Thursday, June 11, 2009

Note Title 6/11/2009

14

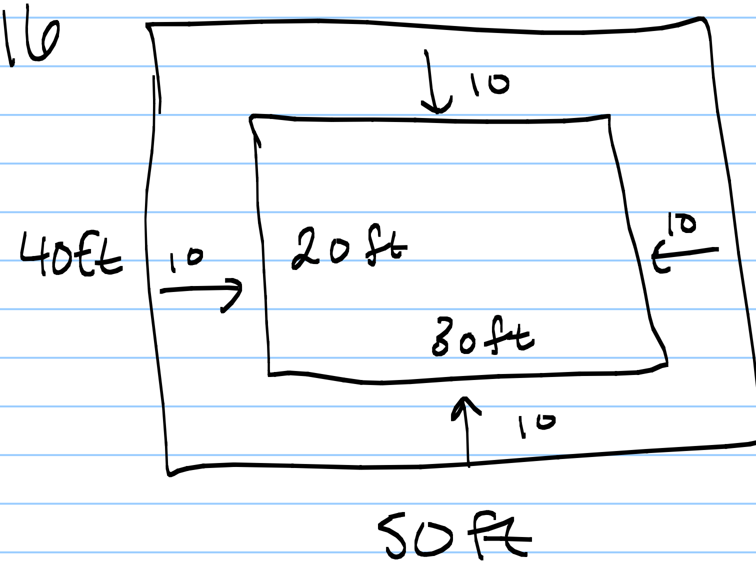
$$\frac{10^7}{10^6} = 10$$

$$\frac{10^{6.8}}{10^{6.6}} = x$$

$$10^{0.2} = x$$

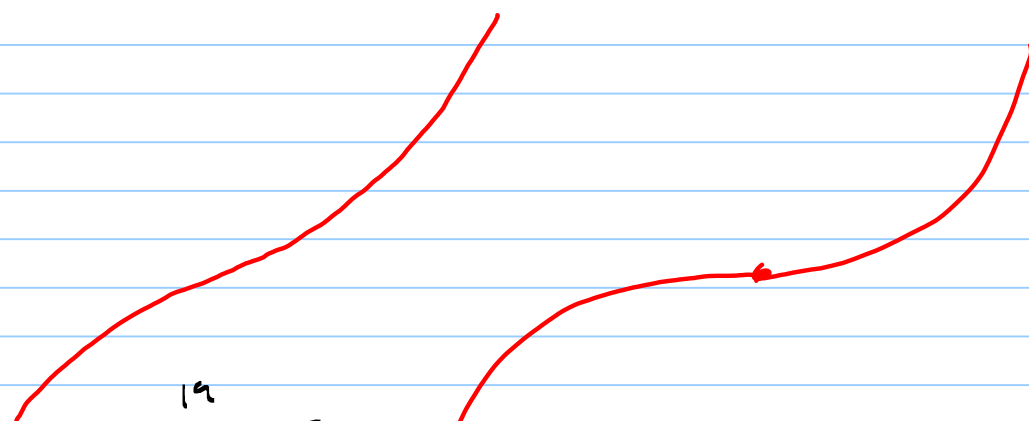
$$= 1.58 \rightarrow 158\%$$

16



$$\frac{20 \times 30}{40 \times 50} = \frac{600}{2000}$$

17



