Data Structures and Algorithms

CS245-2013S-24

Binomial Heaps

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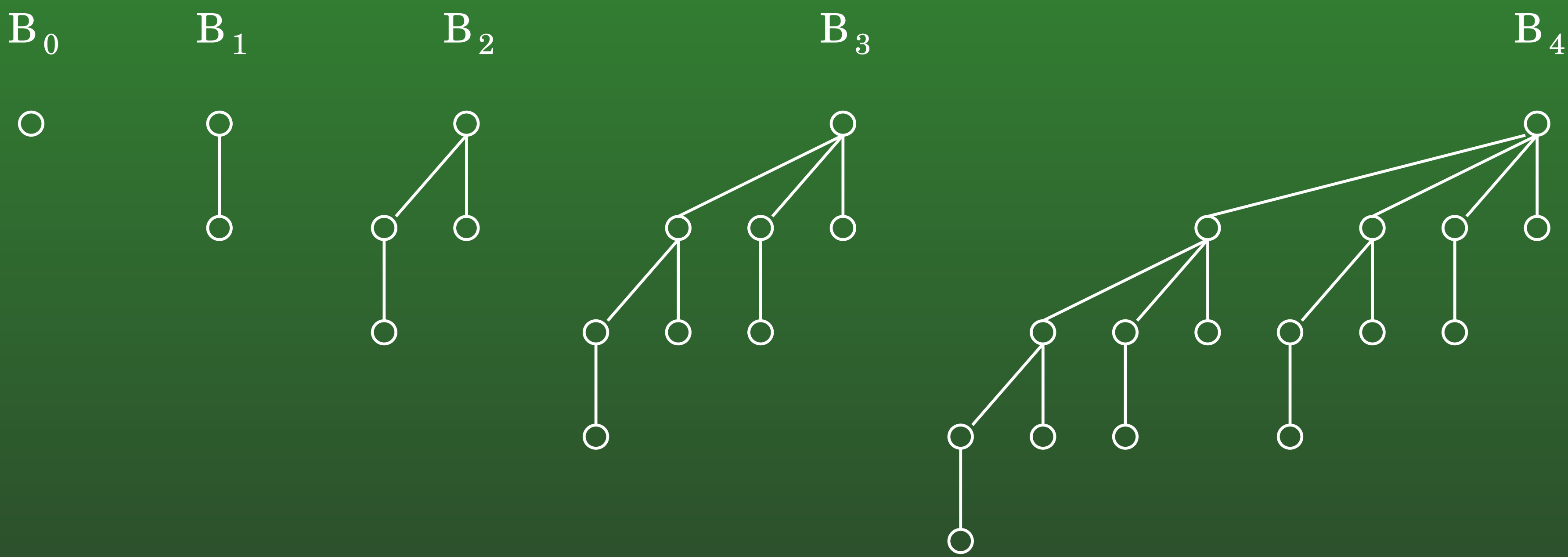
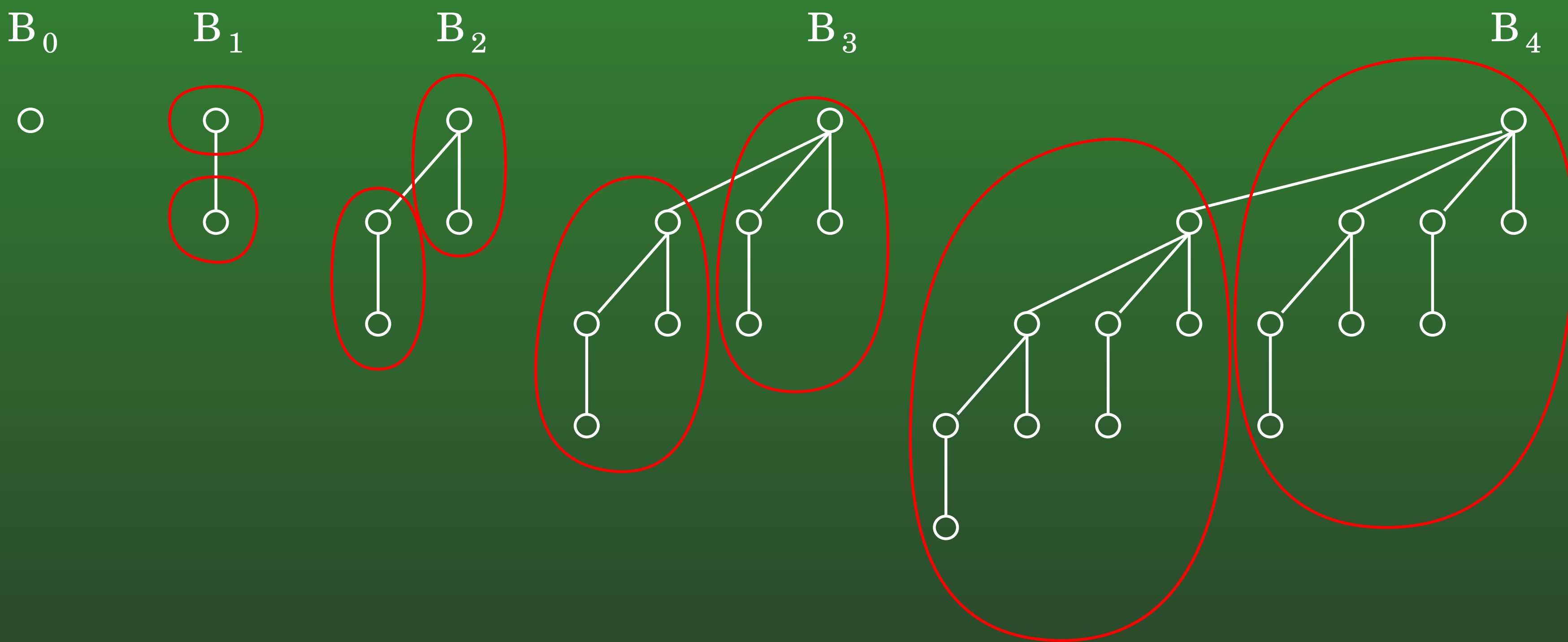
24-0: Binomial Trees

