DISCUSSION #3
FRIDAY APRIL 18TH 2007

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ECS20: Discrete Mathematics

Preliminary Survey Results

Survey Located At:

http://www.surveymonkey.com/s.asp?u=665323704735

Homework #3

Due **Wednesday** April 25th.

Homework #3

- Due date now on Wednesday at 4:00pm
- 31 questions total
- Covers six sections total
  - 2.3: Functions
  - 2.4: Sequences and Summations
  - 3.4: Integers and Division
  - 3.5: Primes and Greatest Common Divisors
  - 3.6: Integers and Algorithms
  - 3.7: Applications of Number Theory

Show versus Prove

- Show:
  - Informal
  - Explanation
  - Diagrams
- Prove:
  - Formal
  - Based on “facts”
  - Uses rules of inference
  - Many methods:
    - By Construction
    - By Contraposition
    - By Contradiction
    - By Counterexample

Homework #3

Section 2.3 hints and examples.

Function Notation

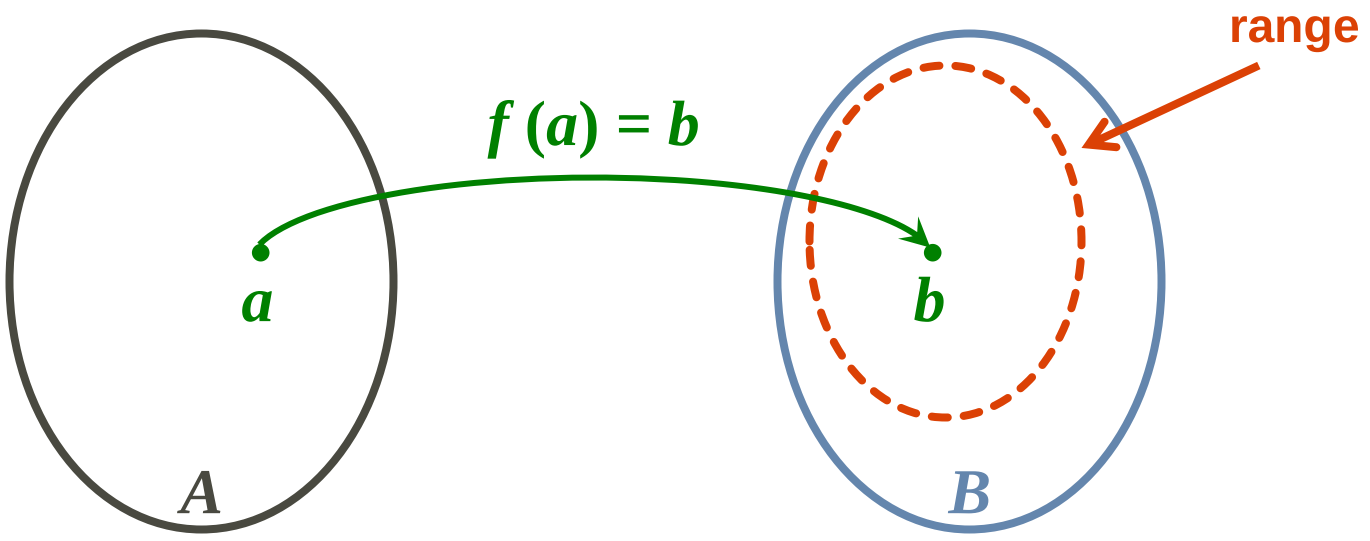
- $f: A \to B$
  - Function f has:
    - domain A
    - codomain B
  - For $f(a) = b$:
    - input $a \in A$
    - output $b \in B$
  - One input variable
- $f: A \times B \to C$
  - Function f has:
    - domain $A \times B$
    - codomain C
  - For $f(a, b) = c$:
    - input $a \in A$
    - input $b \in B$
    - output $c \in C$
  - Two input variables

Function Notation

- $f(m, n) = m + n$
  - Let $m \in \mathbb{N}$ and $n \in \mathbb{N}$:
    - $f(1, 2) = 1 + 2 = 3$
    - $f: \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$
  - Let $m \in \mathbb{Z}$ and $n \in \mathbb{N}$:
    - $f(-4, 1) = -4 + 1 = -3$
    - $f : \mathbb{Z} \times \mathbb{N} \rightarrow \mathbb{Z}$
  - Let $m \in \mathbb{Z}$ and $n \in \mathbb{R}$:
    - $f(2, 0.15) = 2 + 0.15 = 2.15$
    - $f: \mathbb{Z} \times \mathbb{R} \rightarrow \mathbb{R}$

Onto / Surjective

- A function f: A to B is onto iff:
  - For every $b \in B$ there is an $a \in A$ with $f(a) = b$
  - $\forall b \ \exists a \ (f(a) = b)$
  - The codomain is equal to the range



Onto / Surjective

- Determine if the function $f: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto:
  - $f(m, n) = m + n$
    - Onto!
    - For every $p \in \mathbb{Z}$ can we find a pair (m, n) such that $m + n = p$?
      - Let $m = 1$, $n = p - 1$.
  - $f(m, n) = m^2 + n^2$
    - Not onto
    - There is no pair (m, n) such that $m^2 + n^2 = -1$.