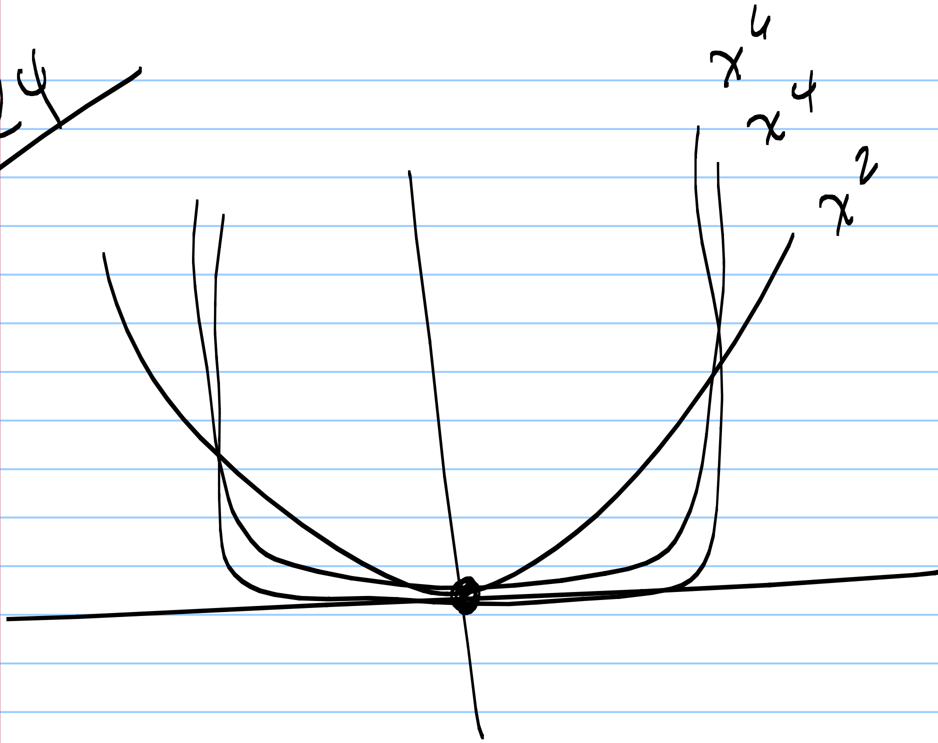
19
13 14 15 16 17 18
7 8 9 10 11 12
1 2 3 4 5 6

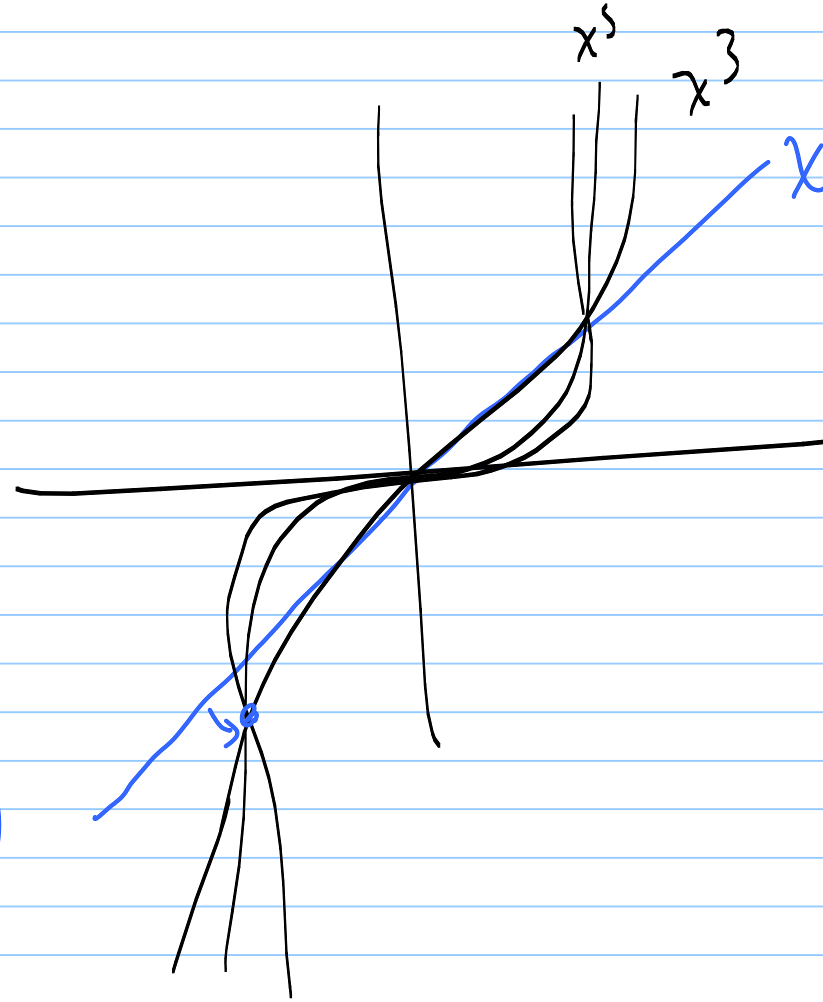
20

$\frac{3}{7} = 0,\overline{428571}$

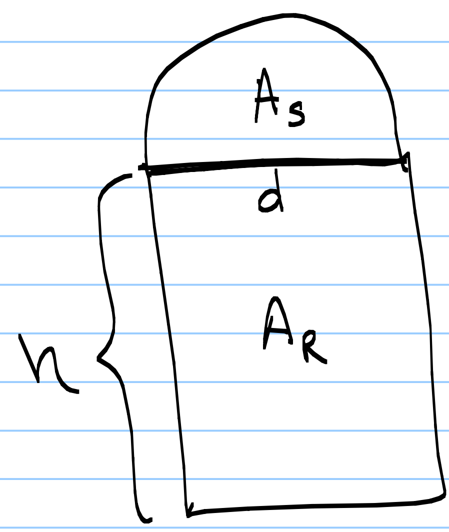
$$
\begin{array}{r}
.42\ldots \\
7\,\overline{)\,3.00} \\
\underline{28} \\
20 \\
\underline{14} \\
6\ldots
\end{array}
$$