- B_0 is a tree containing a single node
- To build B_k:
  - Start with B_{k-1}
  - Add B_{k-1} as left subtree

24-1: Binomial Trees



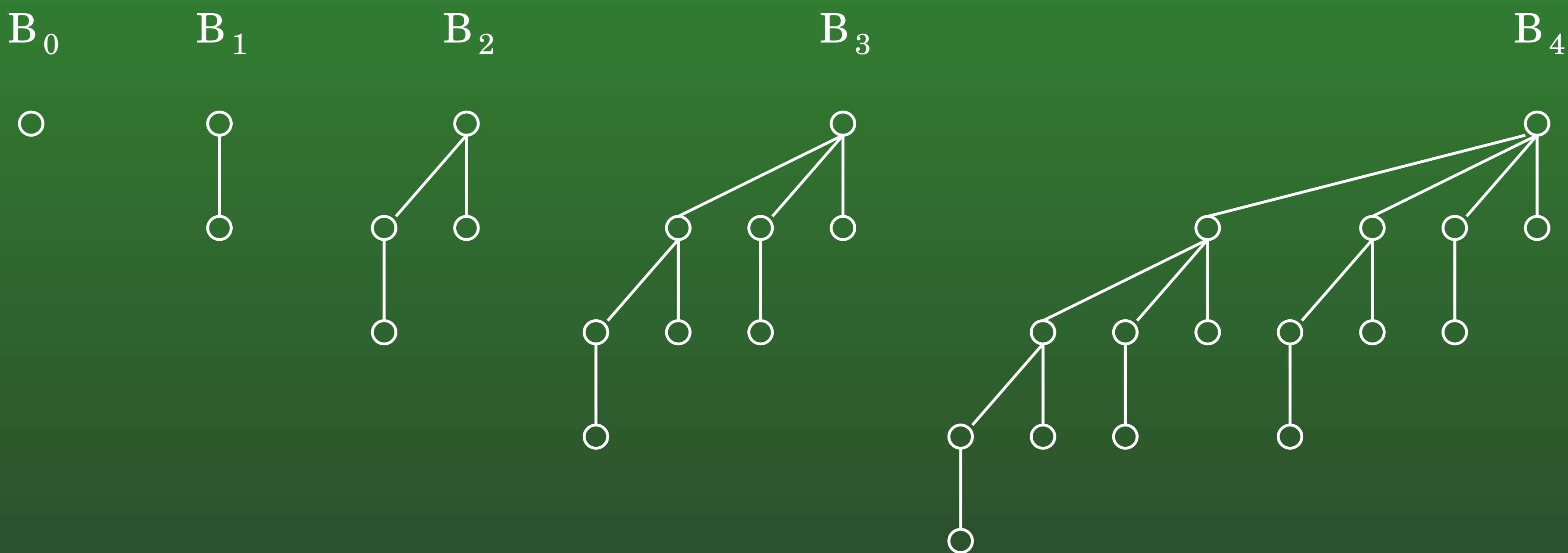
24-2: Binomial Trees



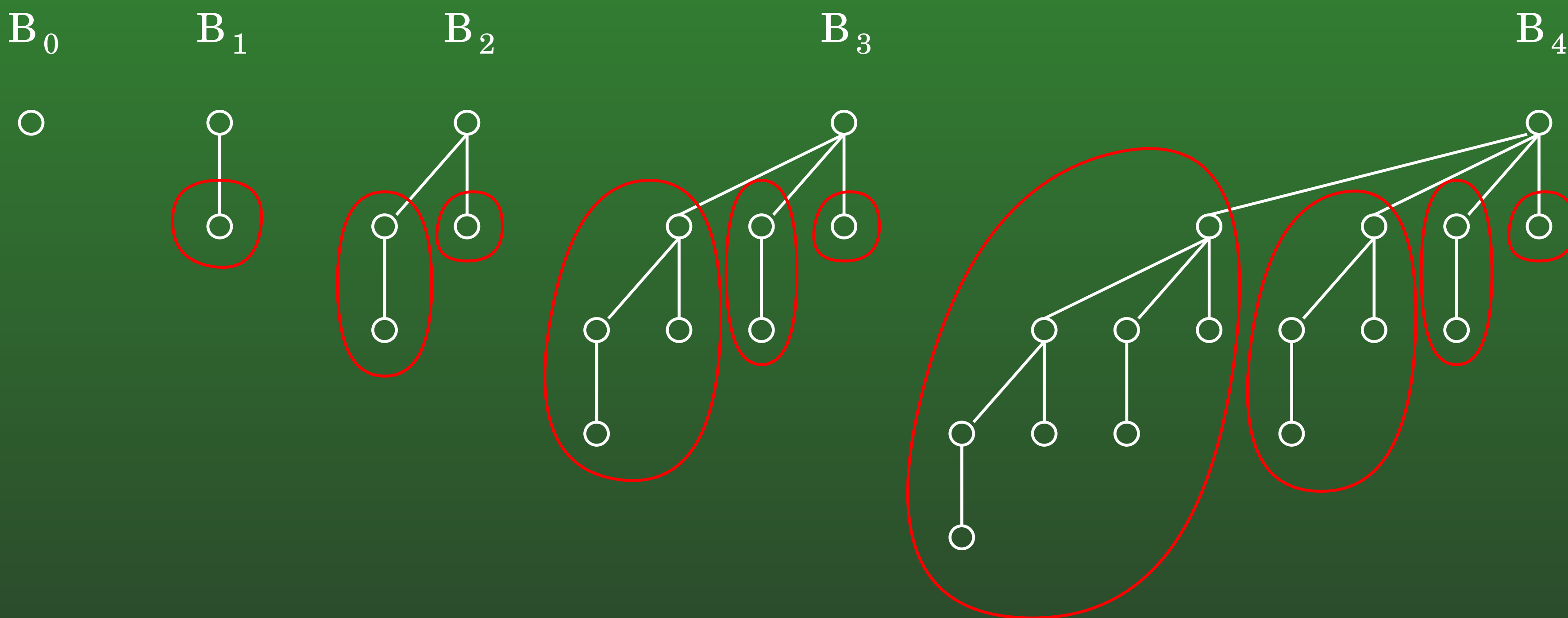
24-3: Binomial Trees

- Equivalent defintion
  - B_0 is a binomial heap with a single node
  - B_k is a binomial heap with k children:
    - $B_0 \ldots B_{k-1}$

