DISCUSSION

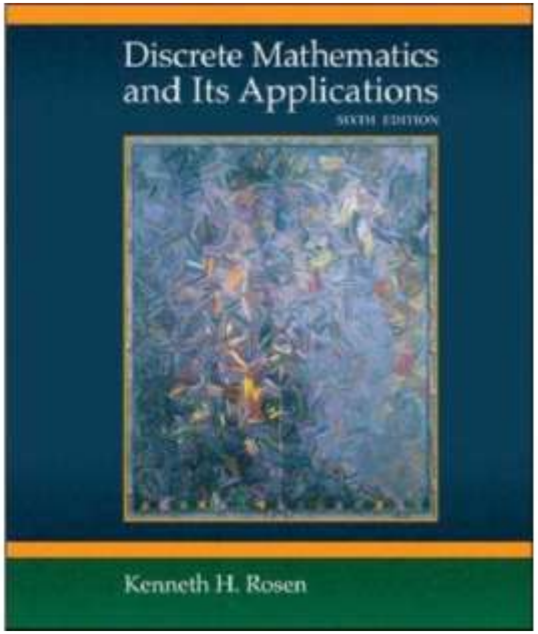
FRIDAY APRIL 13TH 2007

Sophie Engle

ECS20: Discrete Mathematics

Announcements

- Book available at bookstore!

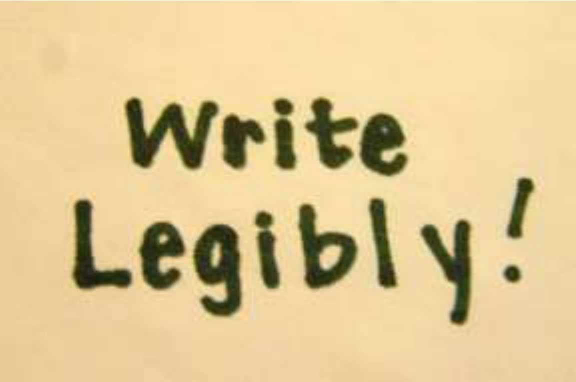


Sixth Edition:

- $140.40 at bookstore

Announcements

- New submission guidelines!



Write Legibly.

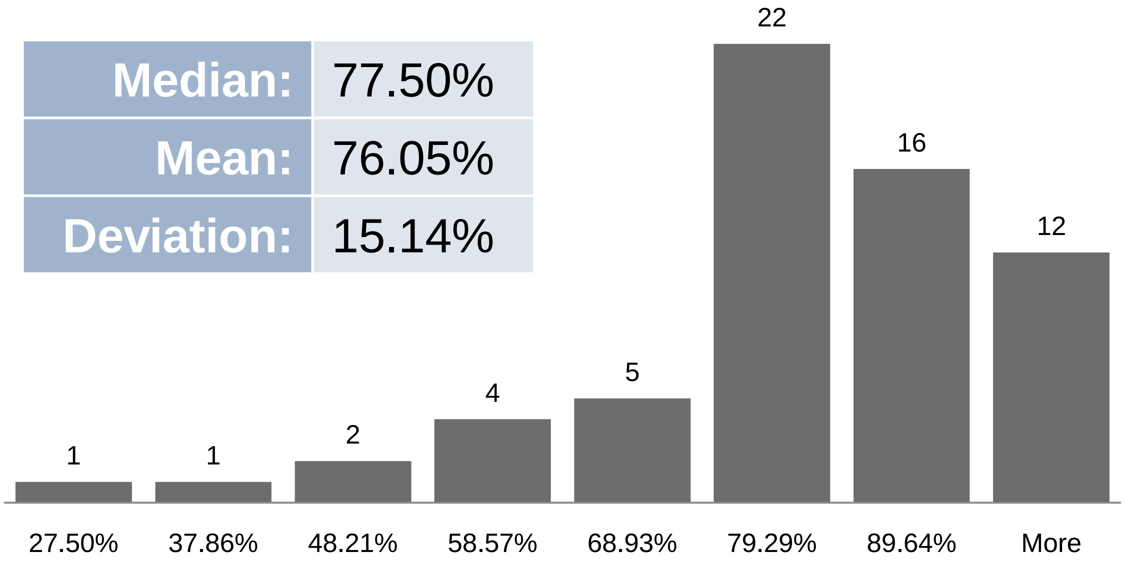
No fringe.

