DISCUSSION #8
FRIDAY MAY 25TH 2007

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ECS20: Discrete Mathematics

Homework 8

Hints and Examples

Section 5.4

Binomial Coefficients

Binomial Theorem

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

- Example:

$$(x+y)^2 = \sum_{j=0}^{2} \binom{2}{j} x^{2-j} y^j$$
$$= \binom{2}{0} x^2 y^0 + \binom{2}{1} x^1 y^1 + \binom{2}{2} x^0 y^2$$
$$= x^2 + 2xy + y^2$$

Binomial Theorem

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

- What is the coefficient of x^9 in $(2 - x)^{19}$?
  - Rewrite as $((-x) + 2)^{19}$
  - We encounter x^9 when $n - j = 9$, or when $j = 10$
  - Therefore that term will look like:

$$\binom{19}{10} * (-x)^9 * 2^{10} = \binom{19}{10} * (-1)^9 * x^9 * 2^{10} = -94{,}595{,}072\, x^9$$

  - Therefore coefficient is -94,595,072.

Example: Expanding $(11_b)^4$

Suppose b is an integer such that $b \geq 7$.
Find the base-b expansion of $(11_b)^4$.

- Hint 1: The numeral 11 in base b represents the number $b + 1$.
  - 11_2 is $2 + 1 = 3$ in binary
  - 11_{10} is $10 + 1 = 11$ in decimal
  - 11_{16} is $16 + 1 = 17$ in hexadecimal

Example: Expanding $(11_b)^4$

- Hint 1: The numeral 11 in base b represents the number $b + 1$.
- Hint 2: Therefore you want to find $(b + 1)^4$
  - Use Binomial Theorem to expand.
  - Use Pascal's Triangle to find coefficients.
- Hint 3: As long as $b \geq 7$, any integer < 7 in base b is that digit.

Example: Expanding $(11_b)^4$

- Hint 3: When $i < b$, then $i = (i)_b$ (meaning there is no change in the digits used.
  - For example: $4 = (4)_{16}$ and $6 = (6)_8$ but $3 = (11)_2$

- Hint 4: The resulting numeral will be the concatenation of the coefficients.
  - For example:
    $13 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = (1101)_2$

Section 6.1

Introduction to Discrete Probability

Finite Probability

- S: Set of possible outcomes
- E: An event such that $E \subseteq S$
- $p(E)$: Probability of event E where

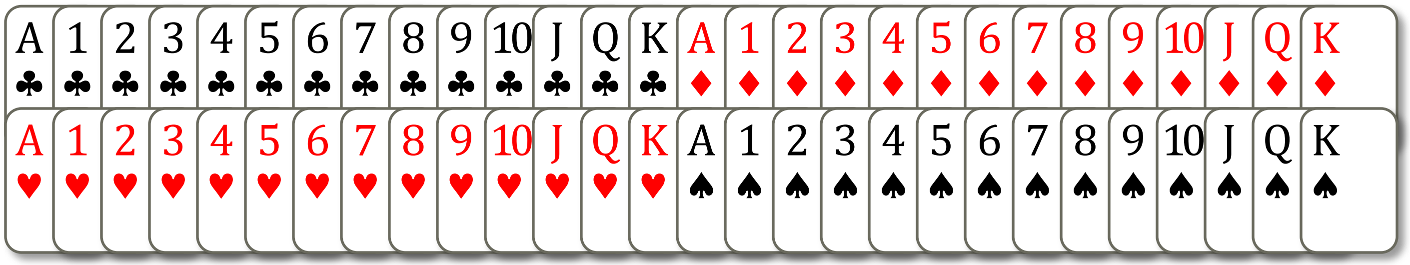
$$p(E) = |E| \div |S|$$

Example: Choosing Cards

What is the probability you choose a king?
A diamond?
A king or a diamond?

Example: Choosing Cards

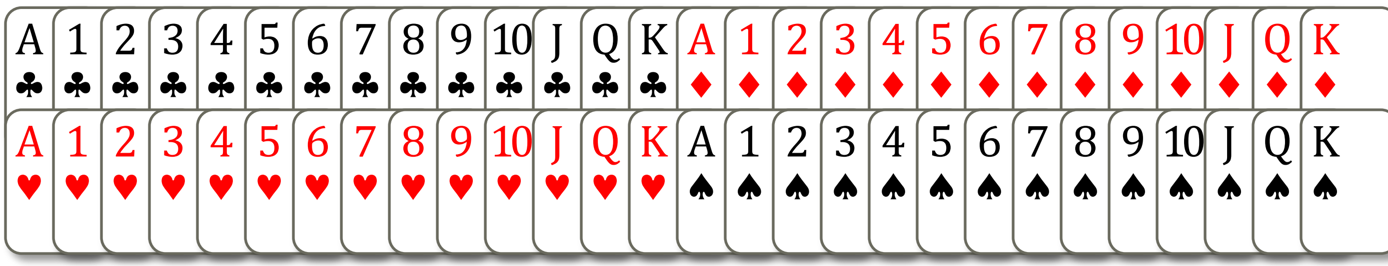
- S: Deck of cards



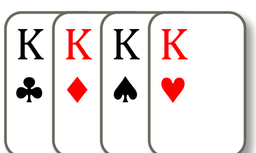
  - What is the size of S?
    - $|S| = 52$ cards total

