Section 2.2

1. Determine the complexity function that measures the number of print statements in an algorithm that takes a positive integer \( n \) and prints one 1, two 2s, three 3s, \ldots, \( n \) ns.

Solution

\[
f(n) = 1 + 2 + 3 + \cdots + n = \frac{n(n + 1)}{2} = O(n^2)
\]

2. Suppose an algorithm takes a sequence of \( n \) (\( \geq 2 \)) integers and determines if it contains an integer that is a repeat of the first integer in the list. Find the complexity function for the:

(a) best case analysis,
(b) worst case analysis,
(c) average case analysis.

Solution

(a) The complexity function for the best case is \( f(n) = 1 \). Making the second integer equal to the first will force the algorithm to terminate after only one comparison.

(b) The complexity function for the worst case is \( f(n) = n \). Having no repeat of the first integer will force the algorithm to terminate after making all \( n - 1 \) comparisons.

(c) The complexity function for the average case is \( f(n) = n \). There might be a repeat of the first integer in any of positions 2 through \( n \), or there may be no repeat. Thus there are \( n \) cases, with respective numbers of comparisons 1, 2, 3, \ldots, \( n - 1 \), \( n - 1 \). The average of these numbers is

\[
\frac{1 + 2 + 3 + \cdots + (n - 1) + (n - 1)}{n} = \frac{n(n - 1)/2}{n} = O(n).
\]

3. Find the complexity function for counting the number of print statements in the following algorithm:

\[
\text{for } i := 1 \text{ to } n \\
\begin{aligned}
\text{begin} \\
\text{for } j := 1 \text{ to } n \\
\quad \text{print } \text{“hello”} \\
\text{for } k := 1 \text{ to } n \\
\quad \text{print } \text{“hello”}
\end{aligned}
\]

Solution For each value of \( i \), both the \( j \)-loop and \( k \)-loop are executed. Thus for each \( i \), \( n + n = 2n \) print statements are executed. Therefore the total number of print statements executed is \( n \cdot 2n = 2n^2 = O(n^2) \).
4. Find the complexity function for counting the number of print statements in the following algorithm:

\[
\text{for } i := 1 \text{ to } n
\begin{align*}
&\text{begin} \\
&\text{for } j := 1 \text{ to } i \\
&\quad \text{print} \text{ “hello”} \\
&\text{for } k := i + 1 \text{ to } n \\
&\quad \text{print} \text{ “hello”} \\
&\text{end}
\end{align*}
\]

\textbf{Solution} For each value of $i$, both the $j$-loop and $k$-loop are executed. Thus for each $i$, $i+j = n$ print statements are executed. Therefore the total number of print statements executed is $n \cdot n = O(n^2)$. 