Staple pages.

Announcements

- Homework 1 graded
  - Solutions posted on my.ucdavis.edu
  - View grades on my.ucdavis.edu

Median:	77.50%
Mean:	76.05%
Deviation:	15.14%



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Announcements

- Homework 2 assigned
  - ~~38 problems total~~
    - Some problems were removed! Be sure to double check the main course website.
  - Due Monday, April 16 **at 4:00pm**

- Extra Exercises posted on TA website for sections 2.1, 2.2, 2.3, and 2.4

Announcements

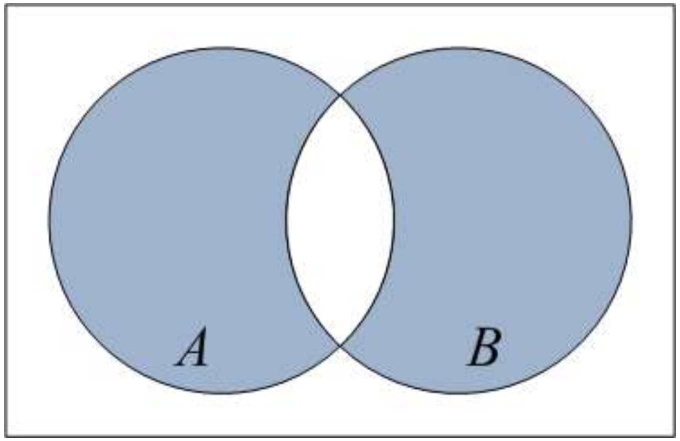
- To get homework questions answered:
  - Submit questions to the newsgroup.
    - Information on how to access newsgroup is on the TA website (linked from the main course website).
    - There is a web-based reader!
  - Questions submitted by Thursday at 4:00pm may be included in Friday's discussion.
    - Otherwise, I will answer the question on the newsgroup.
    - Questions posted after 4:00pm Sunday may not get answered in time.
  - Please do not email homework questions.

Homework 2 Notes

Tips and hints for homework 2.

Symmetric Difference

- $A \oplus B$: the set of those elements in either A or B, but not in both A and B.
  - How can we express this with unions and intersections?
    - $A \oplus B = (A \cup B) - (A \cap B)$



Union and Intersection

$$A_1 \cup A_2 \cup \cdots \cup A_n$$

can be written as:

$$\bigcup_{i=1}^{n} A_i$$

$$A_1 \cap A_2 \cap \cdots \cap A_n$$

can be written as:

$$\bigcap_{i=1}^{n} A_i$$

Union and Intersection

$$A_i = \{1, 2, \ldots, i\}$$

$$\bigcup_{i=2}^{4} A_i = A_2 \cup A_3 \cup A_4$$
$$= \{1, 2\} \cup \{1, 2, 3\} \cup \{1, 2, 3, 4\}$$
$$= \{1, 2, 3, 4\}$$

$$\bigcap_{i=2}^{4} A_i = A_2 \cap A_3 \cap A_4$$
$$= \{1, 2\} \cap \{1, 2, 3\} \cap \{1, 2, 3, 4\}$$
$$= \{1, 2\}$$

$$\bigcup_{i=2}^{\infty} A_i = ?$$

$$\bigcap_{i=2}^{\infty} A_i = ?$$

Union and Intersection

$$A_i = \{1, 2, \ldots, i\}$$

$$\bigcup_{i=2}^{4} A_i = A_2 \cup A_3 \cup A_4$$
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$$= \{1, 2\} \cap \{1, 2, 3\} \cap \{1, 2, 3, 4\}$$
$$= \{1, 2\}$$

$$\bigcup_{i=2}^{\infty} A_i = \{1, 2, 3, \ldots\} = \mathbf{Z}^+$$

$$\bigcap_{i=2}^{\infty} A_i = \{1\}$$

Floor and Ceiling Functions

floor $\lfloor x \rfloor$	Assigns to the real number x the largest integer that is less than or equal to x.

