1. Prove that $\mathbb{R}^n$ with the “usual topology” satisfies the axioms for a topological space.

2. Prove that the $\epsilon - \delta$ definition of continuity is equivalent to the open set definition of continuity (i.e., $f : \mathbb{R}^n \to \mathbb{R}^m$ is continuous iff $f^{-1}(V)$ is open for each open subset $V \subseteq \mathbb{R}^m$).

3. Suppose $(X, \mathcal{U})$ is a topological space and $Y \subseteq X$. Define a family of subsets $\mathcal{V}$ of $Y$ by $V \in \mathcal{V}$ iff there exists a set $U \in \mathcal{U}$ such that $V = U \cap Y$. Prove that $\mathcal{V}$ is a topology on $Y$. $\mathcal{V}$ is called the relative topology on $Y$, and unless we state otherwise, you should assume that a subset of a topological space has the relative topology.

4. Using subsets of euclidean spaces, find topological spaces $X$ and $Y$ and a function $f : X \to Y$, such that

   (a) $f$ is continuous but not open.

   (b) $f$ is open but not continuous.

Can you find a function $f$ satisfying 4a or 4b that is both one-to-one and onto?