24-4: Binomial Trees



24-5: Binomial Trees



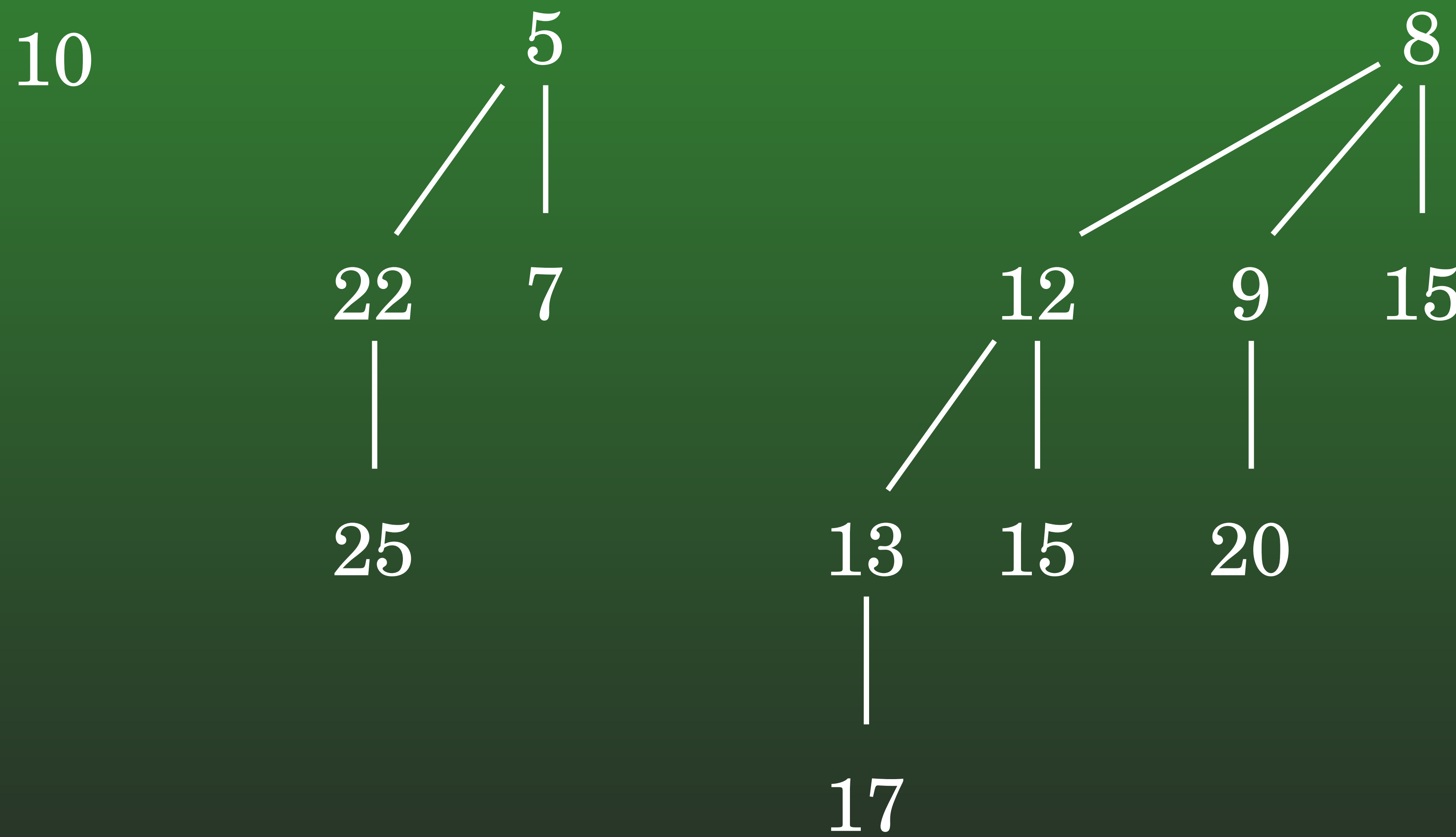
24-6: Binomial Trees

- Properties of binomial trees B_k
  - Contains 2^k nodes
  - Has height k
  - Contains $\binom{k}{i}$ nodes at depth i for $i = 0 \ldots k$

Binomial Heaps

- A Binomial Heap is:
  - Set of binomial trees, each of which has the heap property
    - Each node in every tree is $<=$ all of its children
  - All trees in the set have a different root degree
    - Can't have two B_3's, for instance