Assigns to the real number x the smallest integer that is greater than or equal to x.	**ceil** $\lceil x \rceil$

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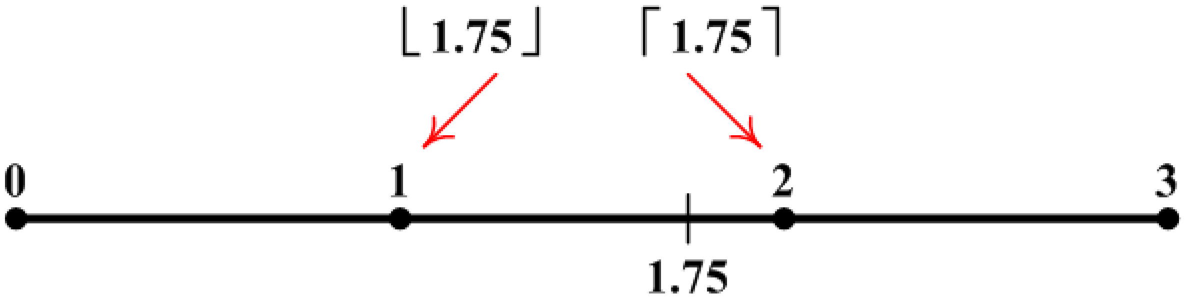

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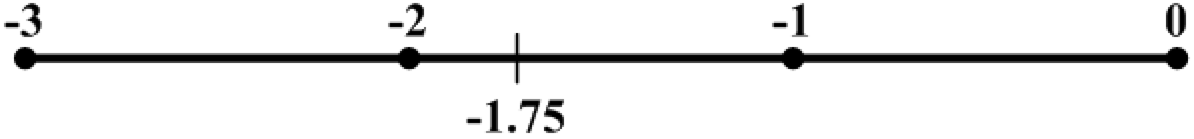