0,428571428957...


x^4
x^4
x^2


x^5
x^3
x^1

25



$$A = A_S + A_R \quad ; \quad A_S = \frac{1}{2}\pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{8} \quad A_R = hd$$

$$A_S = \frac{1}{3}A = \frac{1}{3}(A_S + A_R)$$

$$\boxed{\text{FIND } \frac{h}{d}}$$

$$\frac{\pi d^2}{8} = \frac{1}{3}\left(\frac{\pi d^2}{8} + hd\right)$$

$$\frac{\pi d^2}{8} = \frac{\pi d^2}{24} + \frac{hd}{3}$$

divide by d^2

$$\frac{\pi}{8} = \frac{\pi}{24} + \frac{h\cancel{d}}{3d^{\cancel{2}}}$$

$$3\left(\frac{\pi}{8} - \frac{\pi}{24}\right) = \frac{h}{d}$$

$$\frac{3\pi}{8} - \frac{\pi}{8} = \frac{h}{d}$$

$$\frac{\pi}{4} = \frac{2\pi}{8} = \frac{h}{d}.$$

26

$$\begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 2\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x - y \\ x + 3y \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix} \leadsto \begin{array}{l} x - y = 2x \\ x + 3y = 2y \end{array}$$

$$-x - y = 0 \leadsto x = -y$$

$$x + y = 0 \leadsto y = -x$$

27

$$A(0) = 100$$

$$A(\underbrace{n+1}_{n}) = A(n) + 0.08A(n) + 50$$

$$= 1.08\,A(\underset{n-1}{n}) + 50$$

28

$\times$	a	b
a	b	a
b	a	a

commutative

$$a \cdot (a \cdot b) = (a \cdot a) \cdot b$$

$$a \cdot a = \boxed{b} \checkmark \qquad b \cdot b = \boxed{a} \checkmark$$

NOT ASSOCIATIVE.

29/

$x^2 + 2x + 1 + y^2 = 0 + 1$

$(x-h)^2 + (y-k)^2 = r^2$

$(x+1)^2 + y^2 = 1$

Review of Completing the square

$x^2 + 6x + 9 - 9 \rightarrow (x+3)^2 - 9$

3

1

9

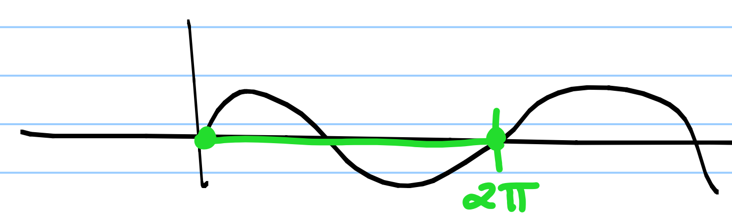
$x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

$\frac{1}{2}$ of this coefficient

square it

add it & subtract it

30/ $y = \frac{1}{3}\sin\left(\frac{1}{2}x + \frac{\pi}{3}\right)$



33

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} 0 & - \\ - & - \end{bmatrix}$$

$ae + bg$

$a = 1$
$e = 1$

$b = 1$

$g = -1$

$$\begin{bmatrix} \ddots & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \ddots \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

34

Given sequence: $\underset{n=1}{1}, \underset{n=2}{3}, \underset{n=3}{6}, \underset{n=4}{10}$

