1. A basis of a topological space $X$ is a family of open subsets $B$ with the property that every open set $U$ in the topological space is a union of elements of $B$.

   (a) Show that the family of open balls forms a basis for the usual topology on $\mathbb{R}^n$.
   (b) Show that a function $f : X \to Y$ is continuous iff $f^{-1}(V)$ is open for each $V$ in a basis $B$ for the topology on $Y$.

2. Suppose $X$ is a finite subset of some euclidean space and it has the relative or subspace topology. If $Y$ is any topological space and $f : X \to Y$ is any function, show that $f$ is continuous.

3. Example 1.7.1 on page 17 of the text. Explain your answer.

4. Example 1.7.2 on page 17 of the text. Explain your answer.

5. Extra Credit. Suppose $X$ is a topological space and $A$ is a closed subset of $X$. Also suppose that $C$ is a subset of $A$ that is closed in the relative topology on $A$. Show that $C$ is a closed subset of $X$.

6. Extra Credit. A topological space $X$ is Hausdorff if for any two points $x, y \in X$, there exist open neighborhoods $U$ of $x$ and $V$ of $y$ such that $U \cap V = \emptyset$.

   (a) Show that $\mathbb{R}^n$ is Hausdorff.
   (b) If $X$ is a Hausdorff space and $C$ is a compact subset of $X$, show that $C$ must be closed.
   (c) Find an example of a topological space $X$ and a compact subset $C$ that is not closed. In order to receive credit, you must explain why your example is correct.