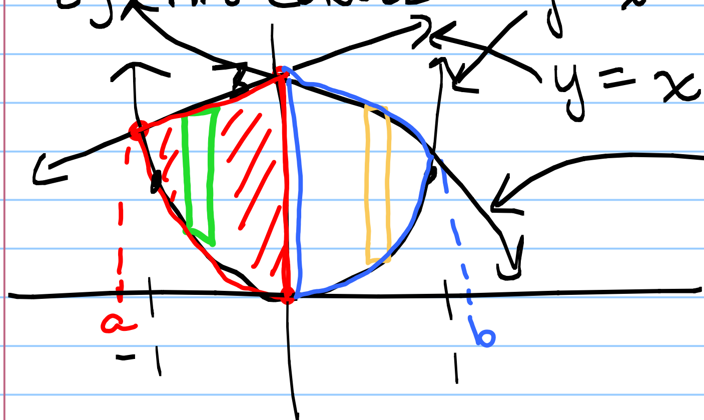
$$2x^2 = \frac{x}{2} + 3$$

$$4x^2 - x - 6 = 0$$

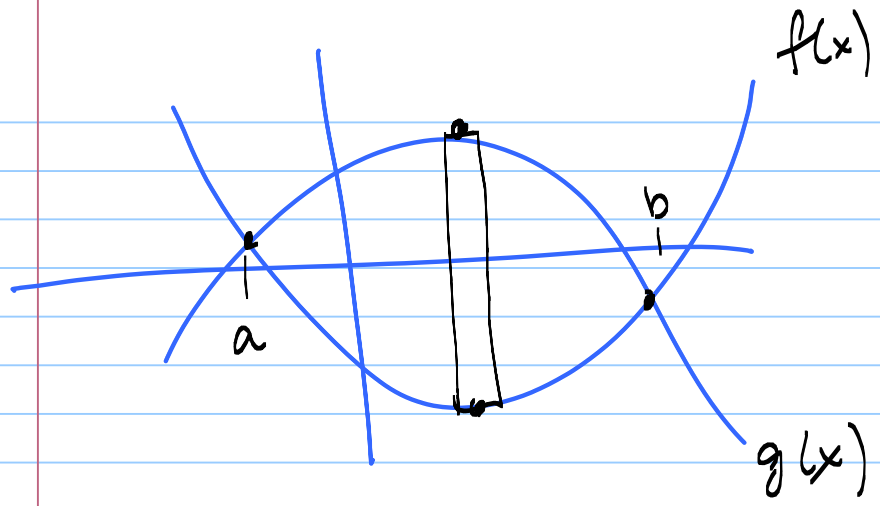
$$x = \frac{1 \pm \sqrt{1+96}}{8} = \frac{1 \pm \sqrt{97}}{8}$$



FIND THE AREA BOUND BY THE CURVES $y = x^2$, $y = x+3$, $y = -x^3 + 3$



$$\int_a^0 (x+3) - x^2\,dx + \int_0^b (-x^3+3) - x^2\,dx$$



$$\left| \int_a^b g(x) - f(x)\, dx \right|$$

$$\int_{x=1}^{x=e} \frac{\ln x}{x}\, dx = \int_0^1 u\, du = \frac{u^2}{2}\Big|_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$$

$u = \ln x$

$du = \frac{1}{x}\, dx$

$$\int_0^{\pi/2} x^2 \cos x \, dx = \left(x^2 \sin x\right|_0^{\pi/2} - \int_0^{\pi/2} 2x \sin x \, dx$$

$u = x^2 \qquad v = \sin x$

$du = 2x\,dx \qquad dv = \cos x\,dx$

$$= \left[\left(\frac{\pi}{2}\right)^2 \cdot 1 - 0\right] - 2\int_0^{\pi/2} x \sin x \, dx$$

$u = x \qquad v = -\cos x$

$du = dx \qquad dv = \sin x\,dx$

$$= \left(\frac{\pi}{2}\right)^2 - 2\left[-x\cos x\Big|_0^{\pi/2} + \int_0^{\pi/2} \cos x \, dx\right]$$

$$= \left(\frac{\pi}{2}\right)^2 - 2\left[0 + \left(\sin x\right|_0^{\pi/2}\right] = \left(\frac{\pi}{2}\right)^2 - 2$$

