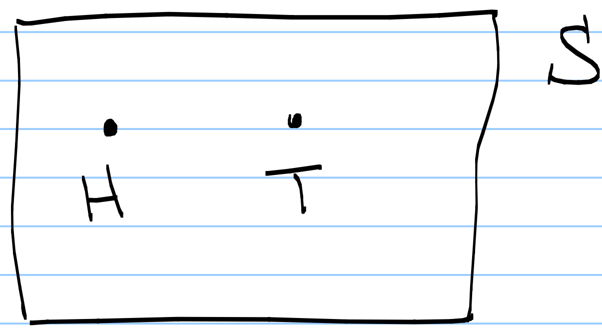
Tuesday, June 9, 2009

Note Title 6/9/2009

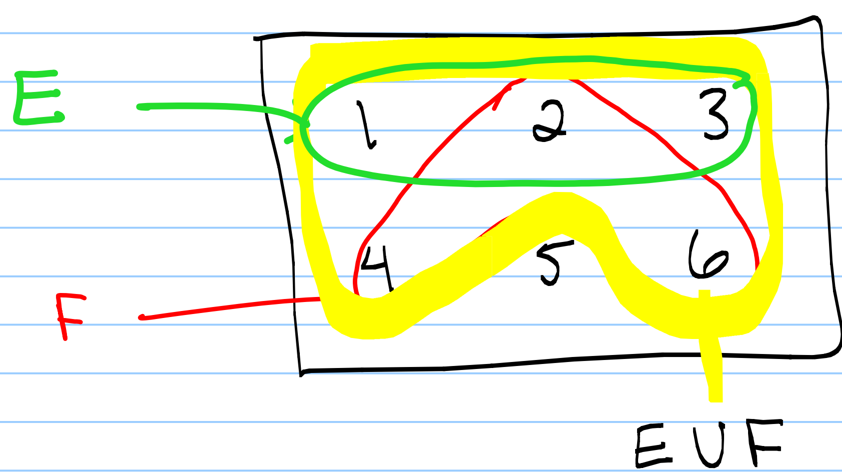
PROBABILITY THEORY.

LIST OF POSSIBLE OUTCOMES OF EXPERIMENT



FAIR COIN

$P(H) = P(T) = \frac{1}{2}$



FAIR 6-SIDED DIE

$P(n) = \frac{1}{6}$

$P(E) = P(1) + P(2) + P(3)$

AND intersection

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

MUTUALLY EXCLUSIVE EVENTS

UNION
OR

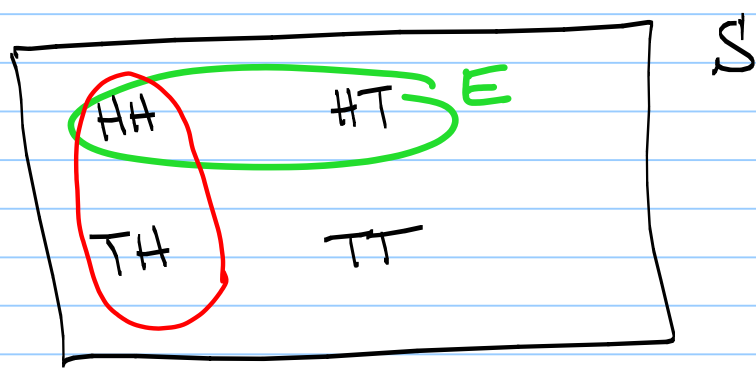
E & F ARE MEE IF $E \cap F = \emptyset$

$\overline{E} \leftarrow$ COMPLEMENT OF EVENT $\equiv$ EVERYTHING IN S THAT'S NOT IN E

E'