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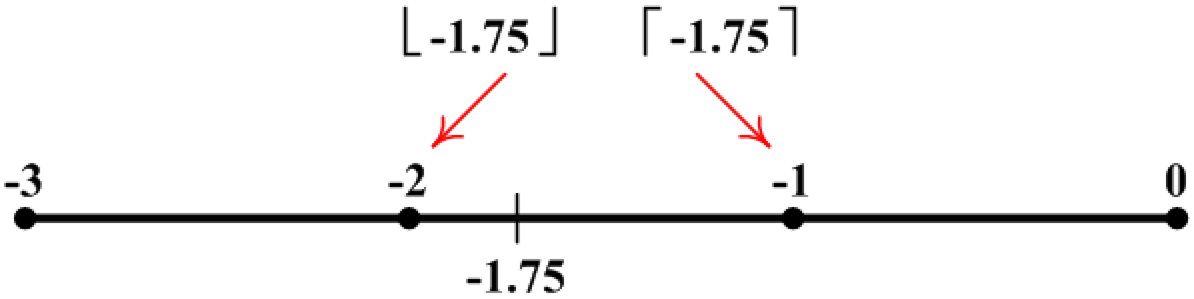
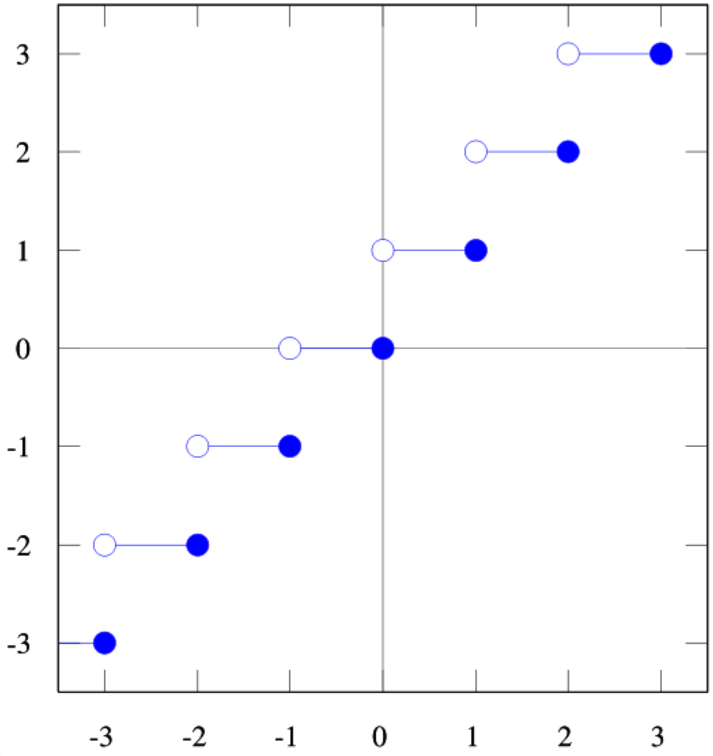
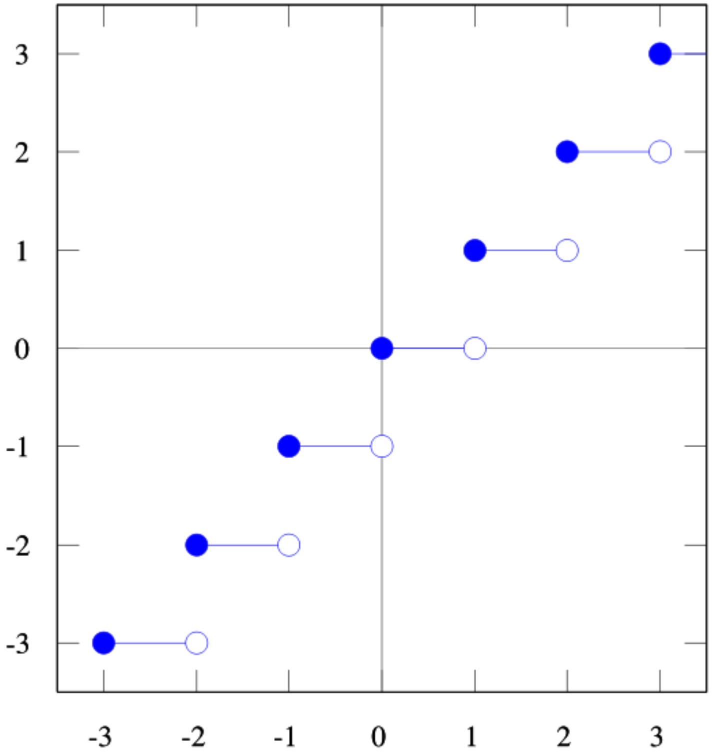


Floor and Ceiling Functions



$y = \lceil x \rceil$



$y = \lfloor x \rfloor$

Floor and Ceiling Functions

- Useful Properties
  - Does $\lfloor x + n \rfloor = \lfloor x \rfloor + n$?
    - TRUE! See proof on page 144.
    - Same for $\lceil x + n \rceil = \lceil x \rceil + n$.
  - Does $\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil$?
    - FALSE! $\lceil 0.5 + 0.5 \rceil = \lceil x \rceil + \lceil y \rceil$

$$\begin{aligned} \lceil 1 \rceil &= \lceil 0.5 \rceil + \lceil 0.5 \rceil \\ 1 &= 1 + 1 \\ 1 &\neq 2 \end{aligned}$$

  - More properties in book.

Proving Equivalence (Example 1)

Let f be a function from the set A to the set B.

Let S and T be subsets of A.

- Show that $f\left(S \cup T\right) = f\left(S\right) \cup f\left(T\right)$
  - Step 1: Show that $f\left(S \cup T\right) \subseteq f\left(S\right) \cup f\left(T\right)$
  - Step 2: Show that $f\left(S\right) \cup f\left(T\right) \subseteq f\left(S \cup T\right)$
- Why does this work?
  - If $A \subseteq B$ and $B \subseteq A$ then $A = B$.