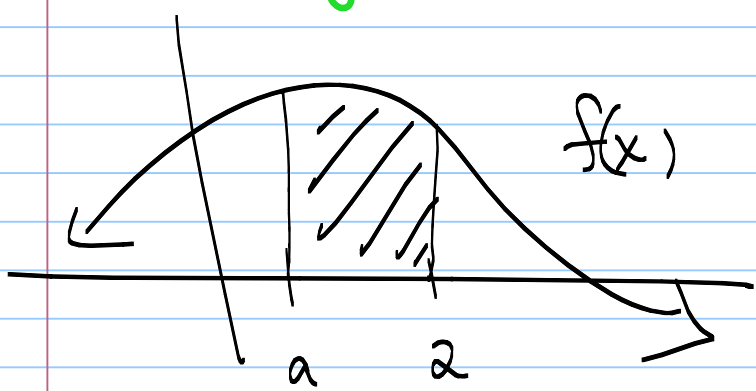
FUNDAMENTAL THM of CALCULUS. II

(PART I WAS: $\int_a^b f(x)\,dx = F(b) - F(a)$)

IF $\underbrace{g(x) = \int_a^x f(t)\,dt}_{}$, THEN $g'(x) = f(x)$.

$g(x) = F(x) - F(a)$

$g'(x) = F'(x) - 0 = f(x)$



$g(x) =$ AREA BETWEEN a AND x, UNDER $f(x)$.

$g(2) = \int_a^2 f(t)\,dt$

Let $g(x) = \int_4^{\sin x} \ln(t) - e^{t^2}\, dt$ — $0 \le x \le \pi$. FIND $g'(x)$

$$= \left(\ln(\sin x) - e^{(\sin x)^2}\right) \cos x$$

CALC II

#15

$$F(x) = \int_1^{\sin x} \ln(4t^2 - t)\, dt \quad , 0 < x < \pi.$$

$$\cos(x) \cdot \ln\left(4\sin^2 x - \sin x\right) = F'(x).$$

#20 (c)

$$\frac{-1}{2}\int_0^1 -2x e^{-x^2}\, dx = -\frac{1}{2} \int_0^{-1} e^u\, du = -\frac{1}{2}\left(e^u \Big|_0^{-1} \right)$$

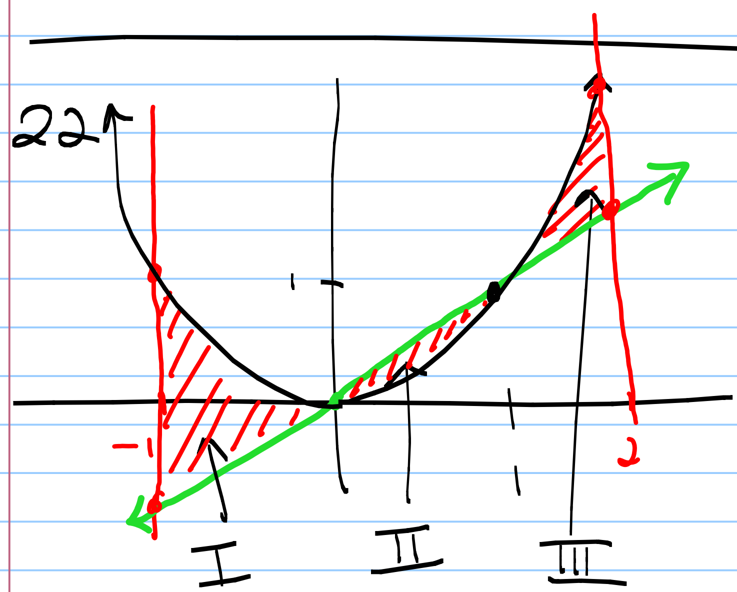
$u = -x^2$

$du = -2x\, dx$

$$= \frac{-1}{2}\left(e^{-1} - 1\right)$$

$$= -\frac{1}{2}\left(\frac{1}{e} - 1\right)$$

$$= \frac{e-1}{2e}$$



$y = x^2$
$y = x$
$x = -1$
$x = 2$

$$\mathrm{I}: \int_{-1}^{0} x^2 - x \, dx = \left(\frac{x^3}{3} - \frac{x^2}{2}\right|_{-1}^{0} = 0 - \left(\frac{-1}{3} - \frac{1}{2}\right) = \frac{5}{6}$$