Homework #3

Section 2.4 hints and examples.

Summation

- Notation: $\sum_{j=m}^{n} a_j = a_m + a_{m+1} + \cdots + a_n$
- Examples:

$$\sum_{k=1}^{5}(k+1) = (1+1)+(2+1)+(3+1)+(4+1)+(5+1)$$

$$= (2)+(3)+(4)+(5)+(6)$$

$$= 20$$

$$S = \{2,4,6,8\}$$

$$\sum_{j \in S} j = 2+4+6+8 = 20$$

(work out on board)

Double Summation

- Example:

evaluate inner sum first

$$\sum_{i=1}^{2}\sum_{j=1}^{3} i+j = \sum_{i=1}^{2}\left(\sum_{j=1}^{3} i+j\right)$$

$$= \sum_{i=1}^{2}\left((i+1)+(i+2)+(i+3)\right)$$

$$= \sum_{i=1}^{2}\left(3i+6\right)$$

$$= (3\cdot 1+6)+(3\cdot 2+6)$$

$$= 3+6+6+6$$

$$= 21$$

(work out on board)

Products

- Notation: $\prod_{j=m}^{n} a_j = a_m \times a_{m+1} \times \cdots \times a_n$
- Examples:

$$\prod_{i=0}^{10} i = 0 \times 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$$

$$= 0$$

$$\prod_{i=1}^{100} (-1)^i = (-1)^1 \times (-1)^2 \times \cdots \times (-1)^{99} \times (-1)^{100}$$

$$= -1 \times 1 \times \cdots \times -1 \times 1$$

$$= 1$$

(work out on board)

Homework #3

Section 3.4 hints and examples.

Number Theory Motivation

- What does it deal with?
  - Studies properties and relationships of specific classes of numbers
  - Most commonly studied classes of numbers:
    - Positive Integers
    - Primes
- What is this stuff good for?
  - Number theory used in cryptography
    - Basis for RSA public-key system
  - Integers often used in programming
    - Array indices

Proofs with Integer Division

- If $a, b \in \mathbb{Z}$ with $a \neq 0$:
  - $a \mid b$ if there exists a k such that $a\,k = b$.

- #7. Show that if a, b, and c are integers with $c \neq 0$, such that $ac \mid bc$, then $a \mid b$.
  - If $ac \mid bc$, then there is an integer k such that:

$$ack = bc$$

$$\tfrac{1}{c}(ack = bc)$$

$$ak = b$$

  - Therefore, we can state that $a \mid b$.

Proofs with Integer Division

#21. Show that if:

- $n \mid m$, where n, m are positive integers > 1, and
- $a \equiv b \pmod{m}$, where a and b are integers

Then:

- $a \equiv b \pmod{n}$

Since $n \mid m$, we know there exists an integer i such that $n\,i = m$ (by definition 1).

Proofs with Integer Division

#21. Show that if:

- $n \mid m$, where n, m are positive integers > 1, and
- $a \equiv b \pmod{m}$, where a and b are integers

Then:

- $a \equiv b \pmod{n}$

Since $a \equiv b \pmod{m}$, we know that there exists an integer $a = b + j\,m$ (by theorem 1).

Proofs with Integer Division

#21. Show that if:

- $n \mid m$, where n, m are positive integers > 1, and
- $a \equiv b \pmod{m}$, where a and b are integers

Then:

- $n\, i = m$
- $a = b + jm$

$$
\begin{aligned}
a &= b + jm \\
&= b + jni \\
&= b + (ji)n \\
&= b + kn \\
&\boxed{= b \pmod{n}}
\end{aligned}
$$

Homework #3

Section 3.5 hints and examples.

Euler ϕ-function

$\phi(n)$ = # of positive integers $\leq n$ that are relatively prime to n

- $\phi(4)$
  - $\gcd(4, 4) = 4$
  - $\gcd(3, 4) = 1$
  - $\gcd(2, 4) = 2$
  - $\gcd(1, 4) = 1$
  - $\phi(4) = 2$
- $\phi(10)$
  - $\gcd(1, 10) = 1$
  - $\gcd(3, 10) = 1$
  - $\gcd(7, 10) = 1$
  - $\gcd(9, 10) = 1$
  - $\phi(10) = 4$

Homework #3

Section 3.6 hints and examples.