Proving Equivalence (Example 1)

- Proof Step 1: $f(S \cup T) \subseteq f(S) \cup f(T)$
  - Let $y \in f(S \cup T)$.
  - Then there exists a $x \in S \cup T$ such that $f(x) = y$.
    - If $x \in S$ then $f(x) \in f(S) \subseteq f(S) \cup f(T)$.
    - If $x \in T$ then $f(x) \in f(T) \subseteq f(S) \cup f(T)$.
  - Therefore $f(x) \in f(S) \cup f(T)$ for all $x \in S \cup T$.
  - This gives us $f(S \cup T) \subseteq f(S) \cup f(T)$.

Proving Equivalence (Example 1)

- Proof Step 2: $f(S) \cup f(T) \subseteq f(S \cup T)$
  - Let $y \in f(S) \cup f(T)$.
  - Then $y \in f(S)$ or $y \in f(T)$.
    - If $y \in f(S)$ then there exists a $x \in S \subseteq S \cup T$ such that $f(x) = y$.
    - If $y \in f(T)$ then there exists a $x \in T \subseteq S \cup T$ such that $f(x) = y$.
  - If there exists such a $f(x) = y$ then $f(x) \in f(S) \cup f(T)$.
  - We also know that $x \subseteq S \cup T$.
  - Therefore $f(x) \subseteq f(S \cup T)$ for all $x \subseteq S \cup T$.
  - This gives us $f(S) \cup f(T) \subseteq f(S \cup T)$.

Proving Equivalence (Example 2)

Let f be a function from the set A to the set B.

Let S be a subset of B.

