Artificial Intelligence Programming

Introducing First-order Logic

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Propositional Logic has several nice features

- Lets us easily express disjunction and negation
  - “There is a pit in (1,1) or in (2,2)”
  - “There is not a pit in (1,1)”
  - This is hard to do in C/Java/Python

- Separates declarative knowledge from inference procedures

- Compositional
  - The meaning of a sentence is a function of the meaning of its parts.
There are several problems with propositional logic:

- Lack of conciseness. (How to say “There is a pit in every square”?)
- Lack of ability to quantify (How to say “There is a square that has a pit”?)
- Lack of ability to deal with large domains.
  - What if there were an infinite number of rooms?
  - What is we wanted to write sentences about the integers?

The problem stems from the fact that propositional logic deals with facts that are either true or false.

No way to talk about relations between facts.
We would like the sorts of structures that are useful in programming languages. In particular, we would like to have:

- Objects: Wumpi, pits, gold, vacuum cleaners, etc.
- Relations: These can include:
  - Unary relations (or properties): smelly(wumpus), shiny(gold), sleepy(student), etc.
  - Binary relations: brother-of(bart, lisa) holding(agent, gold), after(Tuesday, Monday)
  - $n$-ary relations: simpsons(homer, marge, bart, lisa, maggie)
  - These are sometimes called *predicates*
- Functions: father-of(bart) = homer, fall-classes(student) = AI, etc.

*First-order logic* gives us all of this.
Recall that a model is the set of “possible worlds” for a collection of sentences.

In propositional logic, this meant truth assignments to facts.

In FOL, models have objects in them.

The *domain* of a model is the set of objects in that world.

For example, the Simpsons model might have the domain

\{Marge, Homer, Lisa, Bart, Maggie\}

We can then specify relations and functions between these objects

- married-to(marge, homer), baby(maggie), father(bart) = homer
12-4: Terms and sentences

A term is an expression that refers to a single object.
- Bart, Lisa, Homer
- We can also use functions as terms - Saxophone(Lisa) refers to the object that is Lisa’s saxophone

An atomic sentence consists of a predicate applied to terms
- Brother-of(Lisa, Bart), Married(Homer, Marge), Married(Mother(Lisa), Father(Bart))
A Complex sentence uses logical connectives $\neg$, $\lor$, $\land$, $\Rightarrow$, $\Leftrightarrow$ to join atomic sentences.

- $\neg \text{BrotherOf}(\text{Homer}, \text{Bart})$,
- $\text{MotherOf}(\text{Lisa}, \text{Marge}) \Rightarrow \text{MotherOf}(\text{Bart}, \text{Marge})$
- $\text{Oldest}(\text{Bart}) \lor \text{Oldest}(\text{Lisa})$

We can also use equality to relate objects: 
\( \text{homer} = \text{father}(\text{Bart}) \)
Often, it’s not enough to make a statement about particular objects. Instead, we want to make a statement about some or all objects.

- “All of the Simpsons are yellow.”
- “At least one of the Simpsons is a baby.”
- Quantifiers allow us to do this.
- $\forall$ is the symbol for universal quantification
  - It means that a sentence holds for every object in the domain.
  - $\forall x\text{Simpson}(x) \Rightarrow \text{yellow}(x)$
∃ is the symbol for existential quantification
  △ It means that the sentence is true for at least one element in the domain.
  △ \( \exists x \text{female}(x) \land \text{playsSaxophone}(x) \)
  △ What would happen if I said
     \( \exists x \text{female}(x) \Rightarrow \text{playsSaxophone}(x) \)?
In general, \( \Rightarrow \) makes sense with \( \forall \) (\( \wedge \) is usually too strong).

\( \wedge \) makes sense with \( \exists \) (\( \Rightarrow \) is generally too weak.)

Some examples:
- One of the Simpsons works at a nuclear plant.
- All of the Simpsons are cartoon characters.
- There is a Simpson with blue hair and a green dress.
- There is a Simpson who doesn’t have hair.
12-9: *Nesting quantifiers*

- Often, we’ll want to express more complex quantifications. For example, “every person has a mother”
  - $\forall x \exists y \text{mother}(x, y)$
  - Notice the scope - for each $x$, a different $y$ is (potentially) chosen.

- What if we said $\exists y \forall x \text{mother}(x, y)$?

- this is not a problem when nesting quantifiers of the same type.

- $\forall x \forall y \text{brotherOf}(x, y) \Rightarrow \text{siblingOf}(x, y)$ and $\forall y \forall x \text{brotherOf}(x, y) \Rightarrow \text{siblingOf}(x, y)$ are equivalent.

- We often write that as $\forall x, y \text{brotherOf}(x, y) \Rightarrow \text{siblingOf}(x, y)$
6 We can negate quantifiers
   \[ \neg \forall x \text{yellow}(x) \] says that it is not true that everyone is yellow.
   \[ \exists x \neg \text{yellow}(x) \] has the same meaning - there is someone who is not yellow.
   \[ \neg \exists x \text{daughterOf}(Bart, x) \] says that there does not exist anyone who is Bart’s daughter.
   \[ \forall x \neg \text{daughterOf}(Bart, x) \] says that for all individuals they are not Bart’s daughter.

6 In fact, we can use DeMorgan’s rules with quantifiers just like with \( \land \) and \( \lor \).
12-11: More examples

6 A husband is a male spouse
   △ ∀x, y husband(x, y) ⇔ spouse(x, y) ∧ male(x)

6 Two siblings have a parent in common
   △ ∀x, y sibling(x, y) ⇔
      ∃p Parent(x, p) ∧ Parent(y, p)
      ¬(x = y)

6 Everyone who goes to Moe’s likes either Homer or Barney (but not both)
   △ ∀x goesTo(moes, x) ⇒
      (Likes(x, Homer) ⇔ ¬Likes(x, Barney))
Everyone knows someone who is angry at Homer.
\[ \forall x \exists y \text{knows}(x, y) \land \text{angryAt}(y, \text{homer}) \]

Everyone who works at the power plant is scared of Mr. Burns
\[ \forall x \text{worksAt}(\text{PowerPlant}, x) \Rightarrow \text{scaredOf}(x, \text{burns}) \]
Everyone likes Lisa.

Someone who works at the power plant doesn’t like Homer. (both ways)

Bart, Lisa, and Maggie are Marge’s only children.

People who go to Moe’s are depressed.

There is someone in Springfield who is taller than everyone else.

When a person is fired from the power plant, they go to Moe’s.

Everyone loves Krusty except Sideshow Bob.

Only Bart skateboards to school

Someone with large feet robbed the Quickie-mart.