Note Title

June 11, 2009

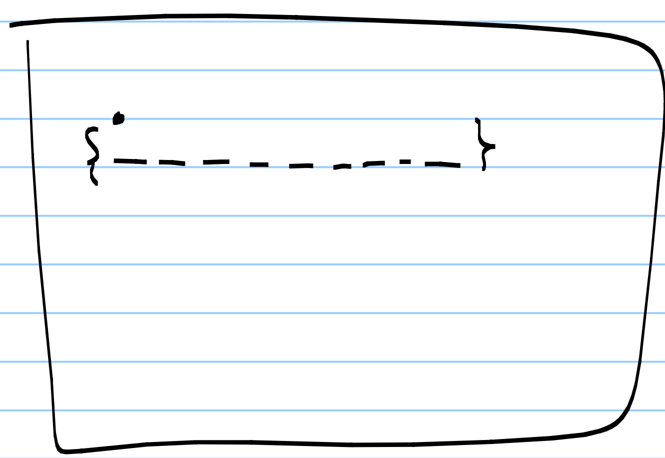
6/11/2009

DISCRETE MATH

Ex 1

$$\frac{\binom{13}{10}\cdot\binom{39}{3}}{\binom{52}{13}} = \frac{{}_{13}C_{10}\cdot{}_{39}C_{3}}{{}_{52}C_{13}}$$

$$= \frac{11\cdot 13\cdot 19\cdot 37}{50\cdot 49\cdot 47\cdot 46\cdot 17\cdot 43\cdot 41\cdot 2}$$



S

2/

$\overline{10} \cdot \overline{10} \cdot \overline{26} \cdot \overline{26} \cdot \overline{26} \cdot \overline{26}$

3/

$_{200}C_5 = \binom{200}{5}$

NO ORDER.

SEQUENCES — EXPLICIT — function of n.

RECURSIVE — where to start $a_0 = k$

& how to keep going. $a_{n+1} =$ function of $a_n, a_{n-1}, \ldots$

FIBONACCI SEQUENCE

$a_0 = 1$

$a_1 = 1$

$a_{n+1} = a_n + a_{n-1}$

$a_2 = a_1 + a_0$

$a_3 = a_2 + a_1$

$n=2$

$\overset{n=0}{1}, \overset{1}{1}, \overset{2}{2}, \overset{a_3}{3}, 5, 8, 13, 21, \ldots$

$\frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \ldots$

$\sigma = \frac{\sqrt{5}+1}{2} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$.

golden mean

golden ratio

ARITHMETIC

$2, 5, 8, 11, 14, 17, 20, 23, 26, 29, \ldots$

+3 +3 +3

WHAT IS a_{100}?

$a_0 = 2$

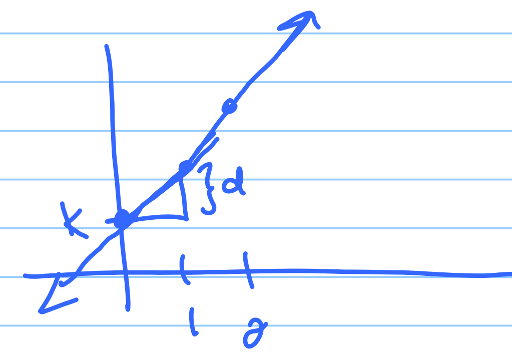
$a_{n+1} = a_n + 3$

$a_{100} = 2 + \underbrace{3 + \ldots + 3}_{100 \text{ times}} = 2 + 3 \cdot 100$

$a_n = 2 + 3n$

ARITHMETIC SEQUENCE   LINEAR GROWTH

$k \xrightarrow{+d} k+d \xrightarrow{+d} k+2d, \ldots$



Recursive

$a_0 = k$

$a_{n+1} = a_n + d$

Explicit

$a_n = k + dn$

GEOMETRIC SEQUENCE

$2 \xrightarrow{\times 5} 10 \xrightarrow{\times 5} 50 \xrightarrow{\times 5} 250, 1250, \ldots$

WHAT IS a_{100}?