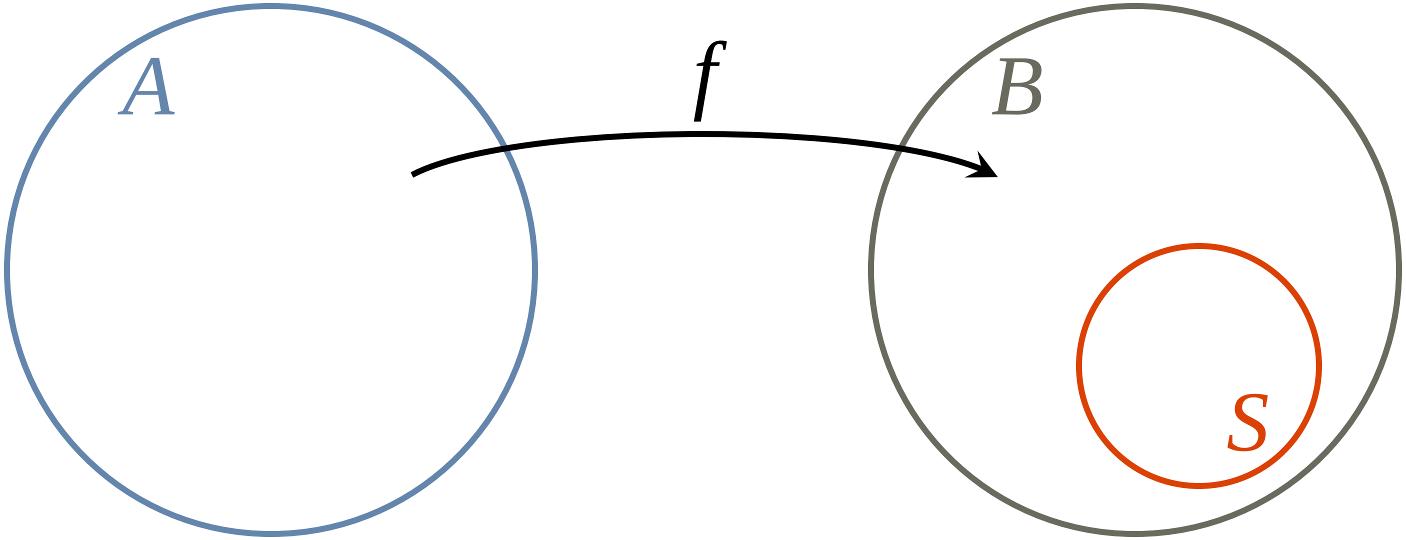
$$f : A \rightarrow B$$

$$S \subseteq B$$

- Show that: $f^{-1}(\overline{S}) = \overline{f^{-1}(S)}$

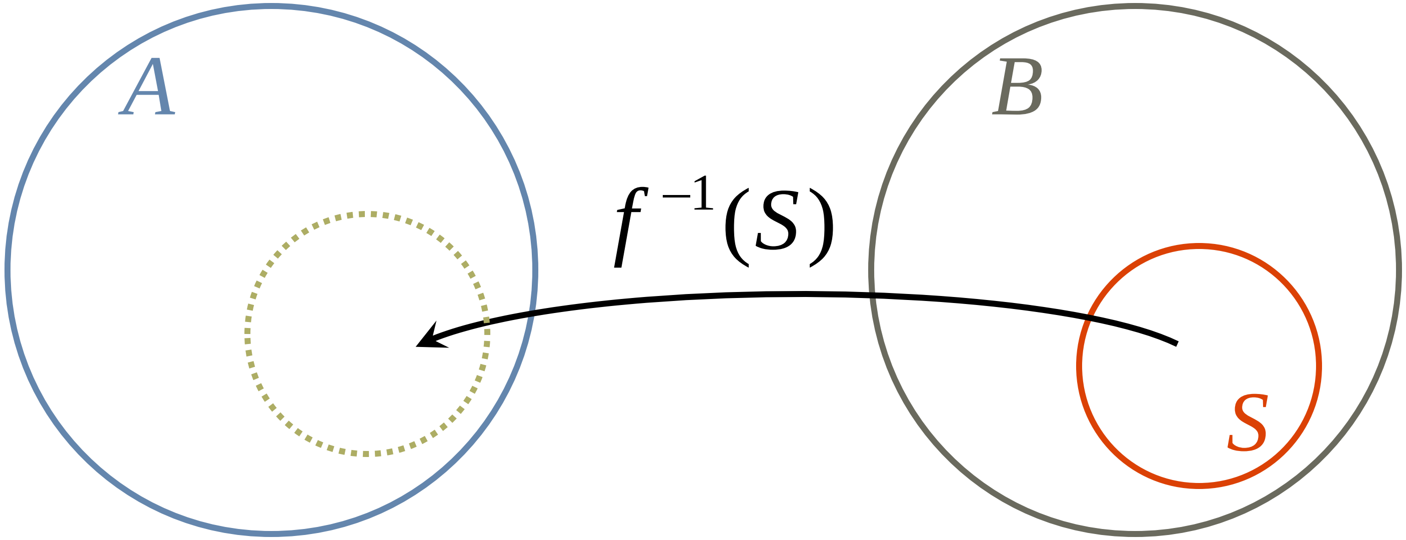
Proving Equivalence (Example 2)