$$\mathrm{II}: \int_{0}^{1} x - x^2 \, dx = \left(\frac{x^2}{2} - \frac{x^3}{3}\right|_{0}^{1} = \left(\frac{1}{2} - \frac{1}{3}\right) - 0 = \frac{1}{6}$$

$$\mathrm{III}: \int_{1}^{2} x^2 - x \, dx = \left(\frac{x^3}{3} - \frac{x^2}{2}\right|_{1}^{2} = \left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - \frac{1}{2}\right) = \frac{5}{6}$$

$$\left.\begin{matrix} \frac{5}{6} \\ + \\ \frac{1}{6} \\ + \\ \frac{5}{6} \end{matrix}\right\} = 11/6$$

mc

$$\frac{d}{dx}10^x = 10^x \cdot \ln 10$$

#6

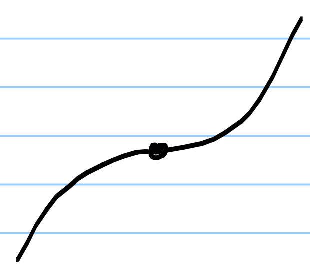
$$g'(x) = \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

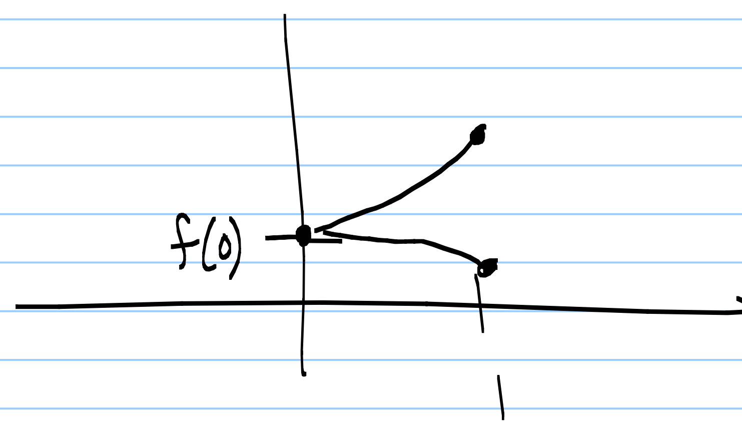
$$g'(2) = \lim_{h\to 0} \frac{g(2+h) - g(2)}{h}$$

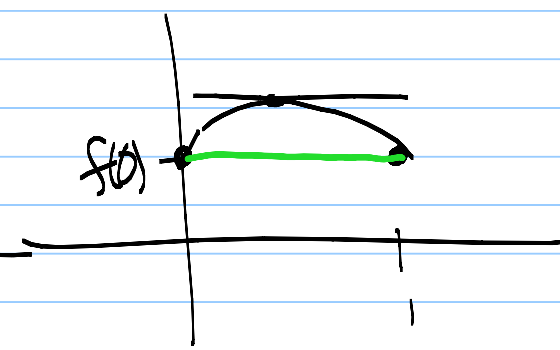
$$g(x) = x^3$$

$$g'(x) = 3x^2$$

$$g'(2) = 12$$







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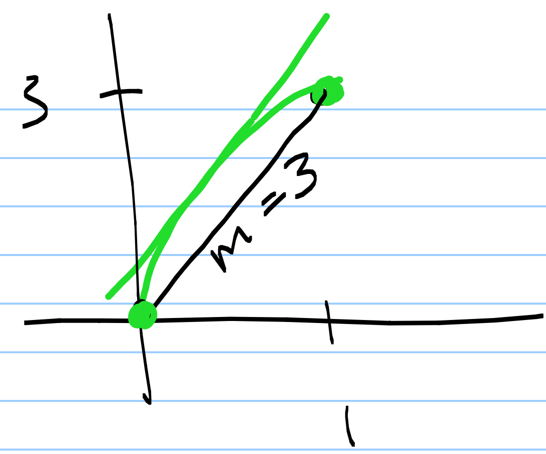


