Homework 3
Grading and Solution Notes
Graded Problems

• Problem 1 (10 possible)
  ▫ 4 points for part (a)
  ▫ 4 points for part (b)
  ▫ 2 points for part (c)

• Problem 3 (15 possible)
  ▫ 5 points for part (a) and (b)
  ▫ 5 points for part (c)
  ▫ 5 points for part (d)

• Problem 4 (10 possible)
Problem 1 (a)

- Unreduced NFA for \((0 \cup 1)^*000(0 \cup 1)^*\)

concatenation removed
the final state here
Problem 1 (b)

- Unreduced NFA for \(((00)^*(11)) \cup 01)^*\)
Problem 3 (c)

• Show that the language $ECHO(A)$ is regular.

  ▫ Assume $A$ is expressed as a DFA $M$.
    • Otherwise, convert the NFA or regex to a DFA.

  ▫ Create a new DFA or NFA $M' = \{ Q', \Sigma, \delta', q_0, F \}$
    • $Q'$ and $\delta'$ defined as...

    • Give the actual components of the new DFA!
    • “Show” still means you have to be formal!
Problem 3 (c)

- $Q' = Q \cup Q \times \Sigma$

$q = \{ a, b \}$  

$q = \{ a, b, \{a,0\}, \{a,1\}, \{b,0\}, \{b,1\} \}$
Problem 3 (c)

\[ \delta'(q, \sigma) = \{ q, \sigma \} \text{ for } q \in Q \]
Problem 3 (c)

- \( \delta'(\{q, \sigma\}, \gamma) = \delta(q, \sigma) \) if \( \sigma = \gamma \)

<table>
<thead>
<tr>
<th></th>
<th>0</th>
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<tbody>
<tr>
<td>{ a, 0 }</td>
<td>a</td>
<td>\Ø</td>
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<tr>
<td>{ a, 1 }</td>
<td>\Ø</td>
<td>b</td>
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<tr>
<td>{ b, 0 }</td>
<td>a</td>
<td>\Ø</td>
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<tr>
<td>{ b, 1 }</td>
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<td>b</td>
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Problem 3 (d)

• Show the language $NOEXTEND(A)$ is regular.

  ▫ Assume $A$ is expressed as a DFA $M$.
    • Otherwise, convert the NFA or regex to a DFA.

  ▫ Create a new DFA $M' = \{ Q, \Sigma, \delta, q_0, F' \}$
    • $F' = \{ q \in F \mid \text{there is no nonempty path from } q \text{ to another final state } f \in F \}.$
Problem 3 (d)

- Example:
  - $A = \{ a, aab \}$
  - $\text{NoExtend}(A) = \{ aab \}$
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  - $\text{NoExtend}(A) = \{ aab \}$

no path from $q_3$ to $q_1$
Problem 3 (d)

• Example:
  ▫ $A = \{ a, aab \}$
  ▫ $\text{NOEXTEND}(A) = \{ aab \}$
Problem 3 (d)

• Show the language $\text{NOEXTEND}(A)$ is regular.

  ▫ Also need to show that $L(M') = \text{NOEXTEND}(A)$.
    • Show $L(M') \subseteq \text{NOEXTEND}(A)$.
      • Argue that if $x \in L(M')$, then $x \in \text{NOEXTEND}(A)$.

    • Show $\text{NOEXTEND}(A) \subseteq L(M')$.
      • Argue that if $x \in \text{NOEXTEND}(A)$, then $x \in L(M')$. 
Problem 4

• Give a decision procedure for \text{ISPREFIX}( A, u ).
  ▫ The input is $A$, $u$.
  ▫ The input is not $w$.
    • There could be an infinite number of strings $w \in A$. 