Note Title Wednesday, June 10, 2009 6/10/2009

MATRICES, PART II.

LAST TIME:

MATRIX ADDITION — EXACTLY THE SAME $r \times c$.
SUBTRACTIONS

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix}$$

SCALAR MULTIPLICATION —

$$3 \begin{bmatrix} 1 & 2 \\ 0 & 7 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 0 & 21 \\ 15 & 18 \end{bmatrix}$$

MATRIX MULTIPLICATION –

$$\underset{3\times 2}{\begin{bmatrix} 2 & 3 \\ 7 & 1 \\ 5 & 6 \end{bmatrix}} \cdot \underset{2\times 1}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}} = \underset{3\times 1}{\begin{bmatrix} 8 \\ 9 \\ 17 \end{bmatrix}}$$

MATRIX INVERSION

$$\underbrace{A \cdot B^{-1}}$$ ← INVERSE of B

THE CLOSEST WE GET TO $A \div B$.

CAN'T ALWAYS DO THIS!

NOT EVERY MATRIX HAS AN INVERSE.

B^{-1} WILL EXIST EXACTLY WHEN $\det B \neq 0$.

IF B IS NOT A 2x2, USE YOUR CALCULATOR FOR B^{-1} & $\det B$.

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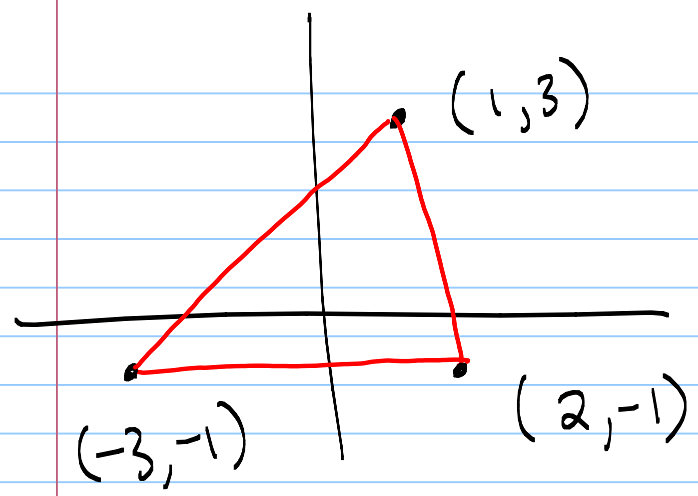
* FOR A 2x2, THERE'S A FAST WAY

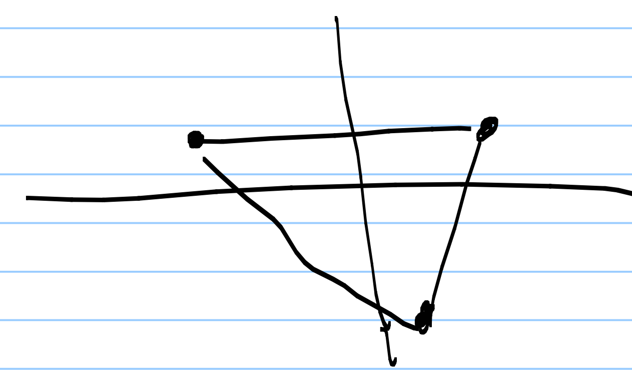
$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc, \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det \begin{bmatrix} a & b \\ c & d \end{bmatrix}} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

GEOMETRIC TRANSFORMATIONS ON THE PLANE



Reflect across x-AXIS

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 & 2 \\ 3 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & +1 & +1 \end{bmatrix}$$



STRETCH BY 2
& FLIP OVER y-AXIS

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \leftarrow \text{STRETCH BY 2}$$

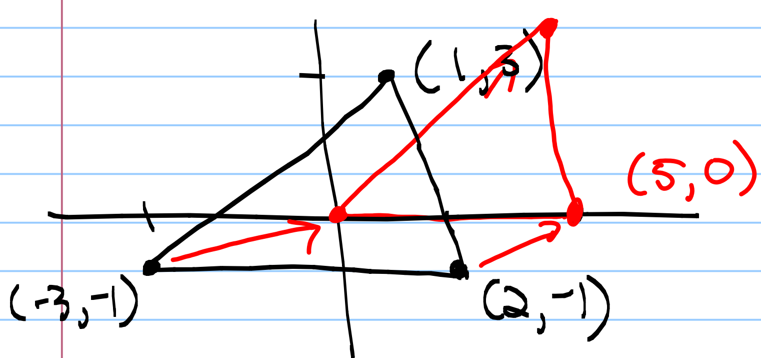
$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \leftarrow \text{FLIP OVER y-AXIS}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$$ $\leftarrow$ STRETCH BY 2 & FLIP OVER y-AXIS

REFLECTION ACROSS $y = x$.