$2 \cdot \underbrace{5 \cdot 5 \cdots \cdot 5}_{100} = 2 \cdot 5^{100}$

Recursive

$a_0 = 2$

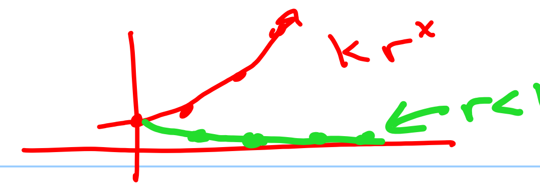
$a_{n+1} = 5a_n$

EXPLICIT

$a_n = 2 \cdot 5^n$

GEOMETRIC SEQUENCE

EXPONENTIAL GROWTH



$k, k\cdot r, k\cdot r\cdot r, \ldots$

RECURSIVE

$a_0 = k$

$a_{n+1} = a_n \cdot r$

EXPLICIT

$a_n = k \cdot r^n$

$$S_n = \underbrace{k + k\cdot r + kr^2 + \ldots + kr^{n-1}}_{\text{sum of first } n \text{ terms}} = \frac{k(1-r^n)}{1-r}, \; r \neq 1$$

PROB #8 ARE THOSE ARITHMETIC / GEOMETRIC / NEITHER?

New Exercise Derive a formula for the sum of the first n terms of arithmetic sequence $k, k+d, k+2d, \ldots, k+d(n-1)$.

__Ex__ # 2, 3,

__MC__ # 1, 5, 8 (see above).

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2. $7, 13, 19, 25, 31, \ldots$

$a_0 = 7$

$a_{n+1} = a_n + 6$

$a_n = 7 + 6n$

$a_0 + a_1 + a_2 + \ldots + a_{n-1}$

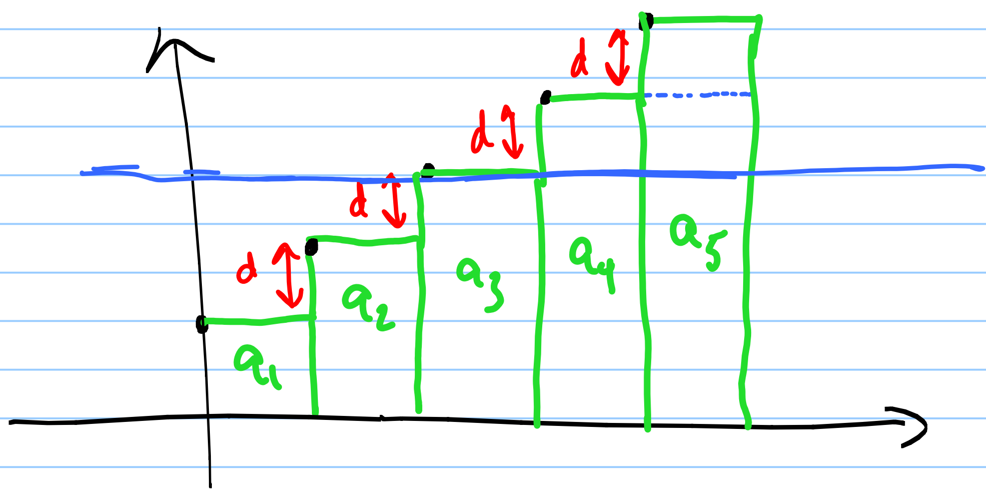
$S_n = (k) + (k+d) + (k+2d) + \ldots + (k + (n-1)d)$

$= nk + d + 2d + 3d + \ldots (n-1)d$

$$= nk + d\,\underbrace{\left(1+2+3+\dots+(n-1)\right)}_{\text{sum of first } n-1 \text{ integers}}$$

$$\frac{(n-1)(n-1+1)}{2} = \frac{(n-1)n}{2}$$

$$S_n = nk + \frac{d\,n(n-1)}{2} = n\left(k + \frac{d(n-1)}{2}\right)$$



$$= \left(\frac{a_1 + a_5}{2}\right)$$

$$\left(\frac{a_1+a_5}{2}\right)\cdot 5$$

add n CONSECUTIVE TERMS OF AN ARITHMETIC SEQ.