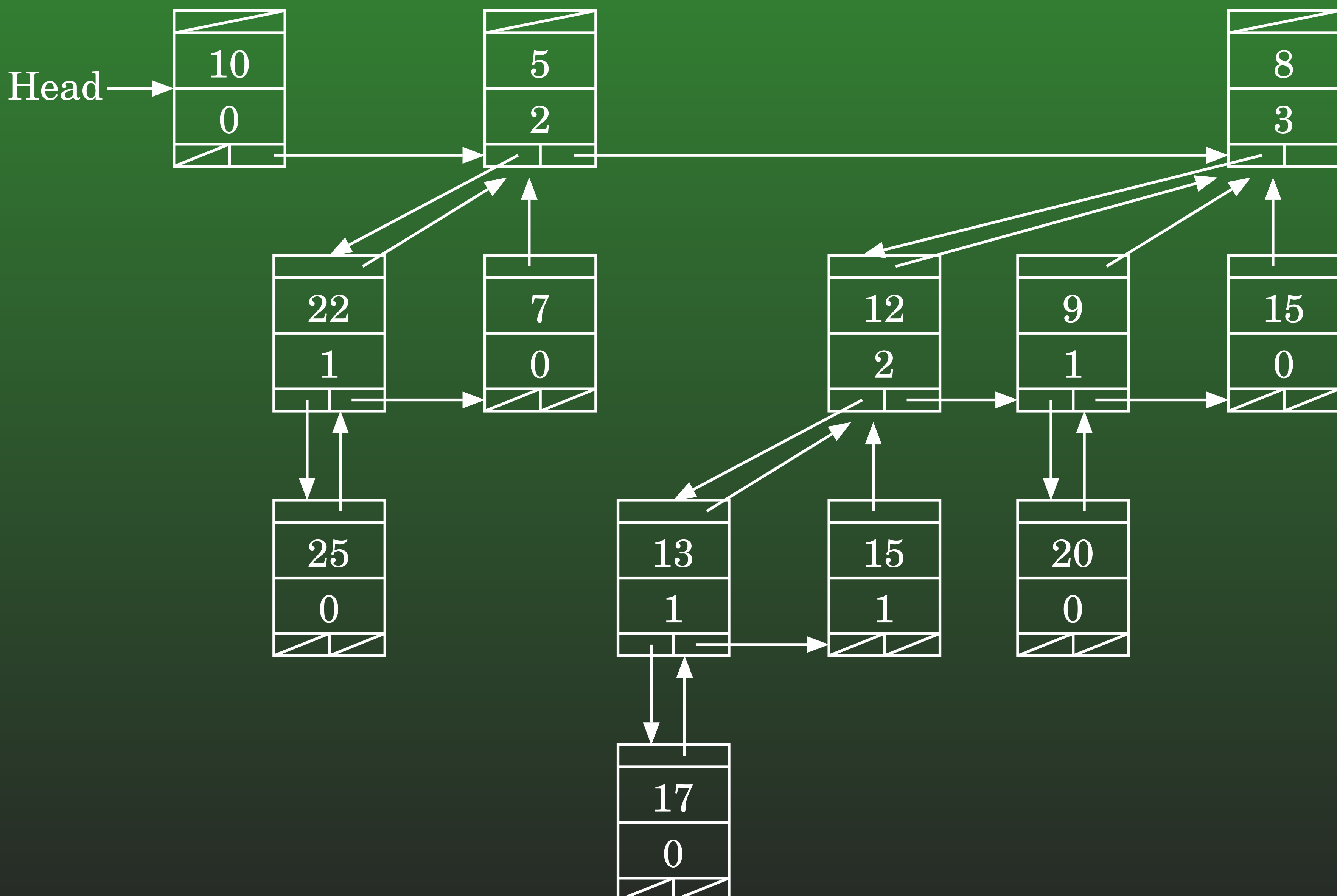
24-8: Binomial Heaps



24-9: Binomial Heaps

- Representing Binomial Heaps
  - Each node contains:
    - left child, right sibling, parent pointers
    - degreee (is the tree rooted at this node B_0, B_1, etc.)
    - data
  - Each list of children sorted by degree

24-10: Binomial Heaps



24-11: Binomial Heaps

- How can we find the minimum element in a binomial heap?
- How long does it take?

Binomial Heaps