Example: Choosing Cards

- S: Deck of cards

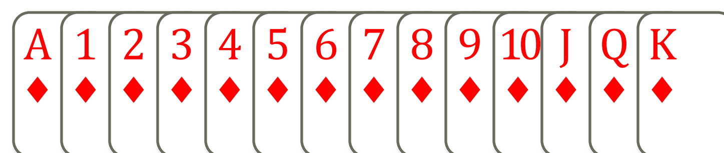


- E_1: King Cards



  - $|E_1| = 4$
  - $p(E_1) = 4 / 52$

- E_2: Diamond Cards

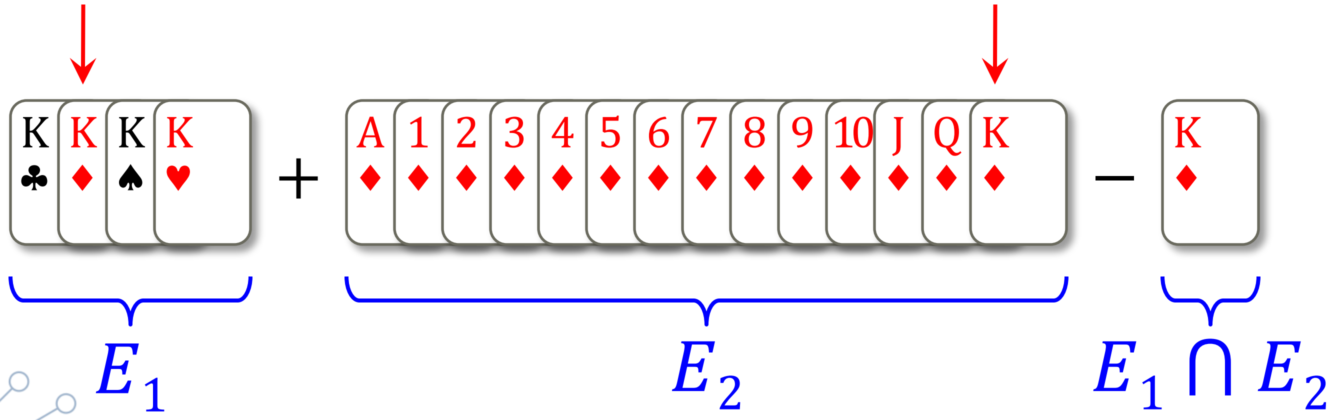


  - $|E_2| = 13$
  - $p(E_2) = 13 / 52$

Example: Choosing Cards

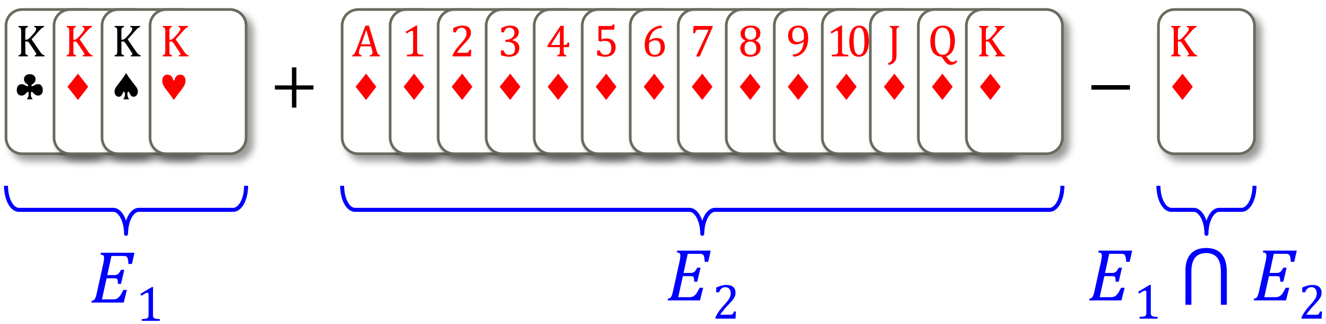
- $p(E_1)$ gives probability we select a king.
- $p(E_2)$ gives probability we select a diamond.

- What about the probability that we select a king or diamond?



Example: Choosing Cards

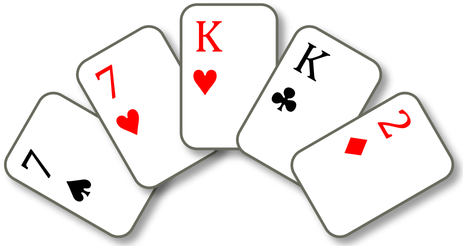
- What about the probability that we select a king or diamond?

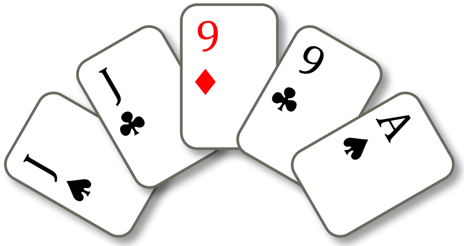


  - $p(E_1 \cup E_2) = p(E_1) + p(E_1) - p(E_1 \cap E_2)$
  - $4/52 + 13/52 - 1/52 = 16/52$

Example: Two Pairs Poker Hands

What is the probability that a five-card poker hand contains two pairs?

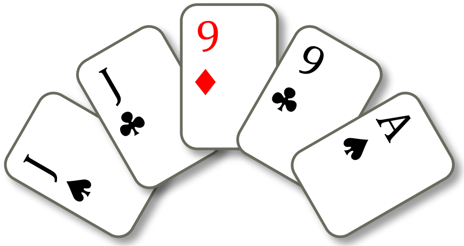




Example: Two Pairs Poker Hands

What about the probability that a five-card poker hand contains two pairs?

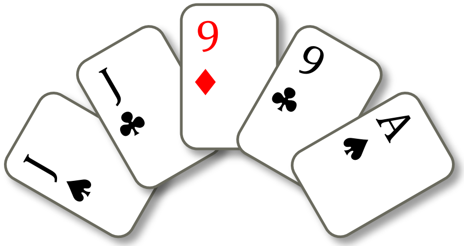
- Looking for two pairs, not a full house (etc.)
- What is a pair?
  - 2 cards with:
    - Same type or number
    - Different suits



Example: Two Pairs Poker Hands

What about the probability that a five-card poker hand contains two pairs?

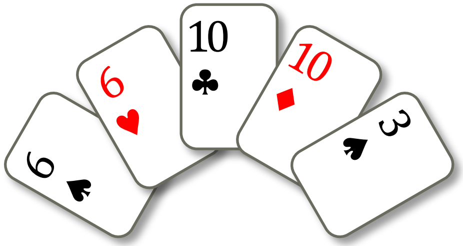
- What is our sample space S?
  - Set of all poker hands
  - $|S| = C(52,5)$
- How do we calculate $|E|$?



Example: Two Pairs Poker Hands

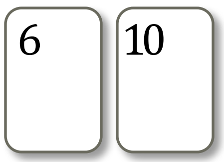
- How do we calculate $|E|$?
  - Use product rule to combine:
    - Possible ways to choose two pairs
    - Possible ways to choose last card

- How do we choose two pairs?
- How do we choose the last card?



Example: Two Pairs Poker Hands

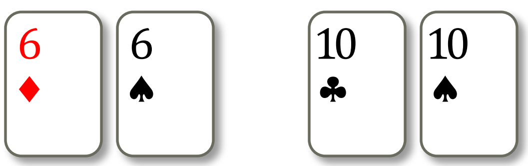
- How do we choose two pairs?
  - (1) Choose two types



Types: { A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K }

$C(13,2)$

  - (2) For each type, choose two suits

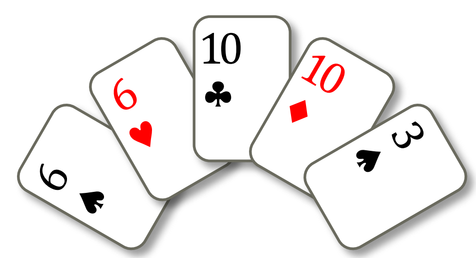


Suits: { ♠, ♦, ♥, ♣ }

$C(4,2)$

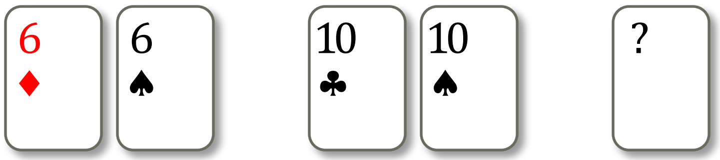
  - (3) Combine using product rule

$C(13,2) \cdot C(4,2) \cdot C(4,2)$

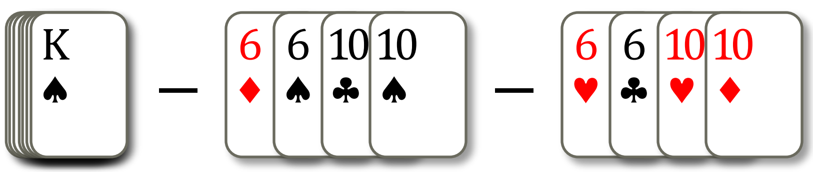


Example: Two Pairs Poker Hands

- How do we choose the last card?