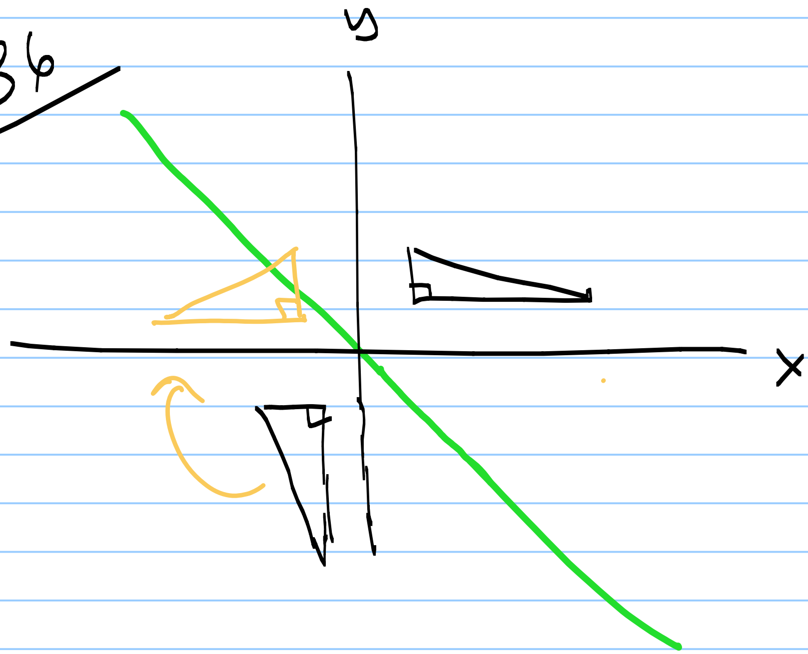
a) 0 ✗

b) $1, \frac{3}{2}$ ✗

✓ c) $1, \frac{6}{2} = 3, 6$?

d) $1, 4-3=1$ ✗

36



D

37

$(0°T, -24°C)$

$(5°T, -16°C)$

$m = \frac{8}{5}$

$y + 16 = \frac{8}{5}(x - 5)$

$80 = \frac{8}{5}x - 24$

$y = \frac{8}{5}x - 8 - 16$

$65°T = 104 \cdot \frac{5}{8} = x$

$P(1) = 2p = \frac{2}{7}$

$P(2) = P(3) = \dots = P(6) = p = \frac{1}{7}$

39

$$\underbrace{5p}_{2,3,4,5,6} + \underbrace{2p}_{1} = 1 \rightsquigarrow p = \frac{1}{7}$$

$$\left.\begin{aligned} P(1,3) &= \frac{2}{7} \cdot \frac{1}{7} \\ P(3,1) &= \frac{1}{7} \cdot \frac{2}{7} \\ P(2,2) &= \frac{1}{7} \cdot \frac{1}{7} \end{aligned}\right\} \frac{2}{49} + \frac{2}{49} + \frac{1}{49} = \frac{5}{49}.$$

$f(x) = x$

40

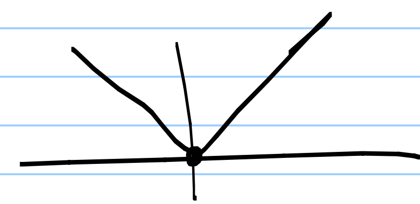
$$y = f(x) \cdot g(x) = \begin{cases} -x & x < 0 \\ 0 & x = 0 \\ x & x > 0 \end{cases}$$

$f \cdot g(x) = f(x) \cdot g(x)$

$f \circ g(x) = f(g(x))$

$g(f(x))$

$y = |x|$



$|f(x)| = |x|$

a) $f(x) = x \neq |x|$

b) $f(-x) = -x \neq |x|$

c) $-|f(x)| = -|x|$

d) $f(|x|) = |x|$ ✓

STRUCTURED RESPONSES

1) $\otimes$ given

a) $\underbrace{(d \otimes a)} \otimes c$

$= b \otimes c = b.$

b) THE $\otimes$ PRODUCT OF ANY PAIR $x, y \in \mathbb{X}$ IS ALSO AN ELEMENT OF $\mathbb{X}$. THIS CLEARLY THE CASE SINCE EACH ENTRY IN THE MULTIPLICATION TABLE FOR $\otimes$ IS AN ELEMENT $\mathbb{X}$.

c) $\otimes$ IS COMMUTATIVE SINCE THE TABLE IS SYMMETRIC ACROSS THE MAIN DIAGONAL.