- How can we find the minimum element in a binomial heap?
  - Look at the root of each tree in the list, find smallest value
- How long does it take?
  - Heap has n elements
  - Represent n as a binary number
  - B_k is in heap iff kth binary digit of n is 1
  - Number of trees in heap $\in O(\lg n)$

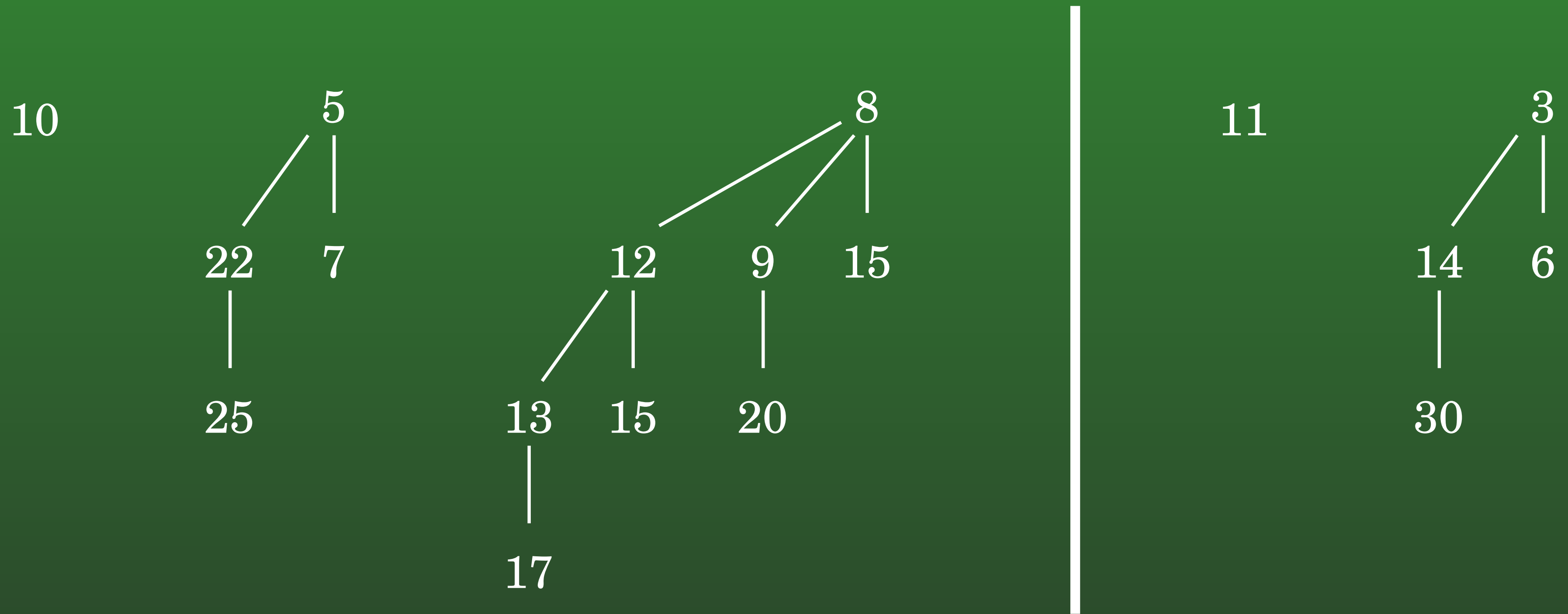
Binomial Heaps

- Merging Heaps H_1 and H_2
  - Merge root lists of H_1 and H_2
  - What property of binomial heaps may be broken?
  - How do we fix it?