MULT n BY avg of 1st & LAST TERMS.

$$S_{37} = \left(\frac{\overbrace{7}^{a_0} + \overbrace{(7 + 6 \cdot 36)}^{a_{36}}}{2} \right) \cdot 37$$

#3 $r = 3$

$a_0 = 4$

$a_{n+1} = a_n \cdot 3$

$a_n = 4 \cdot 3^n$

$$S_{37} = \frac{4 \cdot (1 - 3^{37})}{1 - 3}$$

a_1	a_2	a_3
4	12	36

$a_n = 4 \cdot 3^{n-1}$

MULTIPLE CHOICE

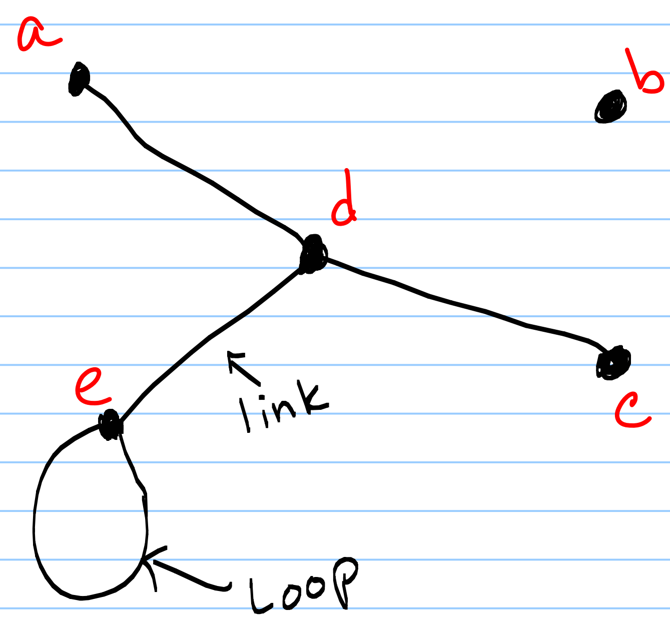
$a_6 = 4$

$a_7 = 5$

$a_n = a_{n-1} + a_{n-2}$

$a_{n+6} = a_{n+5} + d$

GRAPH THEORY



GRAPH
- vertices
- edges

<u>degree</u> of a vertex is # of edges incident to the vertex.

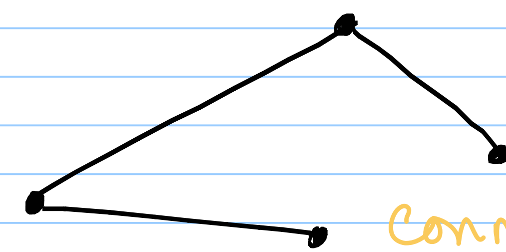
ex $\deg(a) = 1$

$\deg(d) = 3$

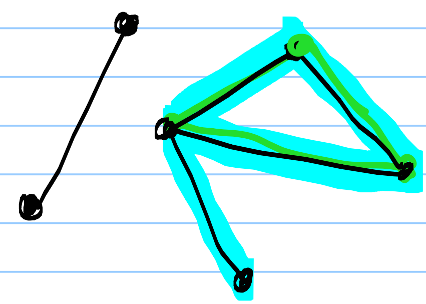
Degree Sum Formula

$$\sum_{v \in G} \deg(v) = 2 \cdot \#\text{edges}.$$

THE sum of the degrees of all the vertices in graph G



Connected.



NOT CONNECTED

A PATH IS A SEQUENCE OF EDGES SO THAT CONSECUTIVE EDGES SHARE A VERTEX.