$\otimes$ IS COMMU'TIVE SINCE FOR ANY x & $y \in \mathbb{X}$, $x \otimes y = y \otimes x$.

d) C IS AN IDENTITY ON $\underline{\overline{X}}$ FOR $\otimes$ BECAUSE FOR ANY $x \in X$, $c \otimes x = x \otimes c = x$.

e) SINCE C ACTS AS AN IDENTITY FOR $\otimes$, THE $x \in \underline{\overline{X}}$ HAS AN INVERSE, x^{-1}, IF THERE IS $x^{-1} \in \underline{\overline{X}}$ SO THAT

$$x \otimes x^{-1} = x^{-1} \otimes x = c$$

identity

$a^{-1} = a$ $\qquad c^{-1} = c$

$b^{-1} = d$ $\qquad d^{-1} = b$

---

(2) arith mean $= \frac{a+b}{2}$ $\qquad$ geom mean $= \sqrt{ab}$

(a) (i) 2, 4 $\qquad$ (ii) 1, 9

(iii) When is $\sqrt{ab} = \frac{a+b}{2}$? Show your work.

$$\sqrt{ab} = \frac{a+b}{2}$$

$$ab = \left(\frac{a+b}{2}\right)^2$$

$$4ab = a^2 + 2ab + b^2$$

$$0 = a^2 - 2ab + b^2$$

$$0 = (a-b)^2$$

$$0 = a - b \longrightarrow a = b$$

$\sqrt{ab} = \frac{a+b}{2}$ WHEN $a=b$. work.

(b) $\sqrt{ab} \le \frac{a+b}{2}$ since

$4ab \le a^2 + 2ab + b^2$, OR SUBT. $4ab$,

$0 \le a^2 - 2ab + b^2$, FACTORING GIVES

$0 \le (a-b)^2$, WHICH IS

TRUE SINCE a AND b are real numbers.

PROOF. $(a-b)^2 \ge 0$.

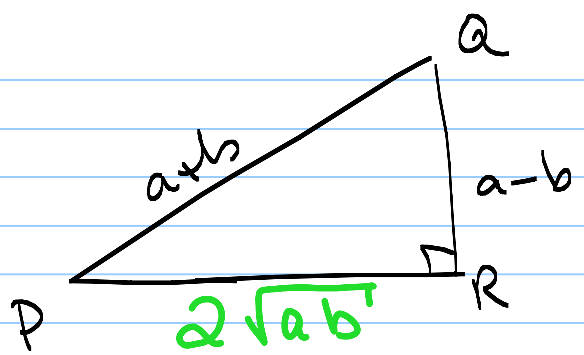
SO $0 \le a^2 - 2ab + b^2$

SO ADDING $4ab$ TO BOTH SIDES yields

$$4ab \le a^2 + 2ab + b^2$$

or $4ab \le (a+b)^2$.

(c)



So then

$$ab \le \frac{(a+b)^2}{4}$$

And thus

$$\sqrt{ab} \le \frac{a+b}{2}$$

$$|PR| = \sqrt{(a+b)^2 - (a-b)^2}$$
$$= \sqrt{4ab} = 2\sqrt{ab}$$

SINCE PQ IS THE HYPOTENUSE OF THE RIGHT TRIANGLE PRQ

$|PQ| > |PR|$. So $a+b > 2\sqrt{ab}$, or dividing by 2

$$\frac{a+b}{2} > \sqrt{ab}.$$

(3)

	Pos	NEG	
SICK	0.0049	0.0001	$0.005 = P(\text{sick})$
WELL	0.0199	0.9751	$0.995 = P(\text{well})$

$$P(\text{pos} \mid \text{sick}) = 0.98 = \frac{P(\text{pos} \cap \text{sick})}{P(\text{sick})}$$

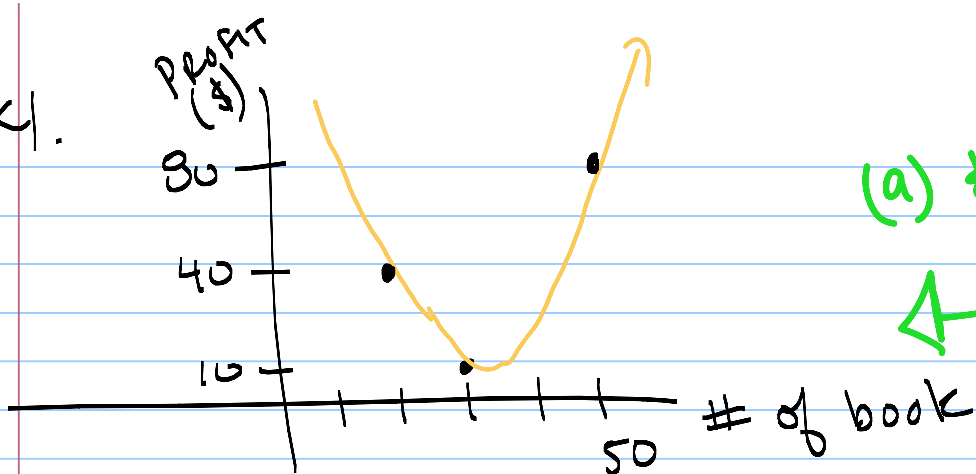
$$P(\text{neg} \mid \text{well}) = 0.98 = \frac{P(\text{neg} \cap \text{well})}{P(\text{well})}$$