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -3 & 2 \\ 3 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & -1 \\ 1 & -3 & 2 \end{bmatrix}$$

HOW CAN MOVE THE TRIANGLE



$$\begin{bmatrix} 3 & 3 & 3 \\ 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -3 & 2 \\ 3 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 5 \\ 4 & 0 & 0 \end{bmatrix}$$

Solving Linear Systems

$$3x + 2y = 7$$
$$x - y = 3$$

$$4x - 3y + 6z + 3w = 8$$
$$-1x + 3z - 2w = 9$$
$$x + y - z + w = 10$$
$$x + y - z - w = 8$$

$$\begin{array}{cccc} x & y & z & w \end{array}$$
$$\begin{bmatrix} 4 & -3 & 6 & 3 \\ -1 & 0 & 3 & -2 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \\ 10 \\ 8 \end{bmatrix}$$

$$A \quad x = b$$

Column vectors

$n \times 1$

$$AX = b$$

$$A^{-1}AX = A^{-1}b$$

$$X = A^{-1}b$$

$$X = \underbrace{\begin{bmatrix} 1/8 & -1/8 & -1/8 & 1/2 \\ -1/12 & 5/12 & 11/12 & -1/6 \\ 1/24 & 7/24 & 7/24 & -1/6 \\ 0 & 0 & 1/2 & -1/2 \end{bmatrix}}_{A^{-1}} \cdot \underbrace{\begin{bmatrix} 8 \\ 9 \\ 10 \\ 9 \end{bmatrix}}_{b}$$

$$X = \begin{bmatrix} 21/8 \\ 131/12 \\ 109/24 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

$x = 21/8$, $y = 131/12$

$z = 109/24$, $w = 1$

MATRIX PROBLEMS

2

$$\begin{pmatrix} 5 & 1.5 \\ 2 & d \end{pmatrix}$$

$$5d - 3 = 0$$

PROOFS

1. x AND y ARE > 0 REAL #'s $x \neq y$, THEN

$$\frac{x}{y} + \frac{y}{x} > 2.$$

#1 $\frac{x}{y} + \frac{y}{x} > 2$

Because $\frac{x^2}{y} + y > 2x$,

so $x^2 + y^2 > 2xy$,

and $x^2 - 2xy + y^2 > 0$,

and $(x-y)^2 > 0$.

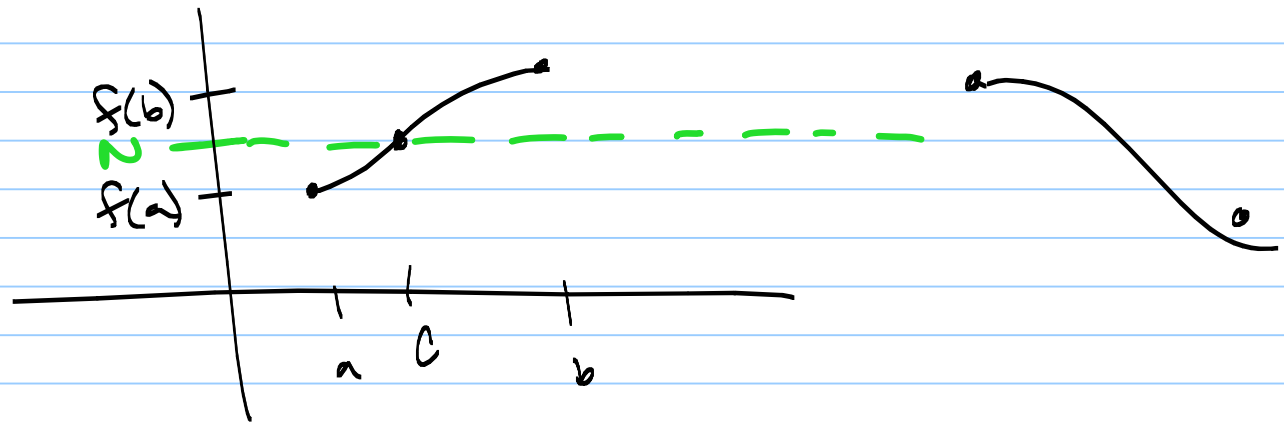
Since $x \neq y$, $x - y \neq 0$, HENCE $(x-y)^2 > 0$.