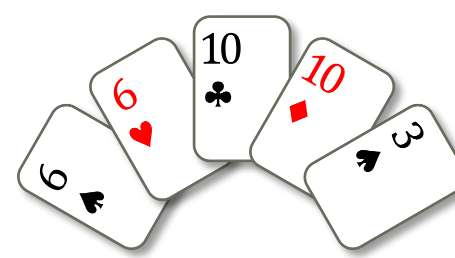
  - Number of choices reduced
    - Can’t choose cards already selected
    - Can’t choose types already selected (no full house)



= 52 – 4 – 4 = 44 cards

  - Choose 1 out of remaining cards

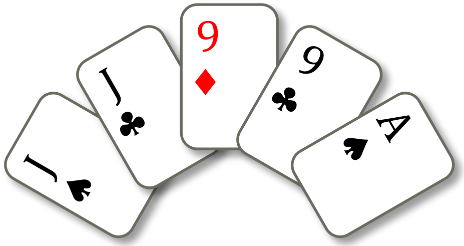
    $C(44,1)$



Example: Two Pairs Poker Hands

> What about the probability that a five-card poker hand contains two pairs?

- Combine all the results
  - $p(E) = \underbrace{C(13,2) \cdot C(4,2) \cdot C(4,2)}_{\text{choose 2 pairs}} \cdot \underbrace{C(44,1)}_{\text{last card}}$



Example: Rolling Dice

Which is more likely:
rolling a total of 9 when two dice are rolled when three dice are rolled?

Example: Rolling Dice

Which is more likely: rolling a 9 when two dice are rolled or when three dice are rolled?

- What is the probability of:
  - Rolling a 9 when two dice are rolled?
  - Rolling a 9 when three dice are rolled?

Example: Rolling Dice

- Probability of rolling a 9 with two dice
  - What is our sample space $|S|$?
    - $6 \cdot 6 = 36$ possible outcomes rolling 2 dice
  - What is our event $|E|$?
    - Enumerate all pairs which sum to 9
      - (6, 3), (3, 6), (5, 4), and (4, 5)
    - 4 possible ways to roll a 9
  - $p(E) = 4 \,/\, 36 \approx 0.111$

Example: Rolling Dice

- Probability of rolling a 9 with three dice
  - What is our sample space $|S|$?
    - $6 \cdot 6 \cdot 6 = 216$ possible outcomes rolling 3 dice
  - What is our event $|E|$?
    - Enumerate all triples which sum to 9
      - Zzzz…
    - 25 possible ways to roll a 9

See Section 5.5 Example 5, etc…

- $p(E) = 25 \,/\, 216 \approx 0.116$

Example: Rolling Dice

Which is more likely: rolling a 9 when two dice are rolled or when three dice are rolled?

- Rolling a 9 with three dice is more likely.

Example: Monty Hall Problem

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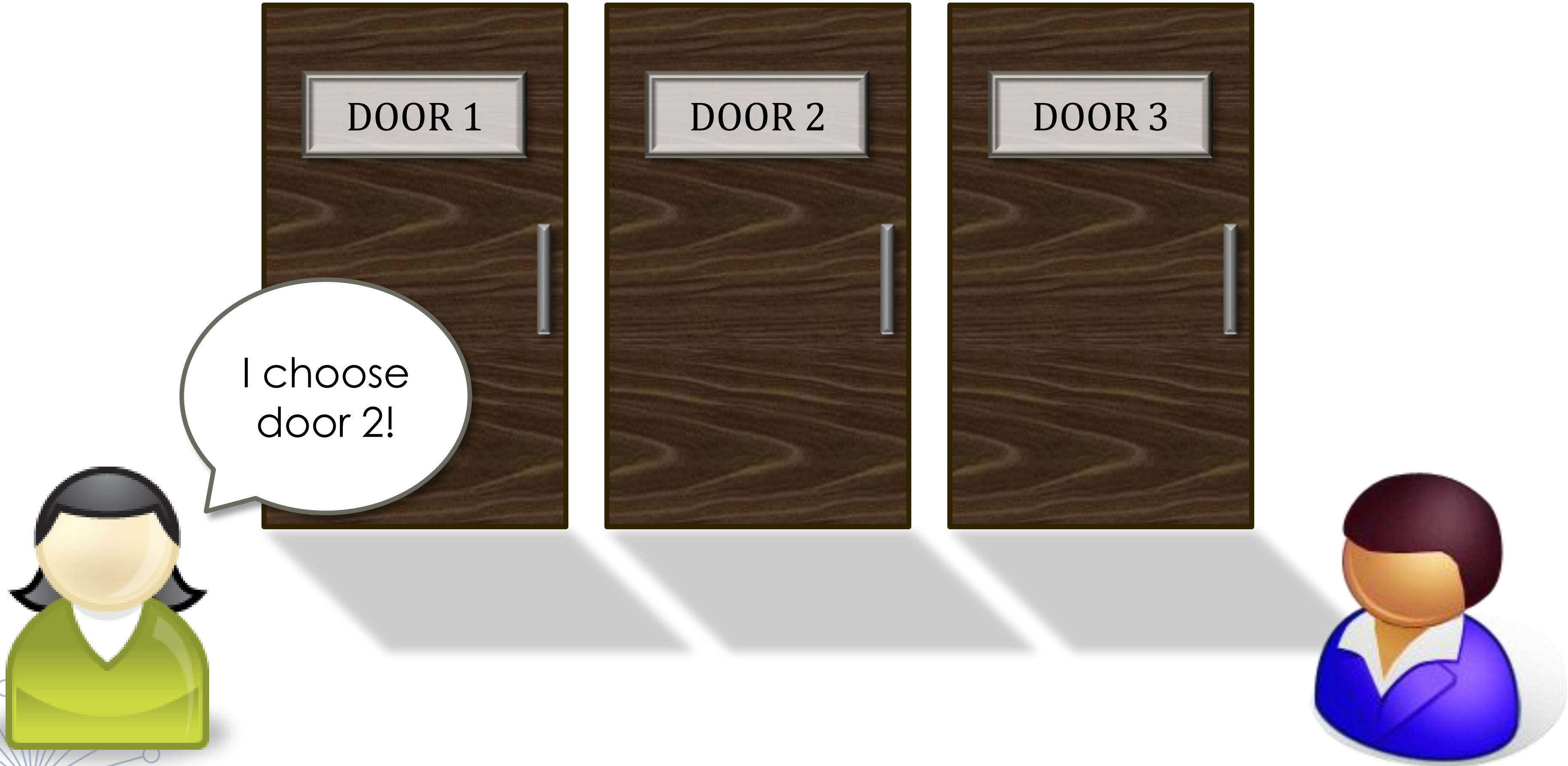