a) $500 \; (= .005 \cdot 100{,}000)$

b) $1990 \; (= 0.0199 \cdot 100{,}000)$

c) $2480 \; (= (0.0049 + 0.0199) \cdot 100{,}000)$

4.



(a) & (b)

(c)

$$80 = 2500a + 50b + c$$

$$10 = 900a + 30b + c$$

$$40 = 400a + 20b + c$$

(d)

$$\underbrace{\begin{bmatrix} 80 \\ 10 \\ 40 \end{bmatrix}}_{B} = \underbrace{\begin{bmatrix} 2500 & 50 & 1 \\ 900 & 30 & 1 \\ 400 & 20 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a \\ b \\ c \end{bmatrix}}_{X}$$

$$AX = B$$

$$X = A^{-1}B$$

rref

$$\begin{bmatrix} A & B \end{bmatrix}$$

(e)

$a = \frac{13}{60}$ $\quad b = \frac{-83}{6}$ $\quad c = 230$

$$y = \frac{13}{60}x^2 + \frac{-83}{6}x + 230$$

DISCRETE MATH

COMBINATORICS

1. FUNDAMENTAL COUNTING PRINCIPLE.

IF THERE ARE n WAYS TO DO THING A AND m WAYS TO DO THING B, THEN THE NUMBERS OF WAYS TO DO BOTH A AND B IS

$n \cdot m$.

2. ADDITION PRINCIPLE

IF THERE ARE n WAYS TO DO THING A AND

m WAYS TO DO THING B, THEN THE NUMBER OF WAYS TO DO A OR B IS
$n + m$.

Permutations – ORDER MATTERS –

$_nP_r$ = # OF WAYS TO SELECT A SEQUENCE OF r THINGS FROM n THINGS.

ORDER MATTERS

$$= n\cdot(n-1)\cdot(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$

$$\boxed{\begin{array}{l} n! = n(n-1)(n-2)\cdots 2\cdot 1 \\ 5! = 5\cdot 4\cdot 3\cdot 2\cdot 1 \end{array}}$$

$$\underset{1}{\underline{n}} \cdot \underset{2}{\underline{n-1}} \cdot \underset{3}{\underline{n-2}} \cdot \underset{4}{\underline{\cdots}} \cdot \underset{5}{\underline{}} \cdots\cdots \cdot \underset{r}{\underline{n-r+1}}$$

$$\frac{n\cdot(n-1)\cdot\ldots\cdot(n-r+1)\cdot\cancel{(n-r)}\cdot\cancel{(n-r-1)}\cdots\cancel{2}\cdot\cancel{1}}{\cancel{(n-r)}\cancel{(n-r-1)}\cdots\cancel{2}\cdot\cancel{1}}$$

COMBINATIONS

ORDER DOESN'T MATTER

$$\underbrace{{}_nC_r = \binom{n}{r}}_{\text{"n choose r"}} = \text{\# OF WAYS TO CHOOSE AN } \underline{\text{SET}} \text{ OF } r \text{ THINGS FROM A SET OF } n \text{ THINGS}$$

y z x

x y z

$\underline{n}$ $\underline{(n-1)}$ $\underline{(n-2)}$ ... $\underline{(n-r+1)}$

NUM OF PERM. OF r things from n

THEN DIVIDE BY # OF ARRANGEMENTS OF THEM.

$${}_rP_r = \frac{r!}{(r-r)!} = r!$$

$${}_nC_r = \frac{\frac{n!}{(n-r)!}}{r!} = \frac{n!}{r!\,(n-r)!}$$

Ex 1; MC 2, 3, 4

#4

$$\left(\begin{array}{c}\text{distinct letters}\\ \underline{26}\cdot\underline{25}\cdot\underline{24}\end{array}\right)\cdot\left(\begin{array}{c}\text{4-numbers}\\ \underline{10}\cdot\underline{10}\cdot\underline{10}\cdot\underline{10}\end{array}\right)$$

(d) $\quad {}_{26}P_3 \cdot 10^4$