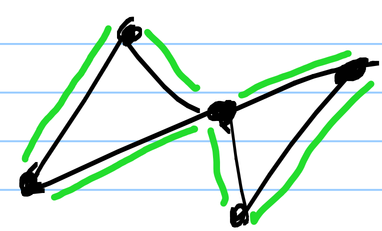
A SIMPLE PATH DOESN'T HAVE THE SAME EDGE TWICE

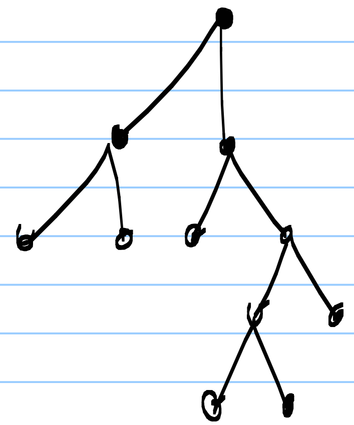
A <u>CIRCUIT</u> IS A PATH THAT STARTS & ENDS IN THE SAME PLACE — NO SIMPLE CIRCUITS ⇒ GRAPH IS A TREE

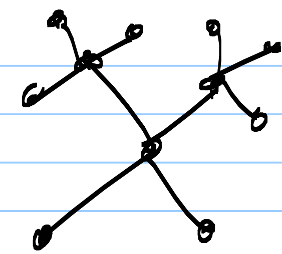
A <u>SIMPLE CIRCUIT</u> IS ONE THAT DOESN'T PASS THE SAME EDGE TWICE.

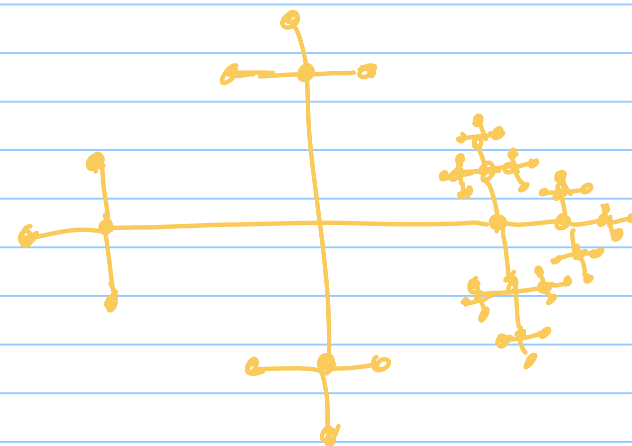
A <u>EULER PATH</u> IS A PATH THAT USES EACH EDGE ONCE. FACT: IF GRAPH HAS MORE THAN 2 VERTICES AND HAS 2 VERTICES OF ODD DEGREE, THERE IS NO EULER PATH.

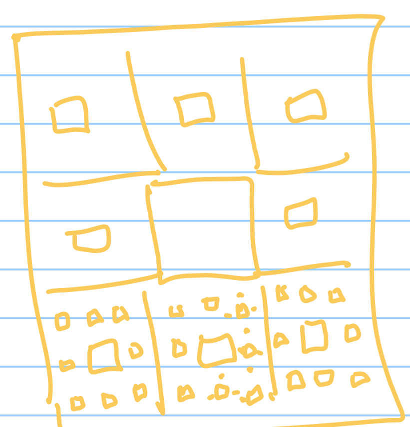
A <u>HAMILTONIAN PATH</u> PASSES EACH VERTEX ONCE.









