1. Consider a B-Tree with maximum degree \( k \) (that is, all interior nodes have \( \lceil k/2 \rceil \ldots k \) children – a 2-3 tree is a B-Tree with maximum degree 3).

   (a) What is the largest number of keys that can be stored in a B-Tree of height \( h \) with maximum degree \( k \)?

   (b) What is the smallest number of keys that can be stored in a B-Tree of height \( h \) with maximum degree \( k \)?

   Show your work!

   (Hint: You may find the following formula helpful:)

   \[
   \sum_{i=0}^{n} x^i = \frac{x^{n+1} - 1}{x - 1}
   \]

2. Leftist Heaps

   (a) The following numbers are inserted (in this order) into a leftist heap. Show the resulting heap after each insert operation (so your answer should contain 9 heaps):

   5 4 6 1 7 9 3 8 2

   (b) Show the result of removing the smallest element from the final heap in part (a) above.

3. What is the smallest number of elements you need to insert into an empty AVL tree to cause a double rotation? Give a sequence of elements to insert that will cause a double rotation, and show the tree right before and after the last insertion.