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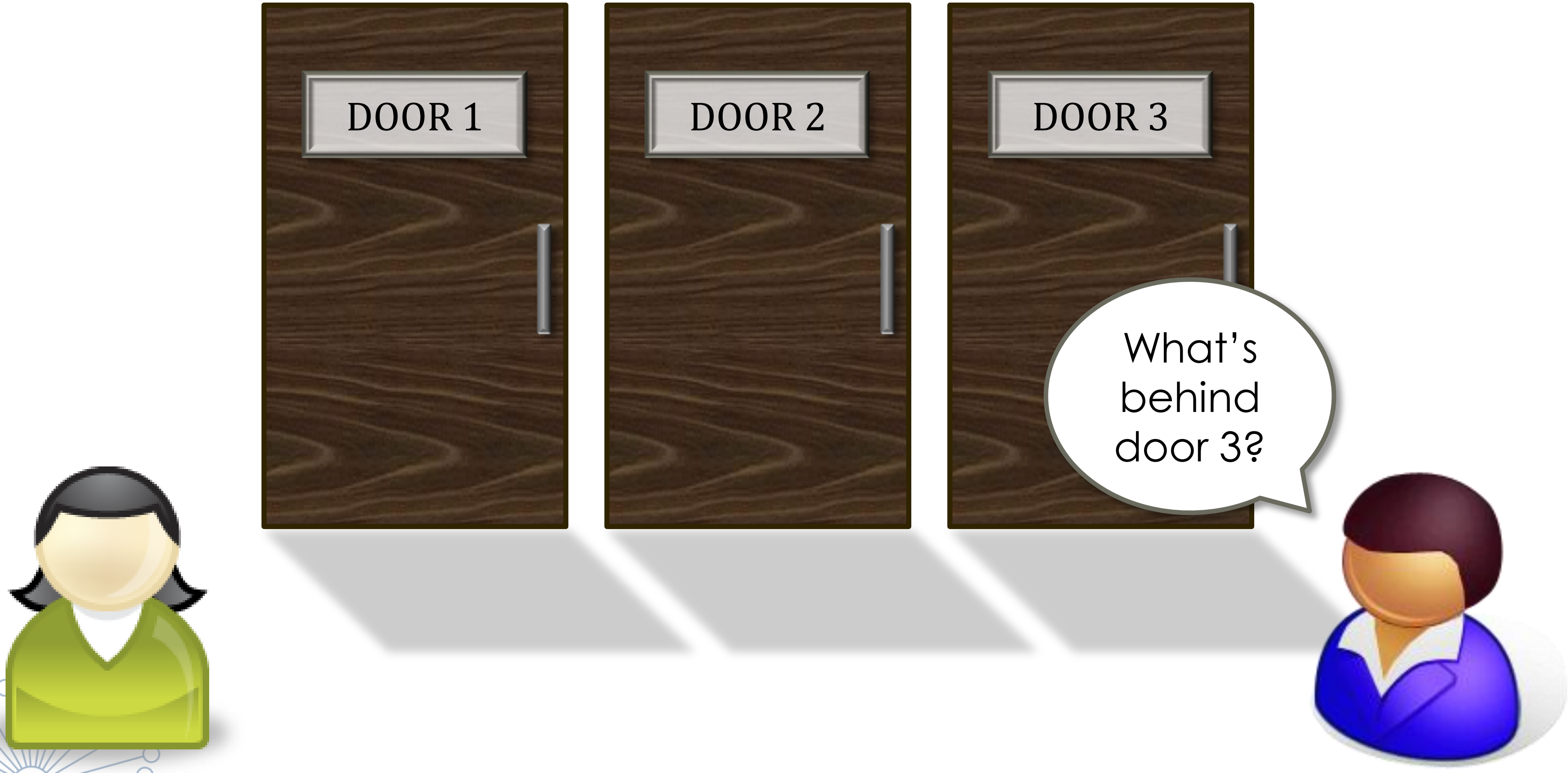

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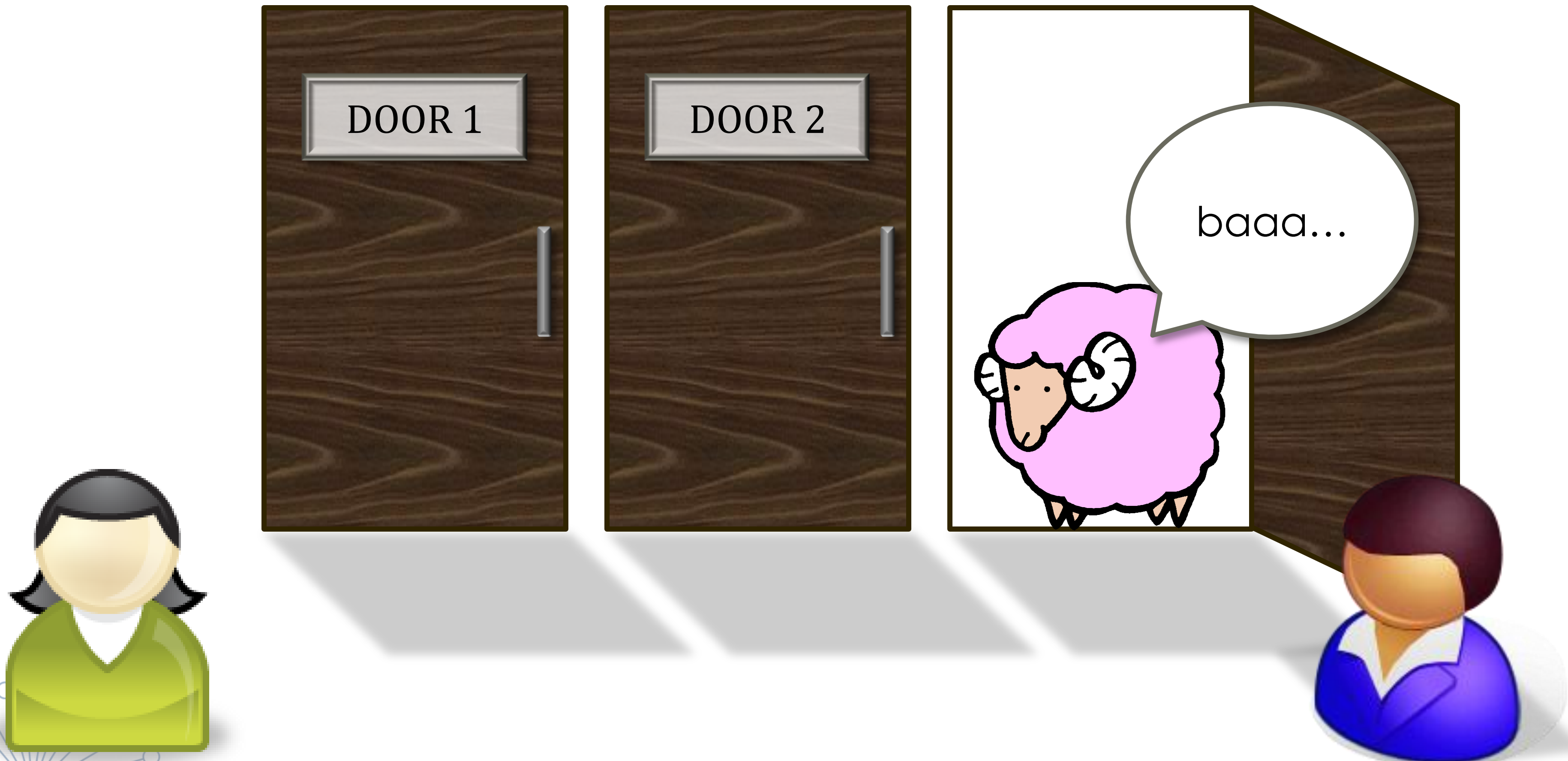

