Automata Theory
Homework 7: Additional Problems

Due, Wednesday, October 29

1. Suppose $M = (K, \Sigma, \delta, s, F)$ is a DFA. Recall the definition of $\hat{\delta} : K \times \Sigma \rightarrow K$:

   (a) $\hat{\delta}(q, e) = q$, for each $q \in K$.
   (b) $\hat{\delta}(q, a) = \delta(q, a)$ for each $q \in K$, and for each $a \in \Sigma$.
   (c) If $w \in \Sigma^*$ and $|w| \geq 1$, then $w = au$ for some $a \in \Sigma$ and for some $u \in \Sigma^*$. If $q \in K$, define

   \[
   \hat{\delta}(q, w) = \hat{\delta}(q, au) = \hat{\delta}(\hat{\delta}(q, a), u).
   \]

   Note that part 1b of the definition is superfluous. If $a \in \Sigma$, then $a = ae$, and we have

   \[
   \hat{\delta}(q, a) = \hat{\delta}(q, ae) = \hat{\delta}(\hat{\delta}(q, a), e) = \hat{\delta}(q, a)\text{ (part 1c)}
   \]

   Show that $\hat{\delta}(q, wa) = \delta(\hat{\delta}(q, w), a)$ for each $q \in K$, for each $a \in \Sigma$, and for each $w \in \Sigma^*$.

2. Identify the equivalence classes of states for the following DFA’s.

   (a) Figure 2-20 on page 94 of the text.
   (b) Figure 2-2 on page 59 of the text.