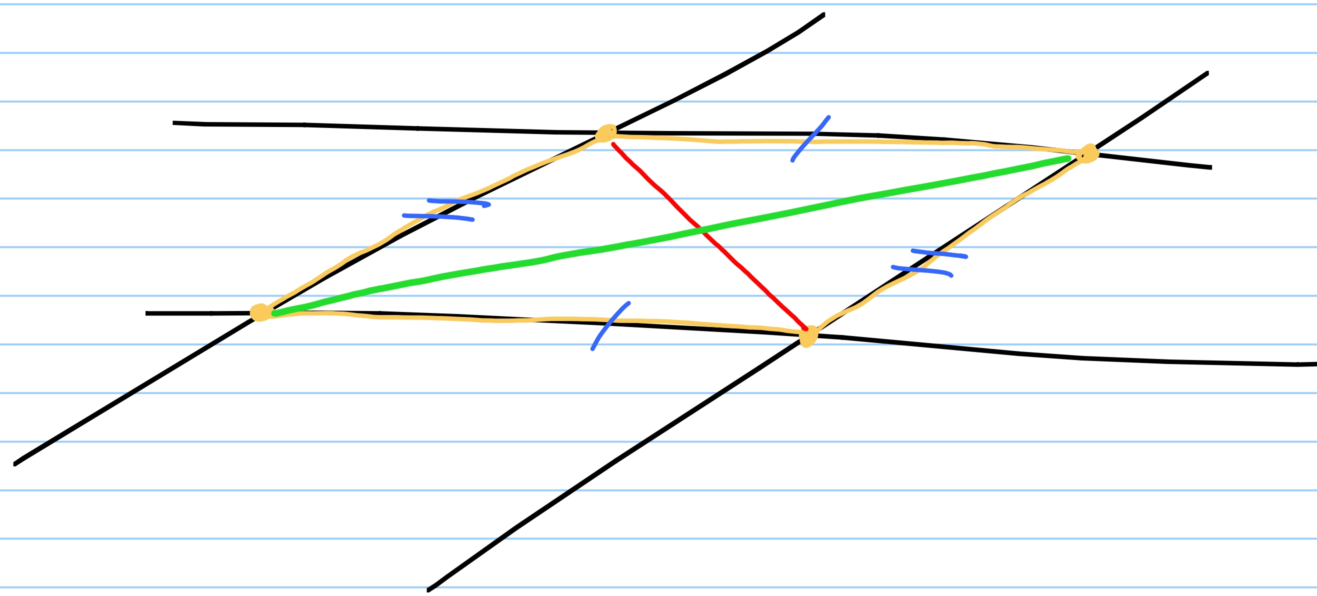


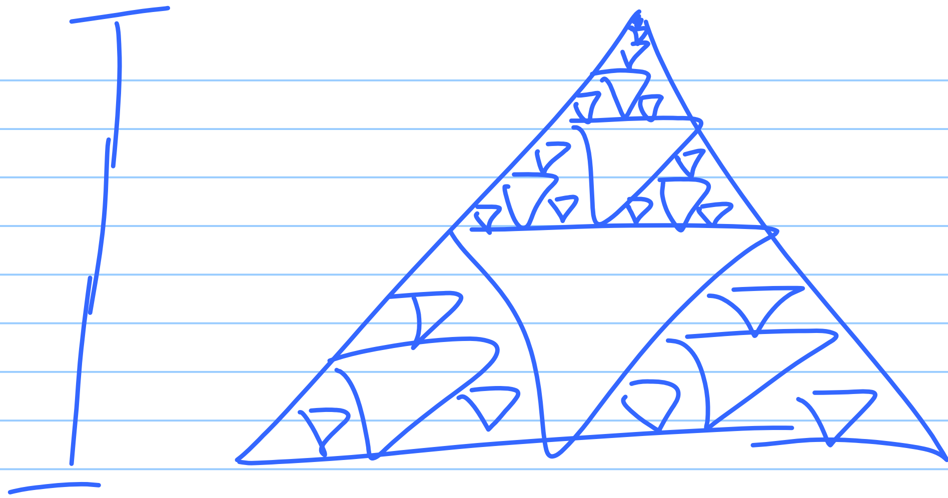
Serpinski's
CARPET.

R.L.
Moore

THM THE DIAGONALS OF A PARALLELOGRAM BISECT EACH OTHER.

Proof

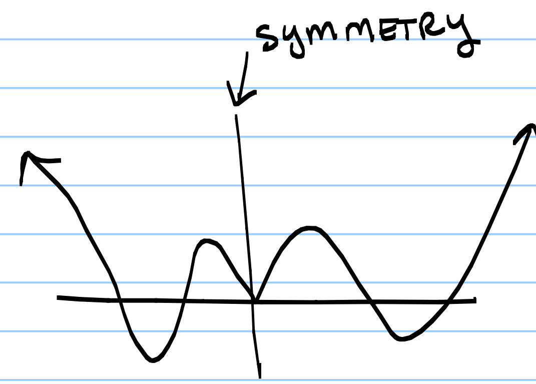




$$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = 1$$

EVEN & ODD FUNCTIONS

EVEN: $f(-x) = f(x)$



ODD: $f(-x) = -f(x)$