Math 202-02
Additional Problems and Solutions to Prepare
Before the First Midterm

Spring, 2015

1. Suppose

\[
A = \begin{bmatrix}
1 & -1 \\
2 & 3
\end{bmatrix},
B = \begin{bmatrix}
2 & -2 \\
3 & 1
\end{bmatrix}, \text{ and } C = \begin{bmatrix}
-1 & 2 \\
0 & 4
\end{bmatrix},
\]

Find \( (AB)C \) and \( A(BC) \).

Solution:

\[
(AB)C = A(BC) = \begin{bmatrix}
1 & -14 \\
-13 & 22
\end{bmatrix},
\]

2. A floating point system uses decimal digits to represent values:

- One digit represents the sign (0 is positive, 1 is negative)
- The mantissa or significand uses three decimal digits.
- The exponent uses two decimal digits: one digit for the sign, and one digit for the absolute value of the exponent.

So, for example, 28.7 would be represented as +.287 +2, since 28.7 = 0.287 \times 10^2. Input values that need more than 3 digits to represent the significand are rounded to 3 digits. For example, 28.748 would be represented as +.288 +2, and 28.751 would be represented as +.288 +2. Arithmetic operations compute the exact result, and then round to the nearest value with a 3-digit mantissa.
(a) Show the result of the floating point operation \((+.287 +2) + (+.353 +4)\).

(b) Show the result of the floating point operation \((+.287 +2) \times (+.353 +4)\).

(c) Show the result of the floating point operation \((+.287 +5) \times (+.353 +6)\).

(d) What is the largest possible positive value that can be stored exactly as a floating point number?

(e) If the first digit to the right of the decimal must be nonzero, what is the smallest possible positive value that can be stored exactly as a floating point number?

Solutions:

(a) In exact arithmetic
\[ +.287 +2 + +.353 +4 = +.35587 +4 \]
This is rounded to a 3-digit mantissa. So the result is \(+.356 +4\).

(b) In exact arithmetic
\[ +.287 +2 \times +.353 +4 = +.101311 +6 \]
This is rounded to a 3-digit mantissa. So the result is \(+.101 +6\).

(c) In exact arithmetic
\[ +.287 +5 \times +.353 +6 = +.101311 +11 \]
This is overflow: the result is too large to be stored as a floating point value, since the exponent is greater than +9, the largest exponent available.

(d) \(+.999 +9\)

(e) \(+.100 -9\)