#2 (a) " IF $a+b=c$ THE $(a+b)^2 = c^2$."

Converse: IF $(a+b)^2 = c^2$ THEN $a+b=c$.

(b) $a=-1$, $b=0$, $c=1$

#3 INTERMEDIATE VALUE THM



SHOW (USING IVT) THAT $f(x) = x^3 + x - 1$ HAS A REAL ROOT, i.e., the graph of $f(x)$ CROSSES THE x-AXIS.

JUST NEED TO FIND a & b so that EXACTLY ONE OF $f(a)$ & $f(b)$ IS NEGATIVE.

IF WE LET $a=0$, $b=1$ THEN

$f(a)=f(0)=-1$ AND $f(b)=f(1)=1$.

Since $f(0)<0$ AND $f(1)>0$, AND $f(x)$ IS continuous everywhere (it's a polynomial), BY IVT THERE IS A c BETWEEN 0 AND 1 SO THAT $f(c)=0$.

#4 $a, b, \& c$ ARE INTEGERS AND $a \mid b$, THEN $a \mid bc$.

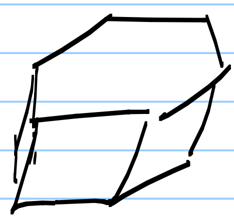
~~Pf~~ $a \mid b$, and $b \mid bc$, so BY TRANSITIVITY, $a \mid bc$.

TRIG IDENTITY

$\dfrac{\cos(2A)}{\quad} = \dfrac{\quad}{\quad}$

$\cos\theta \csc\theta =$

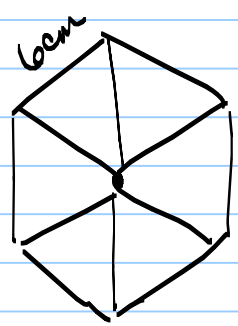
47

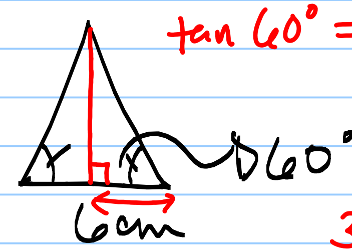


(AREA OF HEXAGON) HEIGHT = VOLUME

$\left(\underbrace{6 \cdot \frac{1}{2} \cdot 6 \cdot 3\sqrt{3}}_{\text{AREA OF } 1\,\Delta}\right) 30\text{cm} = 54\sqrt{3} \cdot 30$

$= 1620\sqrt{3} \approx 2805.9$





$\tan 60° = \dfrac{\text{height}}{3}$

$3\sqrt{3} = h$

$120° =$ measure of an interior ∠ of reg. hex.

$$1 - \left(P\left(\begin{matrix}\text{none share} \\ \text{a birthday}\end{matrix}\right)\right)$$

$$P\left(\begin{matrix}\text{AT LEAST} \\ \text{2 SHARE}\end{matrix}\right) = 1 - \frac{365 \cdot 364 \cdot 363}{365 \cdot 365 \cdot 365}$$

$\leftarrow$ NUMBER OF OUTCOMES IN THE EVENT "NONE SHARE..."

$\leftarrow$ SIZE OF SAMPLE SPACE

A B C

$$P(A=B) = \frac{1}{365}$$

$$P(B=C) = \frac{1}{365}$$

$$P(A=C) = \frac{1}{365}$$

$$P(A=B=C) = \frac{1}{365^2}$$

$$P(A=B \cup B=C \cup A=C) = \frac{1}{365} + \frac{1}{365} + \frac{1}{365} - \overset{\text{overlap}}{\frac{2}{365^2}}$$

#42 (p 122)

T	Pop
0	1000
10	5000

$$\underline{1000}$$

$$Q(t) = Q_0 e^{xt}$$

$$Q(0) = Q_0 e^{x \cdot 0} = Q_0 = 1000$$

$$Q(t) = 1000 e^{xt}$$

$$Q(10) = 5000 = 1000 e^{x \cdot 10}$$

$$5 = e^{10x}$$

$$\ln 5 = 10x$$

$$\frac{\ln 5}{10} = x$$