Keck Cluster
Differential Equations

June 15, 2001

Write a program that implements Euler’s method in C or C++ on a conventional, 1-processor computer. The program should accept as input the limits of integration \((a \text{ and } b)\), the initial value of \(y\) \((y_0)\), and the number of timesteps \((n)\).

Output should include values of \(t\), the independent variable, values of \(\phi(t_i)\), the exact solution, values of \(y_i\), the numerical solution, the error and the relative error. Since the program could generate huge amounts of output if there are many timesteps, it might be a good idea to include an additional input, \(m\), the number of timesteps to compute between outputs. For example, if \(n\) is 100 and \(m\) is 10, the program would only print the output for every tenth timestep.

Test your programs with the following initial value problems.

\[
\begin{align*}
y' &= 2, \quad y(0) = 1 \\
\phi(t) &= 2t + 1
\end{align*}
\]

\[
\begin{align*}
y' &= -2t + 5, \quad y(0) = 0 \\
\phi(t) &= -t^2 + 5t
\end{align*}
\]

\[
\begin{align*}
y' &= 1 - t + 4y, \quad y(0) = 1 \\
\phi(t) &= \frac{1}{4}t - \frac{3}{16} + \frac{19}{16}e^t
\end{align*}
\]
\[ y' = \frac{3t^2 + 4t + 2}{2(y - 1)}, \quad y(0) = 1 \]
\[ \phi(t) = 1 - \sqrt{t^3 + t^2 + 2t + 4} \]