18 $v(t) = 16 - t^2$ int. from $t=0$ 'Till $t=4$

$$\int_0^4 16 - t^2\,dt = \left(16t - \frac{t^3}{3}\right)\Big|_0^4 = 64 - \frac{64}{3} = 42\tfrac{2}{3}$$

19 $$\lim_{x\to 2} \frac{x^2-4}{2-x} = \lim_{x\to 2} \frac{-(x+2)\cancel{(x-2)}}{\cancel{(2-x)}} = -4.$$

25. $$\lim_{x\to 0} \frac{\sin 2x}{5x} = \lim_{x\to 0} \frac{2\cos 2x}{5} = \frac{2}{5} = 0.4$$

$\frac{0}{0}$

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$$

SEQUENCES

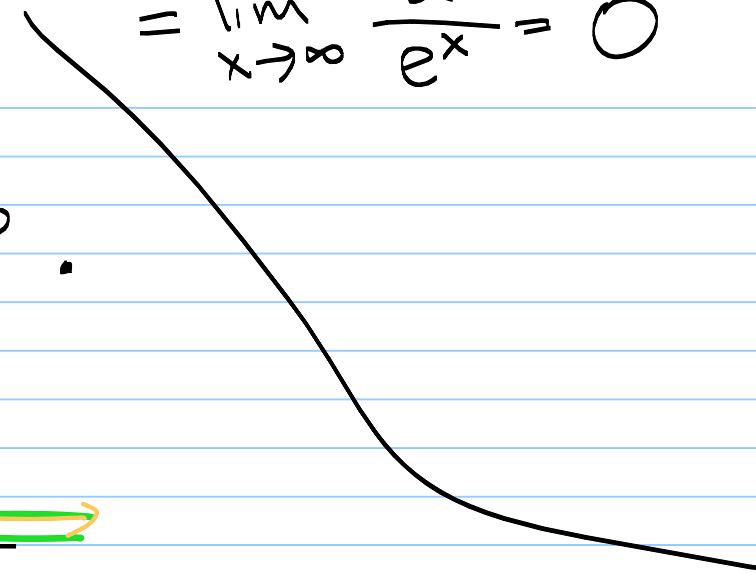
$1, 1, 2, 3, 5, 8, 13, 21, \ldots$

$a_0, a_1, a_2, \ldots$

$\frac{\infty}{\infty}$

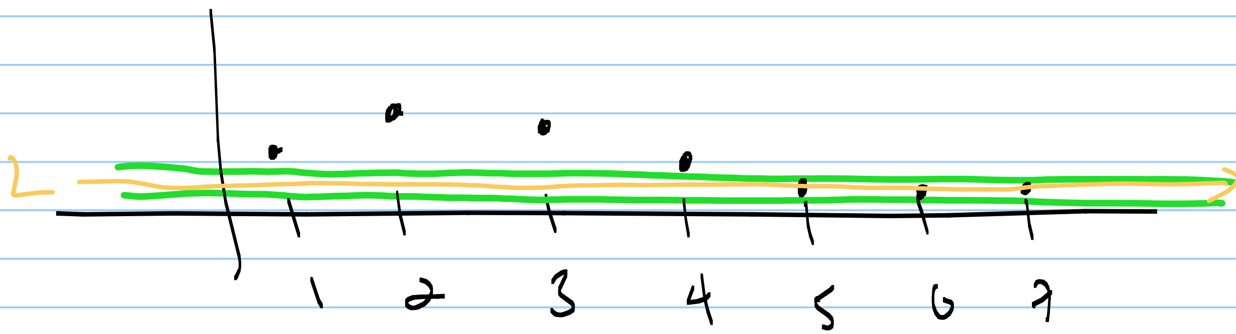
$$\left\{ a_n = \frac{n^2}{e^n} \right\}_{n=0}^{\infty} \longleftarrow \lim_{n\to\infty} \frac{n^2}{e^n} = \lim_{x\to\infty} \frac{x^2}{e^x} = \lim_{x\to\infty} \frac{2x}{e^x} =$$

$$= \lim_{x \to \infty} \frac{2}{e^x} = 0$$



A SEQUENCE CONVERGES IF

$$\lim_{n \to \infty} a_n = L < \infty .$$



HARMONIC SEQUENCE $\left\{ \frac{1}{n} \right\}_{n=1}^{\infty} = 1, \frac{1}{2}, \frac{1}{3}, \ldots$

$\lim_{x \to \infty} \frac{1}{x} = 0$ so, THE SEQUENCE

CONVERGES TO 0.