Number Conversion Motivation

- Binary:
  - Low-level language of computers
  - Easy to represent in electrical systems (“on” versus “off”)
  - Can implement Boolean logic
- Octal:
  - File permissions in Unix often use an octal representation
- Decimal:
  - Number representation used in most modern languages
- Hexadecimal:
  - Used by HTML/CSS to represent colors
  - Character codes often represented in hexadecimal

Decimal Expansion

- $(\,0101\ 1111\,)_2 =$

0	1	0	1	1	1	1	1
7	6	5	4	3	2	1	0

$$0 \times 2^{7} + 1 \times 2^{6} + 0 \times 2^{5} + 1 \times 2^{4} + 1 \times 2^{3} + 1 \times 2^{2} + 1 \times 2^{1} + 1 \times 2^{0}$$

digit

Decimal Expansion

- $(0101\ 1111)_2 =$

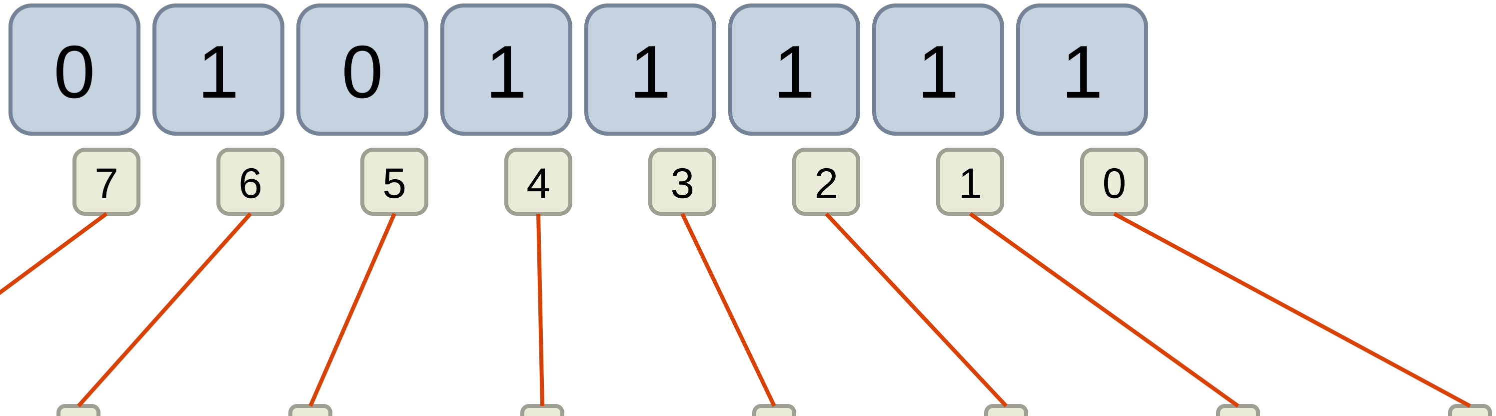
0	1	0	1	1	1	1	1
7	6	5	4	3	2	1	0

$$0 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

"base"
binary = base 2

Decimal Expansion

- $(\ 0101\ 1111\ )_2 =$



$$0 \times 2^{7} + 1 \times 2^{6} + 0 \times 2^{5} + 1 \times 2^{4} + 1 \times 2^{3} + 1 \times 2^{2} + 1 \times 2^{1} + 1 \times 2^{0}$$

position

Decimal Expansion

- $(\,0101\ 1111\,)_2 =$

0	1	0	1	1	1	1	1
7	6	5	4	3	2	1	0

$$0 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

$2^6 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 = 64 + 16 + 8 + 4 + 2 + 1 = 95$

Hexadecimal Expansion

- $177130 = (?)_{16}$

$$177130 \div 16 = 11070.625$$

$$177130 = 16 \times 11070 + 10$$

$$11070 = 16 \times 691 + 14$$

Hexadecimal Expansion

- $177130 = (\ ?\ )_{16}$

177130	=	16	×	11070	+	10	A
11070	=	16	×	691	+	14	E
691	=	16	×	43	+	3	3
43	=	16	×	2	+	11	B
2	=	16	×	0	+	2	2

2 B 3 E A

Hexadecimal Expansion

- 177130 = (?)16

177130	=	16	×	11070	+	10	A	↑
11070	=	16	×	691	+	14	E	
691	=	16	×	43	+	3	3	
43	=	16	×	2	+	11	B	
2	=	16	×	0	+	2	2	

$(2B3EA)_{16}$

Homework #3

Section 3.7 hints and examples.

Examples

- See PDF example for:
  - Euclidean Algorithm
  - Greatest Common Divisor
  - Modular Inverses

Fermat's Little Theorem

- Show that $2^{340} \equiv 1 \ (\bmod\ 11)$:
  - By Fermat's Little Theorem: $a^{10} \equiv 1 \ (\bmod\ 11)$
  - We can rewrite $2^{340} = (2^{10})^{34}$
  - Therefore we get:

$$2^{340} = (2^{10})^{34}$$
$$\equiv (1)^{34} (\bmod 11)$$
$$\equiv 1 (\bmod 11)$$