NOTE THAT: $P(\overline{E}) = 1 - P(E)$

Ex TOSS A COIN TWICE

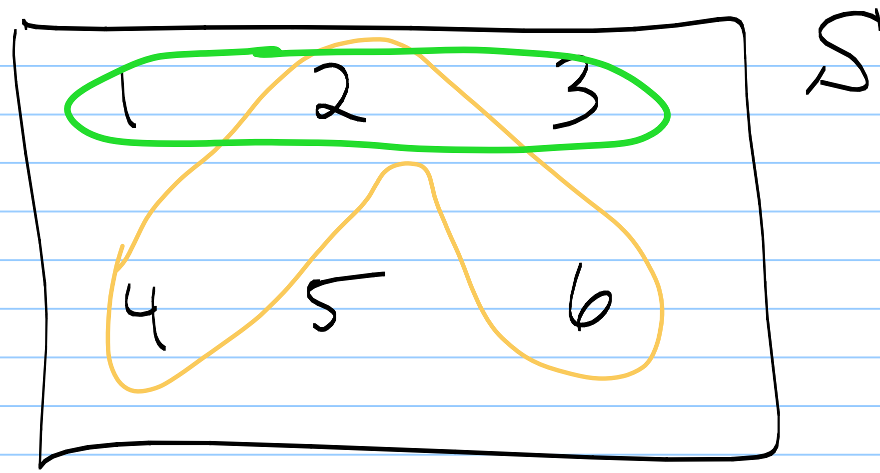


INDEPENDENT.

E – HEADS ON FIRST TOSS

F – HEADS ON SECOND TOSS

$P(E \cap F) = P(E) \cdot P(F)$.



E: roll ≤ 3

F: roll an even number

$$P(E|F) = \frac{P(2)}{P(2)+P(4)+P(6)}$$

Conditional Probability

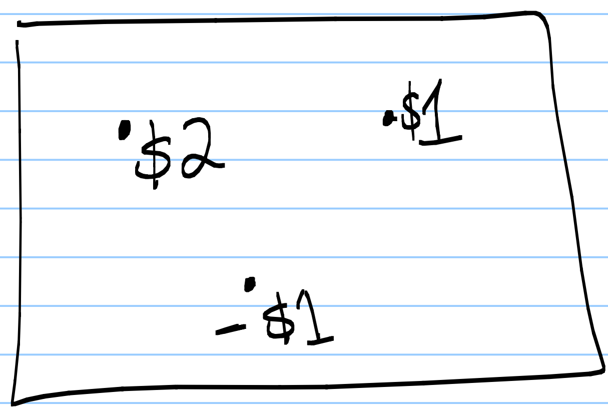
$$\underbrace{P(E|F)} = \frac{P(E \cap F)}{P(F)}$$

"THE PROB. THAT E HAPPENED GIVEN THAT F HAPPENED."

$\cap$ AND INTERSECTION

$\cup$ OR UNION

EXPECTED VALUE



DRAW A CARD OUT OF A STANDARD DECK.

WIN $2 FOR AN ACE

WIN $1 FOR K, Q, J

ANYTHING ELSE - LOSE $1.

$$E = \$2 \cdot P(\$2) + \$1\, P(\$1) + (-1) P(-\$1).$$

$$= 2 \cdot \frac{1}{13} + 1 \cdot \frac{3}{13} + -1 \cdot \frac{9}{13} = \frac{2}{13} + \frac{3}{13} - \frac{9}{13} = \frac{-4}{13} \approx -0.31$$

<u>ODDS</u> OF E HAPPENING $\equiv \dfrac{\#\text{ WAYS } E \text{ CAN HAPPEN}}{\#\text{ WAYS } \bar{E} \text{ CAN HAPPEN}}$

PROBABILITY PROBLEMS.

#2 $P(H) = 2 \cdot P(T)$

$P(H) + P(T) = 1$

$2P(T) + P(T) = 1$

$3(P(T)) = 1$

$P(T) = \frac{1}{3}$.