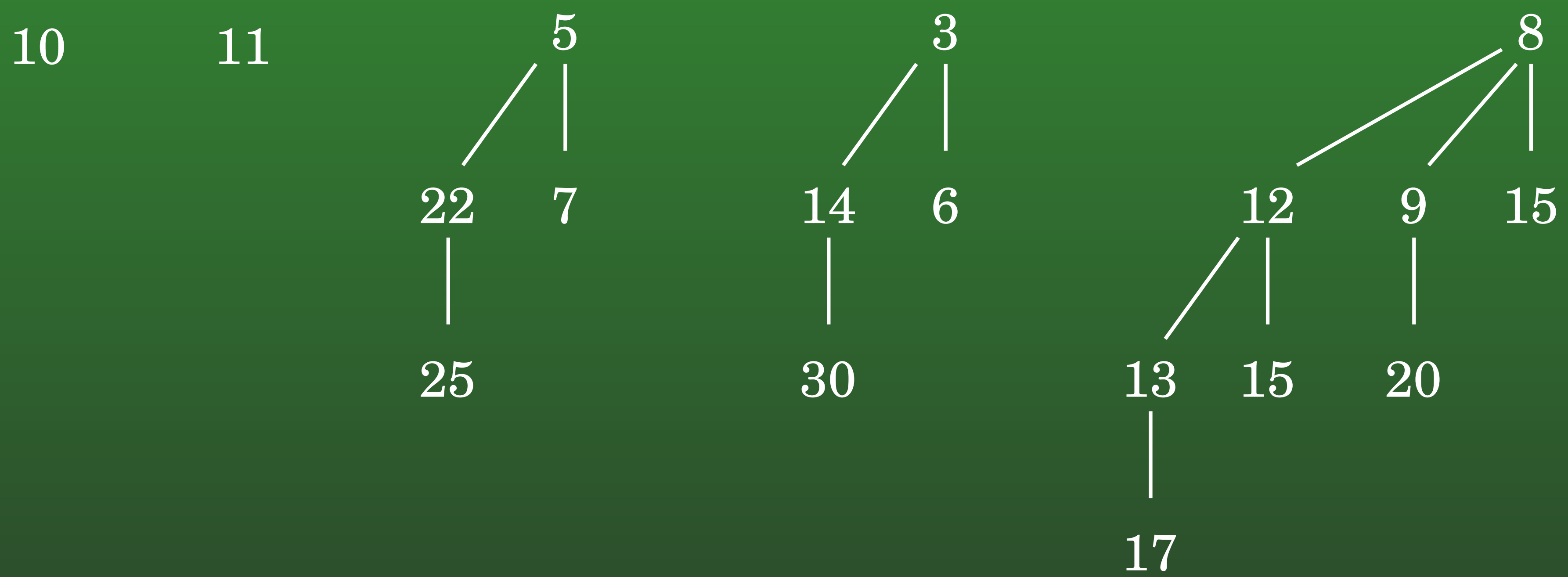
24-14: Binomial Heaps

- Merging Heaps H_1 and H_2
  - Merge root lists of H_1 and H_2
    - Could now have two trees with same degree
  - Go through list from smallest degree to largest degree
    - If two trees have same degree, combine them into one tree of larger degree
    - If three trees have same degree (how can this happen?) leave one, combine other two into tree of larger degree

