Computer Science 245

Homework 2

Algorithm Analysis II

Due Wednesday, February 11th, 2015

1. For each of the following recursive functions:

   - (1 point) Describe what the function computes (careful, some of these are tricky!)
   - (1 point) Give a recurrence relation that describes the running time of the function (Give both base and recursive cases)
   - (1 point) Solve the recurrence to get a Θ running time for the function. Use either the repeated substitution method, or the recursion tree method (which is essentially the same as the repeated substitution method, just a little more graphical). *Do not* use the master method for this question (you will have a chance to use the master method on later questions!)

   (a)
   ```
   int recursive1(int n)
   {
     if (n == 0)
       return 0;
     else
       1 + recursive1(n-1);
   }
   ```

   (b)
   ```
   int recursive2(int n)
   {
     if (n == 0)
       return 1;
     else
       recursive2(n-1) +  recursive2(n-1);
   }
   ```

   (c)
   ```
   int recursive3(int n)
   {
     if (n == 0)
       return 1;
     else
       2 * recursive3(n-1);
   }
   ```

   (d)
   ```
   int recursive4(int n)
   {
     int no_op;
     if (n > 1)
     {
        for (int i = 0; i < n; i++)
        {
           no_op++;
        }
        return recursive4(n/2) * recursive4(n/2);
     }
     else
     {
        return 0;
     }
   }
   ```

2. Use the substitution method (that is, proof by induction) to prove the following bounds:

   (a) (3 points) $O(n^2)$ bound for:

$$\begin{aligned} T(0) &= C_1 \\ T(1) &= C_1 \\ T(n) &= T(n-2) + C_2 n \end{aligned}$$

   (b) (3 points) $O(2^n)$ bound for:

$$\begin{aligned} T(0) &= C_1 \\ T(1) &= C_1 \\ T(n) &= 2T(n-1) + C_2 \end{aligned}$$

3. Use the master method to find Θ bounds for the following recurrence relations:

   (a) (1 point)

$$\begin{aligned} T(0) &= C_1 \\ T(1) &= C_1 \\ T(n) &= 4T(n/4) + n^2 \end{aligned}$$

   (b) (1 point)

$$\begin{aligned} T(0) &= C_1 \\ T(1) &= C_1 \\ T(n) &= 16T(n/2) + n^4 + 2n^2 + n \end{aligned}$$

   (c) (1 point)

$$\begin{aligned} T(0) &= C_1 \\ T(1) &= C_1 \\ T(n) &= 4T(n/2) + n \end{aligned}$$