$P(H) = \frac{2}{3}$

$TTH \leftarrow \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{3}$

or $+$

$THT \leftarrow \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{3}$

or $+$

$HTT \leftarrow \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$

$= \frac{2}{27} + \frac{2}{27} + \frac{2}{27}$

$= \frac{6}{27} = \frac{2}{9}$

#5 $3 : (5 + 1 + 15)$

$3 : 21$

$1 : 7$

6

	Pos	Neg	
Sick	.0099	0.0001	$1\% = 0.01$
well	0.0099	.9801	$99\% = 0.99$

$$P(pos \mid well) = 0.01 = \frac{P(pos \cap well)}{P(well)}$$

$$0.0099 = 0.01 \cdot 0.99 = P(pos \cap well)$$

$$P(neg \mid sick) = 0.01 = \frac{P(neg \cap sick)}{P(sick)}$$

$$P(neg \cap sick) = 0.01 \cdot 0.01 = 0.0001$$

$$P(sick \mid pos) = \frac{P(sick \cap pos)}{P(pos)} = \frac{1}{2}$$

MATRICES

size $\equiv$ dimensions $\equiv$ rows $\times$ columns

$$3\left\{\overbrace{\begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 3 & -7 & 1 \\ 6 & 8 & 2 & 7 \end{bmatrix}}^{4}\right.$$

SQUARE IF rows = columns

3×4 matrix.

DIAGONAL MATRIX

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

IDENTITY MATRIX

↳ SQUARE MATRIX w/ 1's on DIAG, zeros elsewhere

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

ZERO MATRIX

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

MATRIX ADDITION

- TO ADD $A + B$, A & B HAVE TO HAVE THE SAME DIMENSIONS.

$$\begin{bmatrix} 2 & 3 & 1 \\ 7 & 1 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 7 & 1 \\ 0 & 6 & 3 \end{bmatrix} = \begin{bmatrix} \underline{1} & \underline{10} & \underline{2} \\ \underline{7} & \underline{7} & \underline{5} \end{bmatrix}$$

MULTIPLICATION

$$\underset{n \times m}{A} \times \underset{m \times p}{B} = \underset{n \times p}{C}$$

# of cols = # of rows

ex

$$\underset{3\times 2}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 0 & 7 \end{bmatrix}} \cdot \underset{2\times 2}{\begin{bmatrix} -1 & 6 \\ 0 & 2 \end{bmatrix}} = \underset{3\times 2}{\begin{bmatrix} -1 & 10 \\ -3 & 26 \\ 0 & 14 \end{bmatrix}}$$

$$A = \underset{3\times 3}{\begin{bmatrix} 1 & 0 & 2 \\ 4 & 6 & 3 \\ -1 & 1 & 6 \end{bmatrix}} \qquad B = \underset{3\times 1}{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}} \qquad C = \underset{1\times 3}{\begin{bmatrix} 1 & -1 & 2 \end{bmatrix}}$$

ex

$AA =$

$AB =$

$AC = DNE$

$BA = DNE$

$BB = DNE$

$$BC = \underset{3 \times 3}{\begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & 0 \\ 1 & -1 & 2 \end{bmatrix}}$$

$CC = DNE$

$$CB = \underset{1 \times 3}{\begin{bmatrix} 1 & -1 & 2 \end{bmatrix}} \underset{3 \times 1}{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}} = \underset{1 \times 1}{\begin{bmatrix} 3 \end{bmatrix}}$$

$CA =$

ex

$$\begin{bmatrix} a & 3 \\ x & 7 \end{bmatrix} + \begin{bmatrix} 2 & y \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 5 & c \end{bmatrix}$$

FIND a, y, x, AND c.

SCALAR MULTIPLICATION

$$3 \cdot \begin{bmatrix} 1 & 7 & -1 \\ 2 & 6 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 21 & -3 \\ 6 & 18 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2c & 2d \\ 2a & 2b \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 8 & 10 & 12 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix}$$

$$2 \cdot \frac{1}{2} = 1$$

MULTIPLICATIVE INVERSES

identity for $\cdot$

- NOT ALL MATRICES HAVE AN INVERSE.
- ENTER MATRIX — "-1" "INV" — ERROR — NO INVERSE / INVERSE

$$A \cdot A^{-1} = I_n = A^{-1} \cdot A$$

SQUARE MATRIX

$n \times n$

- FOR 2×2 MATRICES ONLY

Determinant of matrix $= 0 \Rightarrow$ NO INVERSE

└ EASY TO CALC. FOR A 2×2.

$$\det\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right) = 1\cdot 4 - 2\cdot 3 = 4-6 = -2 \neq 0$$

so this matrix has an inverse

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$$

$$A\cdot A^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

TRY p43 #2, 5, 6, 7

<u>AND</u> PRACTICE TEST <u>II</u>. FROM CLIFF'S.