$$f^{-1}(\overline{S}) = \overline{f^{-1}(S)}$$



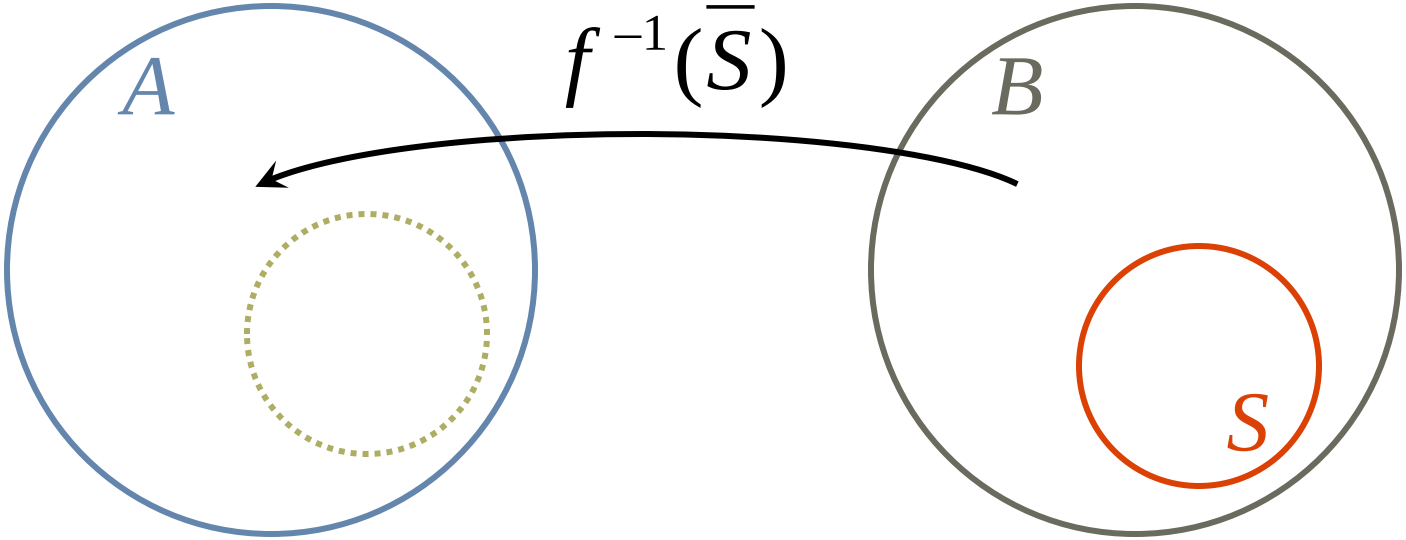
Proving Equivalence (Example 2)

$$f^{-1}(\overline{S}) = \overline{f^{-1}(S)}$$



Proving Equivalence (Example 2)

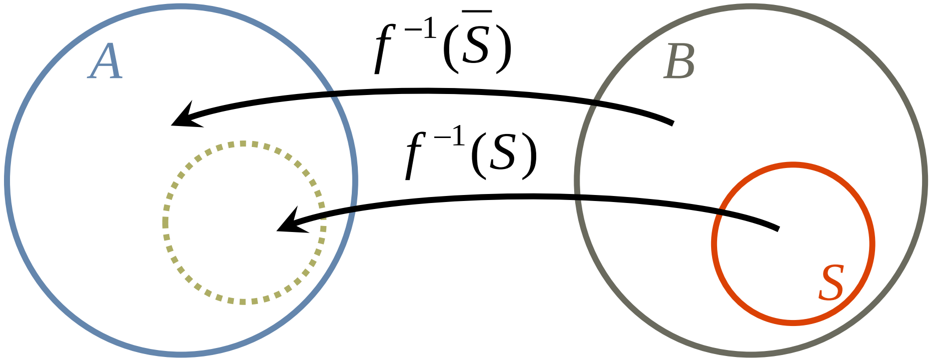
$$f^{-1}(\overline{S}) = \overline{f^{-1}(S)}$$



Proving Equivalence (Example 2)

$$f^{-1}(\overline{S}) = \overline{f^{-1}(S)}$$

$$\begin{aligned} f^{-1}(\overline{S}) &= \{x \in A \mid f(x) \notin S\} \\ &= \overline{\{x \in A \mid f(x) \in S\}} \\ &= \overline{f^{-1}(S)} \end{aligned}$$



Homework 1 Notes

Notes and solutions from homework 1.

Homework 1 Notes

- Converse and contrapositive

Implication:	$p \to q$	**Converse:**	$q \to p$
Inverse:	$\neg p \to \neg q$	**Contrapositive:**	$\neg q \to \neg p$

- Example:
  - $\underbrace{\text{I go to the beach}}_{\text{result } (q)}$ whenever $\underbrace{\text{it is a sunny day.}}_{\text{condition } (p)}$

Homework 1 Notes

- Converse and contrapositive

Implication:	$p \rightarrow q$	**Converse:**	$q \rightarrow p$
Inverse:	$\neg p \rightarrow \neg q$	**Contrapositive:**	$\neg q \rightarrow \neg p$

- Example:
  - $\underbrace{\text{I go to the beach}}_{\text{result } (q)}$ whenever $\underbrace{\text{it is a sunny day}}_{\text{condition } (p)}$.
  - Whenever it is a sunny day, I go to the beach.
    - Same meaning, get $p \rightarrow q$.

