A comparison of condition numbers of the full rank least squares problem

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Condition numbers of the full rank least squares (LS) problem $\min_x \|Ax - b\|_2$ are considered theoretically and their computational implementation is compared. These condition numbers range from a simple normwise measure that may overestimate by several orders of magnitude the true numerical condition of the LS problem, to refined componentwise and normwise measures. Inequalities that relate these condition numbers are established, and it is concluded that the solution x_0 of the LS problem may be well-conditioned in the normwise sense, even if one of its components is ill-conditioned. It is shown that the refined condition numbers are ill-conditioned in some circumstances, the cause of this ill-conditioning is identified, and its implications are discussed.

Refined condition numbers of the matrix-vector (MV) product $\hat{b} := Ax_0$ are considered and it is shown that as these condition numbers of the MV product increase, the corresponding condition number of the LS problem decreases. The motivation for the derivation of this result is discussed, and it is shown that it cannot be obtained if the simple normwise condition number is used.

All the theoretical results are verified computationally by considering the regression of different sets of data points by a linear combination of radial basis functions.