Automata Theory, Homework Problems

Due September 17, 2003 at 11 am

1. Show that the composition of two injective functions is injective.

2. Show that the inverse of a bijection is a bijection.

3. Suppose \( h : A \to B \) is a bijection, and \( f : A \to A \) is injective, but not surjective. Show that the composite function \( h \circ f \circ h^{-1} : B \to B \) is injective, but not surjective.

4. Suppose \( X \) is a set, and \( U \subseteq X \). Also suppose that \( q : U \to U \) is injective but not surjective. Define \( r : X \to X \) by

\[
r(x) = \begin{cases} 
q(x), & x \in U \\
x, & x \in X - U 
\end{cases}
\]

Show that \( r \) is injective but not surjective.

5. Extra Credit. Show that any subset \( A \) of \( \mathbb{N} \) is either finite or countably infinite. (Hint: enumerate the elements of \( A \) by finding the smallest element, then the second smallest, etc.)

6. Suppose \( B \subseteq Y \), and \( f : X \to Y \) is a bijection. Find a subset \( A \subseteq X \), and a bijection \( g : A \to B \).