Homework 1 Notes

- Converse and contrapositive

Implication:	$p \to q$	**Converse:**	$q \to p$
Inverse:	$\neg p \to \neg q$	**Contrapositive:**	$\neg q \to \neg p$

- Example:
  - $\underbrace{\text{I go to the beach}}_{\text{result } (q)} \text{ whenever } \underbrace{\text{it is a sunny day.}}_{\text{condition } (p)}$
  - I don’t go to the beach whenever it isn’t a sunny day.
    - Inverse, get $\neg p \to \neg q$.

Homework 1 Notes

- Converse and contrapositive

Implication:	$p \rightarrow q$
Inverse:	$\neg p \rightarrow \neg q$

Converse:	$q \rightarrow p$
Contrapositive:	$\neg q \rightarrow \neg p$

- Example:
  - $\underbrace{\text{I go to the beach}}_{\text{result } (q)}$ whenever $\underbrace{\text{it is a sunny day.}}_{\text{condition } (p)}$
  - It is a sunny day whenever I go to the beach.
    - Converse, get $q \rightarrow p$.

Homework 1 Notes

- Converse and contrapositive

Implication:	$p \rightarrow q$
Inverse:	$\neg p \rightarrow \neg q$

Converse:	$q \rightarrow p$
Contrapositive:	$\neg q \rightarrow \neg p$

- Example:
  - $\underbrace{\text{I go to the beach}}_{\text{result } (q)}$ whenever $\underbrace{\text{it is a sunny day.}}_{\text{condition } (p)}$
  - It isn’t a sunny day whenever I don’t go to the beach.
    - Contrapositive, get $\neg q \rightarrow \neg p$.

Homework 1 Notes

- ☐ Make **standard** truth tables!

~~*p*~~	~~*q*~~	~~*r*~~
~~T~~	~~T~~	~~T~~
~~F~~	~~F~~	~~T~~
~~T~~	~~F~~	~~T~~
~~F~~	~~T~~	~~T~~
~~F~~	~~F~~	~~F~~
~~F~~	~~T~~	~~F~~
~~T~~	~~T~~	~~F~~
~~T~~	~~F~~	~~F~~

p	*q*	*r*
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Homework 1 Notes

- Negation
  - When distributing a ¬ in a proposition, you must:
    - Negate every variable:
      - p becomes $\neg p$
      - $\neg p$ becomes p
    - Negate every operator:
      - $\vee$ becomes $\wedge$
      - $\wedge$ becomes $\vee$
    - Negate every quantifier:
      - $\forall x\ (\cdots)$ becomes $\exists\ x\ \neg\ (\cdots)$
      - $\exists\ x\ (\cdots)$ becomes $\forall x\ \neg\ (\cdots)$

Homework 1 Notes

- Negation Example

$$\neg\ \exists x\ [\ \forall y\ P(x, y)\ \wedge\ \forall z\ [\ \neg Q(x, y)\ \vee\ \exists y\ R(x, y, z)\ ]\ ]$$

$$\forall x\ \neg[\ \forall y\ P(x, y)\ \wedge\ \forall z\ [\ \neg Q(x, y)\ \vee\ \exists y\ R(x, y, z)\ ]\ ]$$

$$\forall x\ [\ \exists y\ \neg P(x, y)\ \vee\ \exists z\ \neg[\ \neg Q(x, y)\ \vee\ \exists y\ R(x, y, z)\ ]\ ]$$

$$\forall x\ [\ \exists y\ \neg P(x, y)\ \vee\ \exists z\ [\ Q(x, y)\ \wedge\ \forall y\ \neg R(x, y, z)\ ]\ ]$$