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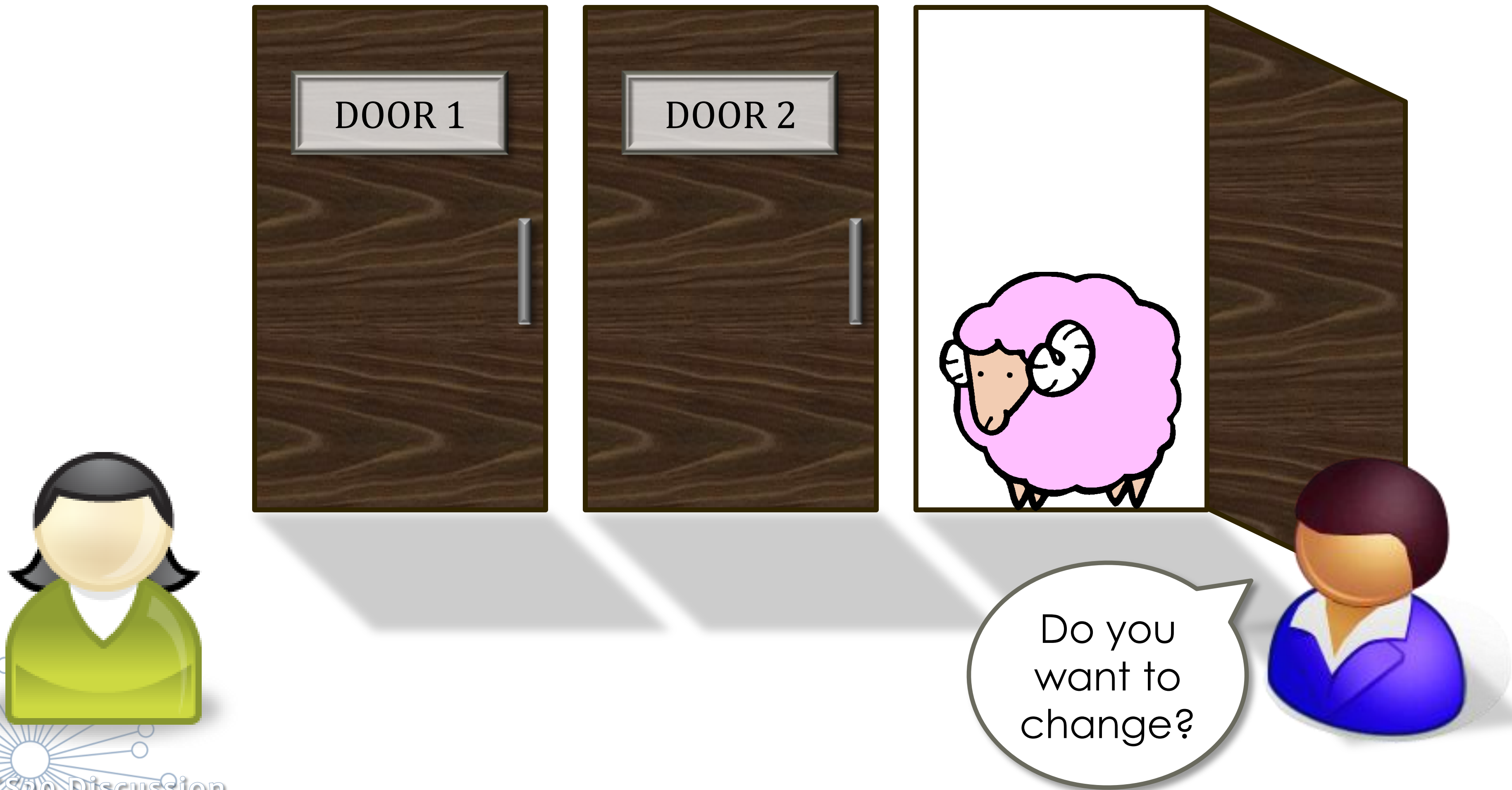

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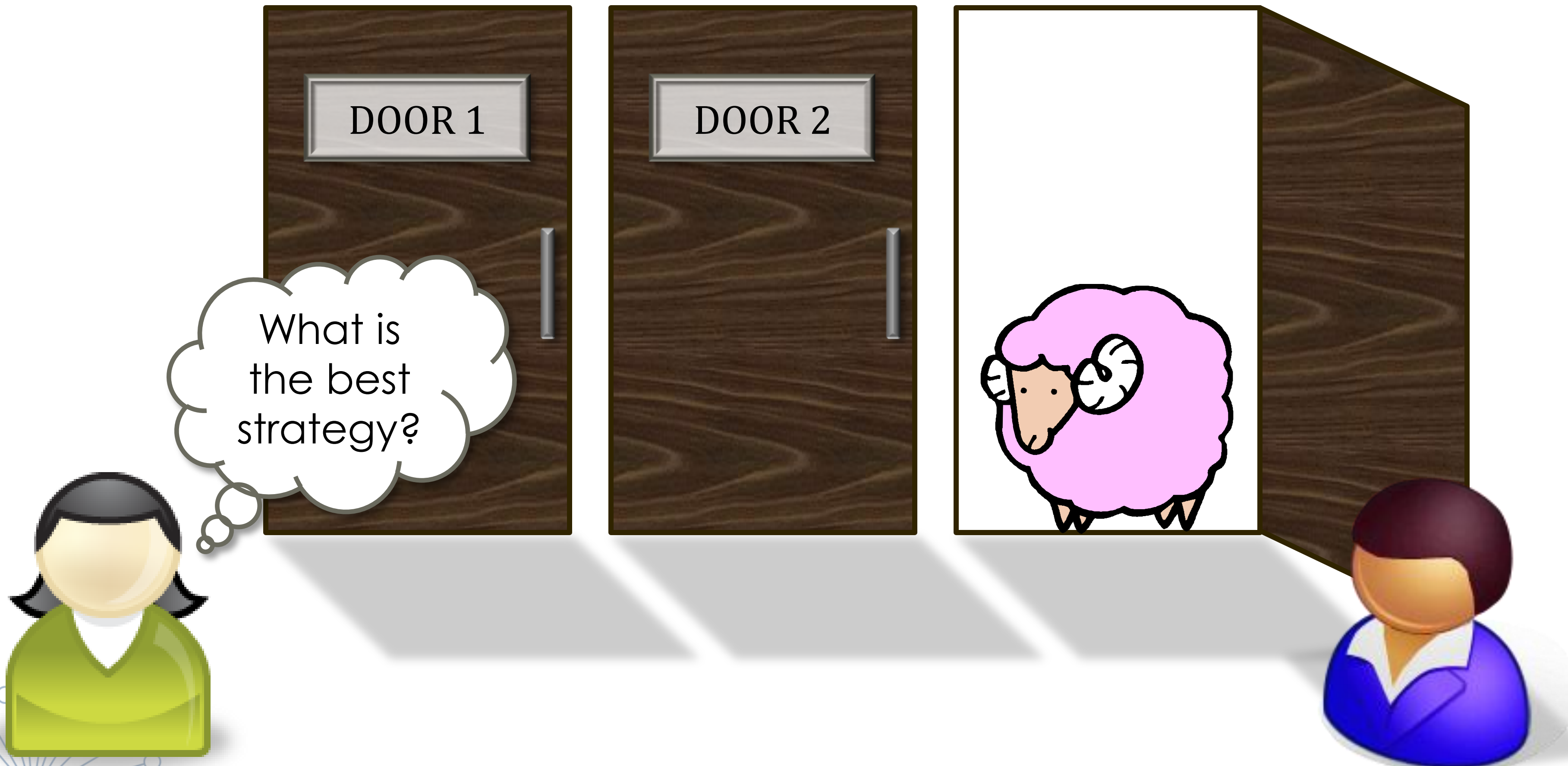