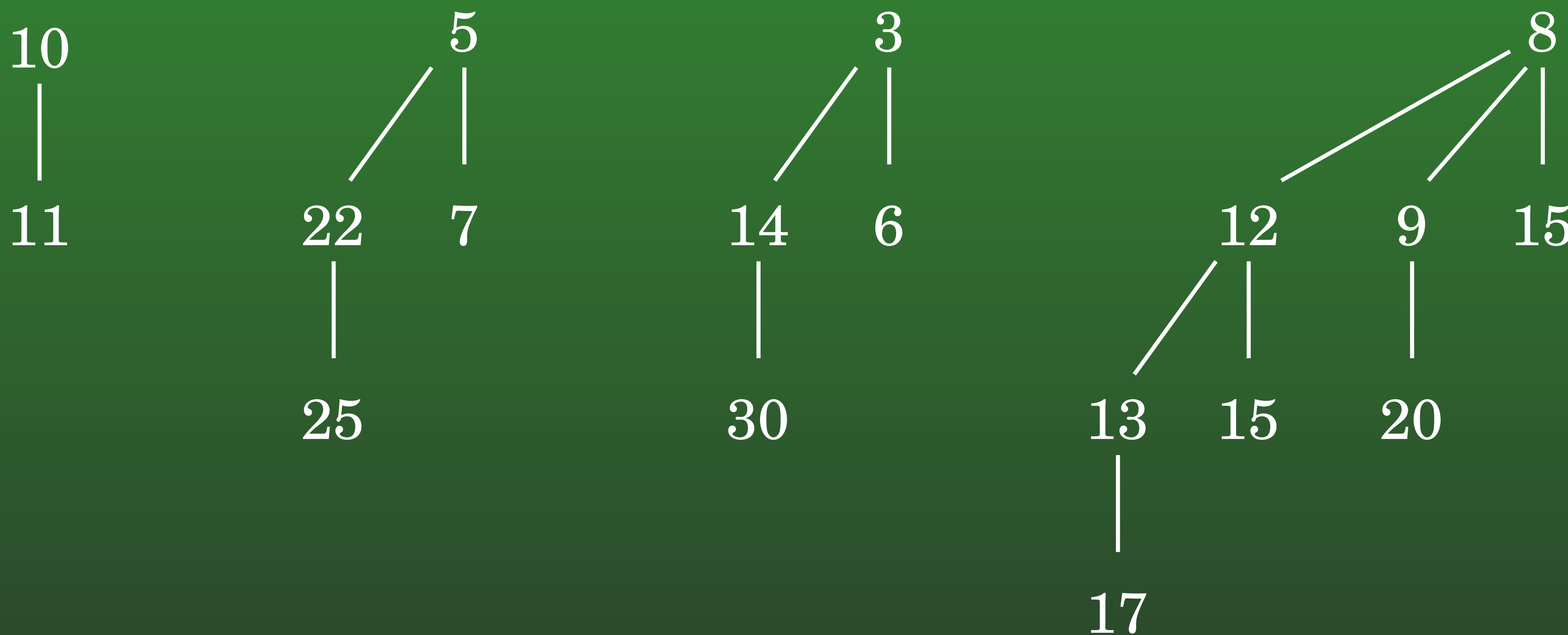
24-15: Binomial Heaps



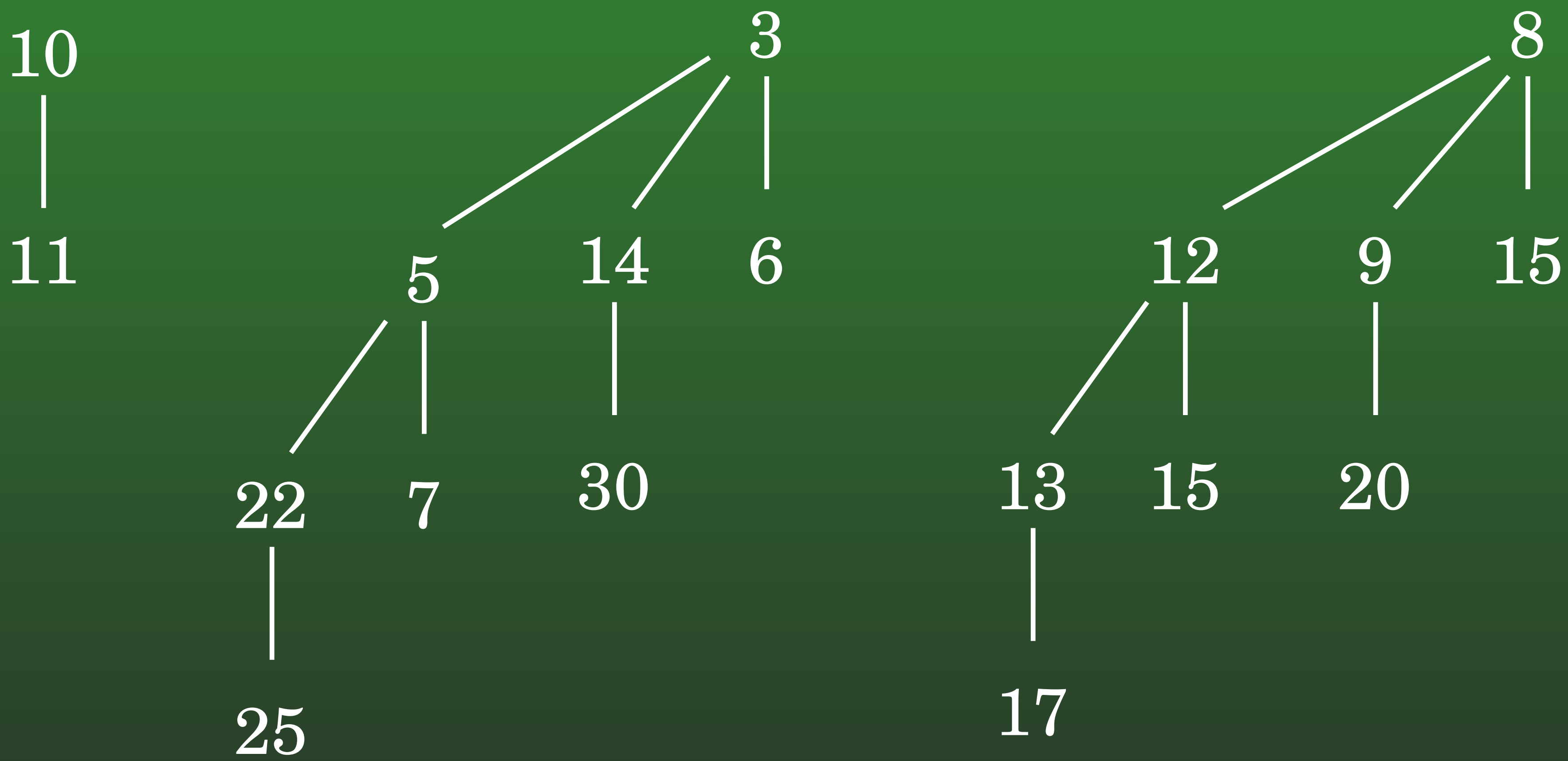
24-16: Binomial Heaps