GEOMETRIC SEQ. $\{r^n\}_{n=0}^{\infty} = 1, r, r^2, r^3, \ldots$

Converge

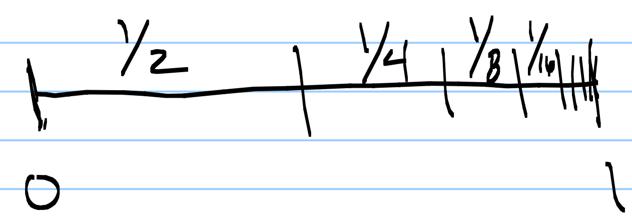
$\{2^n\}_{n=0}^{\infty} = 1, 2, 4, 8, 16, \ldots$

IF

$|r|<1$ OR $r=1$

$\left\{\left(\frac{-1}{2}\right)^n\right\}_{n=0}^{\infty} = 1, \frac{-1}{2}, \frac{1}{4}, \frac{-1}{8}, \frac{1}{16}, \ldots$

SERIES $a_0 + a_1 + a_2 + a_3 + \ldots = \sum_{n=0}^{\infty} a_n$



$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$

$$\overbrace{\underbrace{\underbrace{\underbrace{a_0}_{S_0} + a_1}_{S_1} + a_2}_{S_2} + a_3 + a_4 + \dots + a_n}^{S_n} + \dots = \sum_{n=0}^{\infty} a_n$$

A series converges if if the corresponding seq of partial sums $\{S_n\}_{n=0}^{\infty}$ converges.

Geometric Series

$$1 + r + r^2 + \dots = \sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

Converges if $|r| < 1$

$$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{2} + \frac{1}{4} + \dots$$

$$= \left(1 + \frac{1}{2} + \frac{1}{4} + \dots\right) - 1$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n - 1$$

$$= \frac{1}{1-\frac{1}{2}} - 1 = 1$$

SUM/DIFF/CONSTANT MULT RULE.

- $\displaystyle\sum_{n=0}^{\infty}(a_n \pm b_n) = \sum_{n=0}^{\infty} a_n \pm \sum_{n=0}^{\infty} b_n$, PROVIDED BOTH OF THESE CONVERGE.

- $\displaystyle\sum_{n=0}^{\infty} k\cdot a_n = k\sum_{n=0}^{\infty} a_n$

FIND THE SUM $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{2^n} - \frac{5}{3^{n+1}}\right) = \sum_{n=1}^{\infty}\frac{1}{2^n} - \sum_{n=1}^{\infty}\frac{5}{3^{n+1}}$

$$= 1 - 5\underbrace{\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^{n+1}}_{\left(1+\frac{1}{3}+\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^3+\cdots\right) - \frac{4}{3}}$$

$$= 1 - 5\left[\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n - \frac{4}{3}\right]$$

$$= 1 - 5\left[\frac{1}{1-\frac{1}{3}} - \frac{4}{3}\right] = 1 - 5\left[\frac{3}{2} - \frac{4}{3}\right]$$

$$= 1 - 5 \cdot \frac{1}{6} = \frac{1}{6}.$$

HARMONIC SERIES $\sum_{n=1}^{\infty} \frac{1}{n}$ DIVERGES.

p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$

DIVERGENCE TEST

IF $\lim_{n\to\infty} a_n \neq 0$, THEN $\sum^{\infty} a_n$ DIVERGES.

($\sum \frac{1}{n}$ IS A counterexample to the converse)

$\lim_{n\to\infty} a_n = k \neq 0.$

$a_0 + a_1 + a_2 + \ldots + k + k + k + k + k + \ldots$

$\sum_{n=1}^{\infty} k$ Diverges.

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pg 33 #26 ; pg 34 #7, 8.