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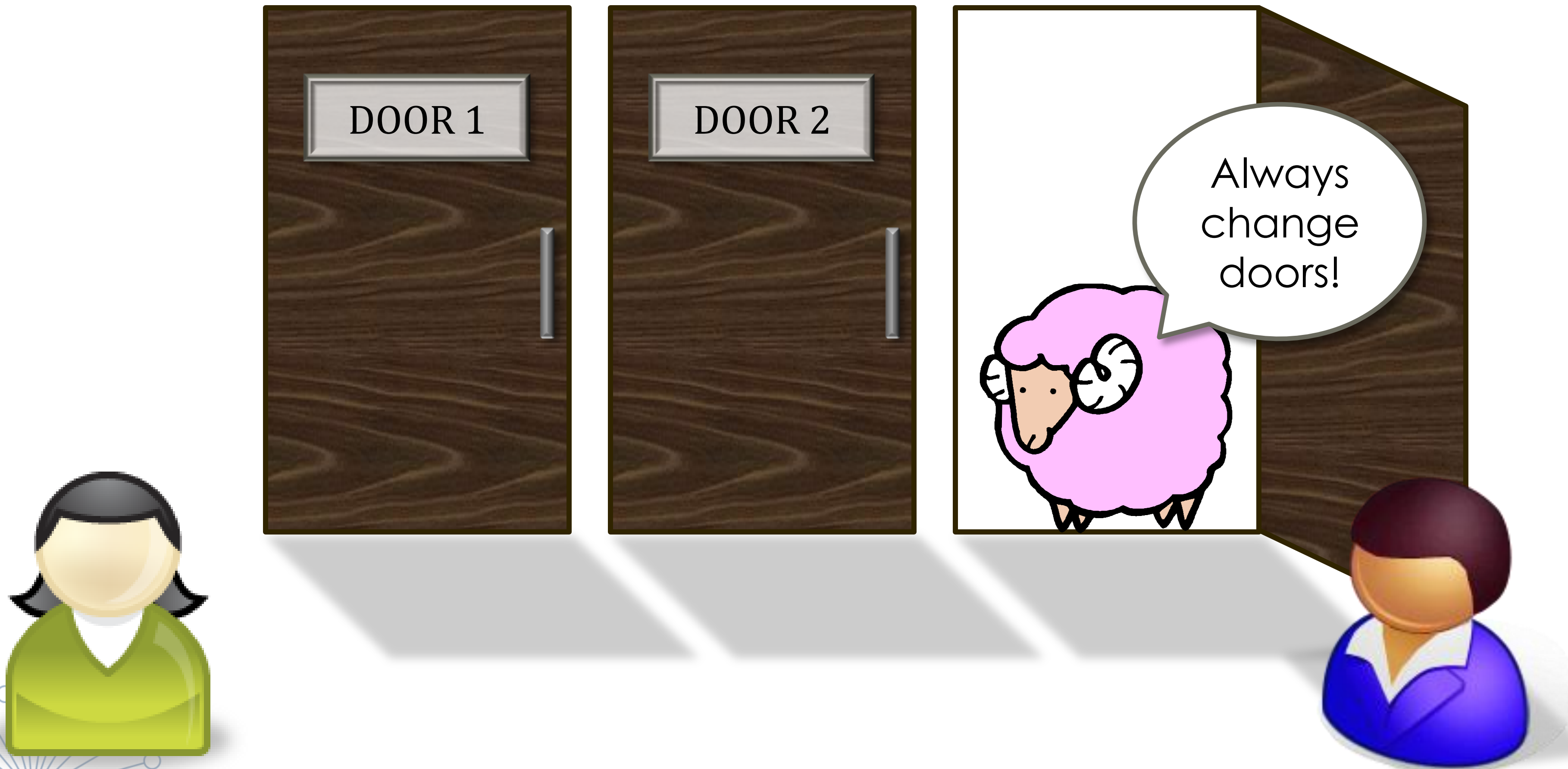

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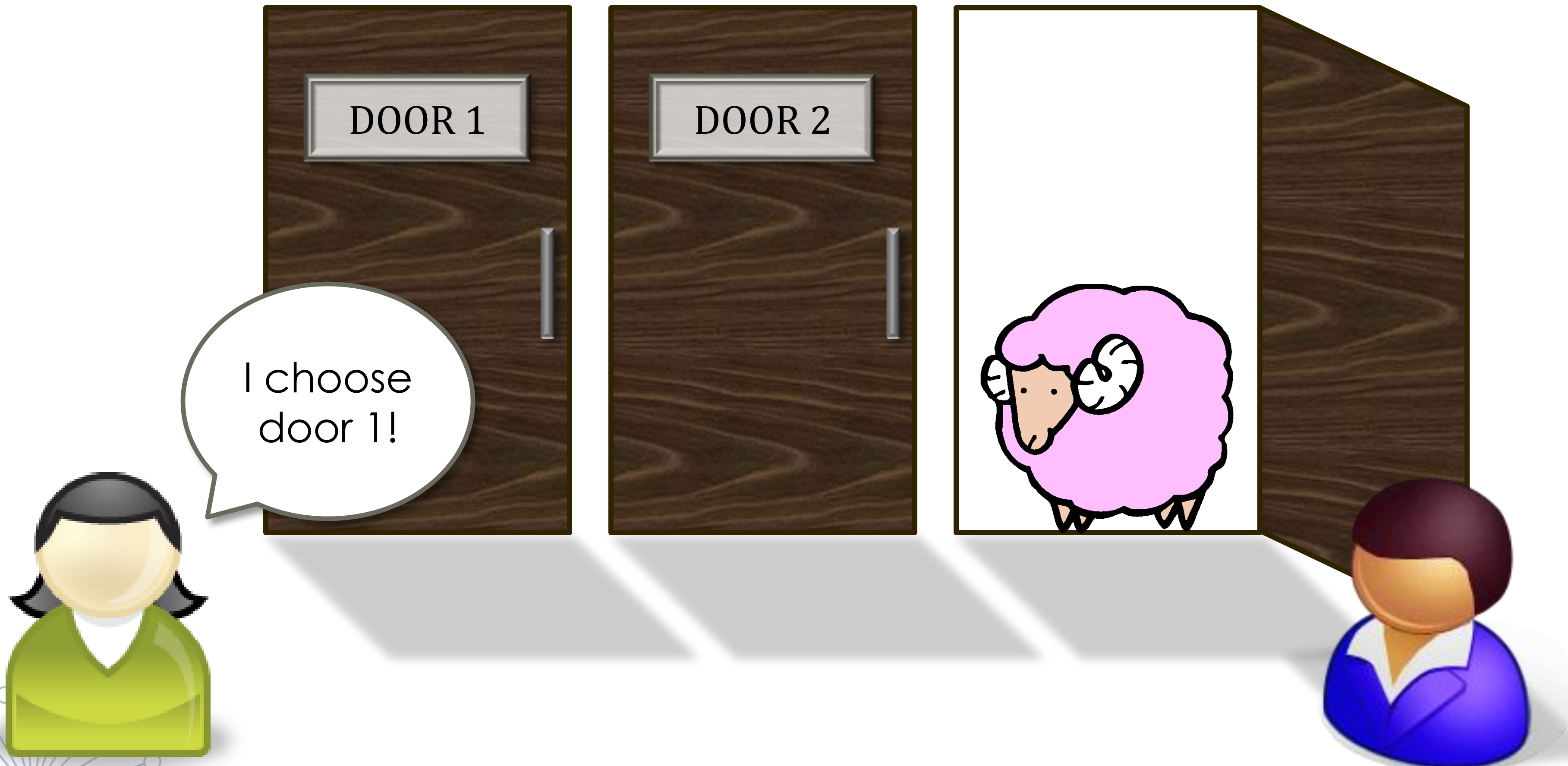
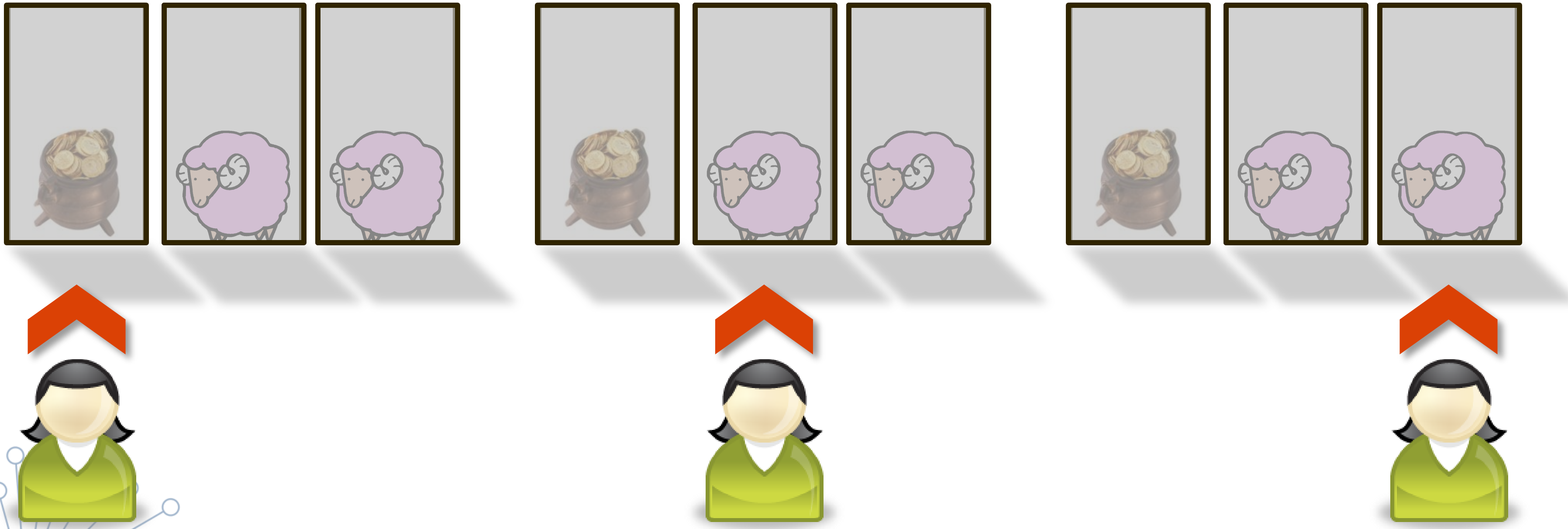


Example: Monty Hall Problem

Example: Monty Hall Problem

- Why is this the best strategy?
  - Look at overall outcomes!



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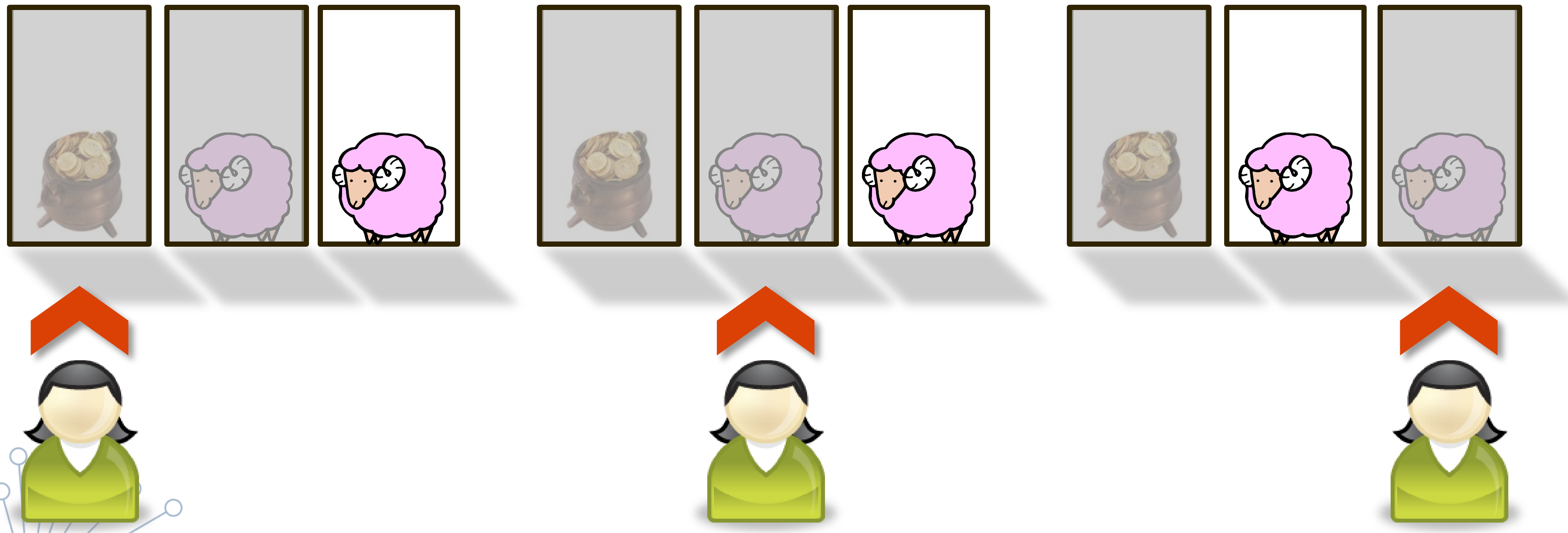

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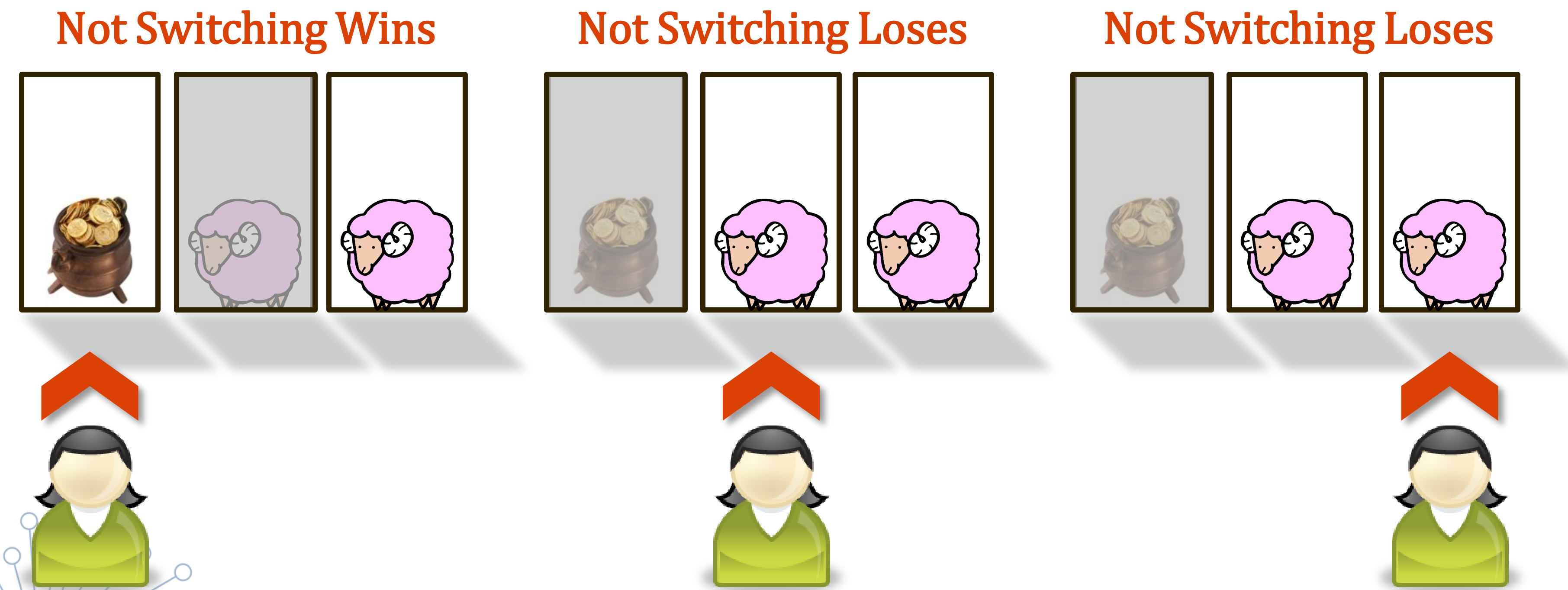

Example: Monty Hall Problem

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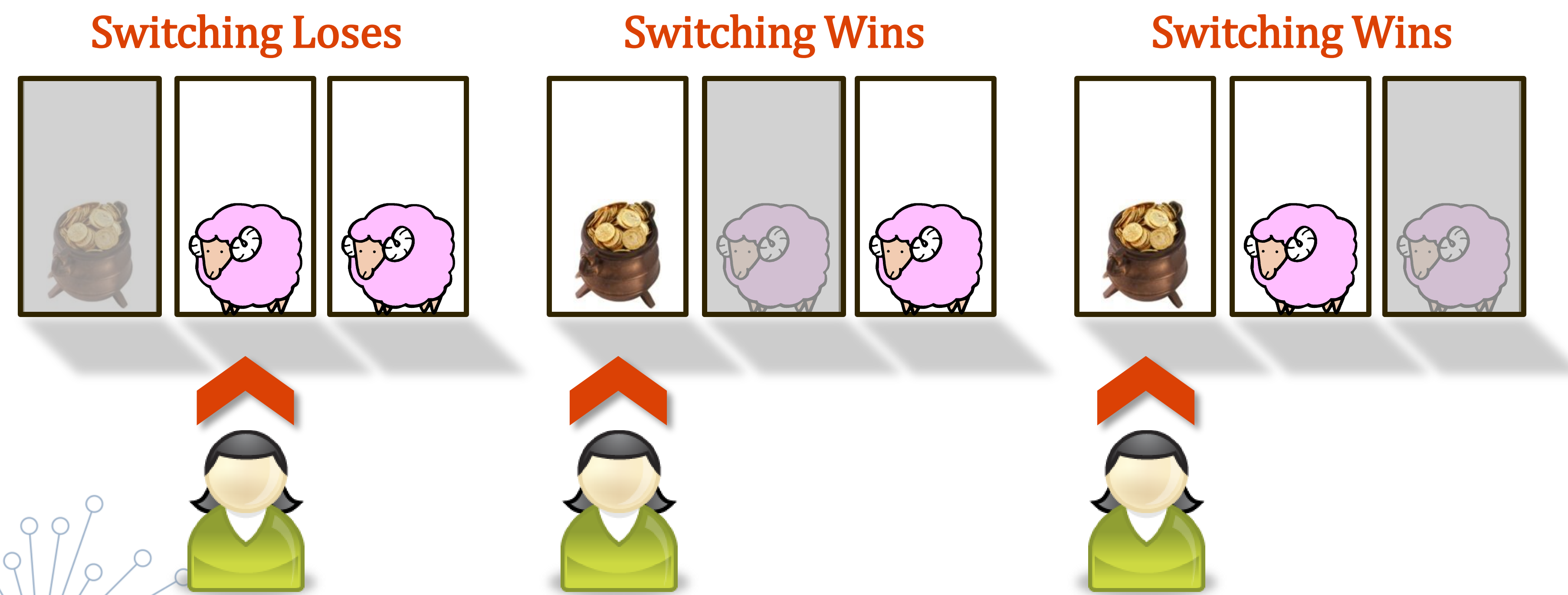

Example: Monty Hall Problem

- Why is this the best strategy?
  - Not switching wins 1/3 times
  - Switching wins 2/3 times

- What happens when you have four doors?
  - What is probability you win when switching?
  - What is probability you win when not switching?