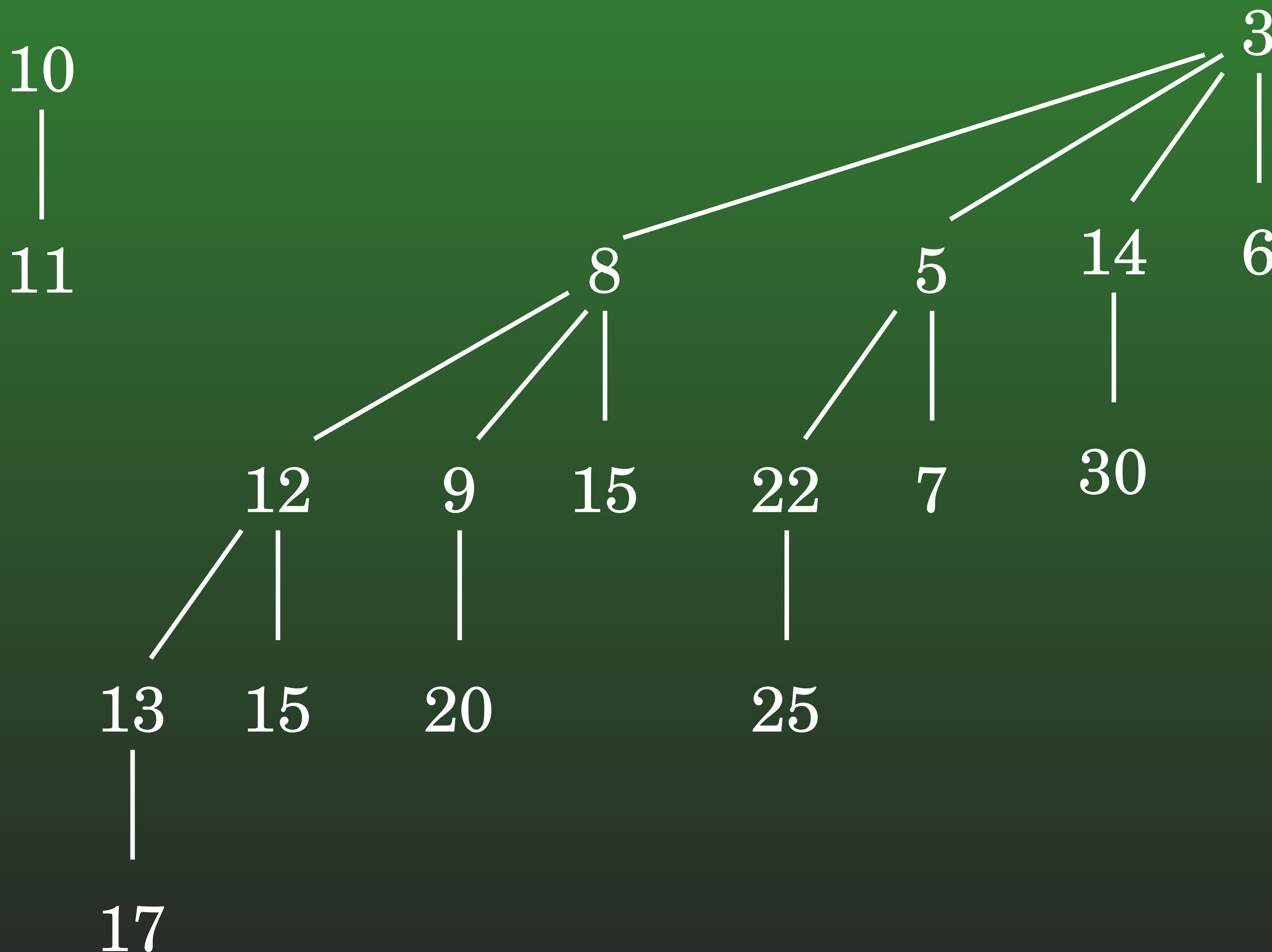
24-17: Binomial Heaps



24-18: Binomial Heaps



24-19: Binomial Heaps

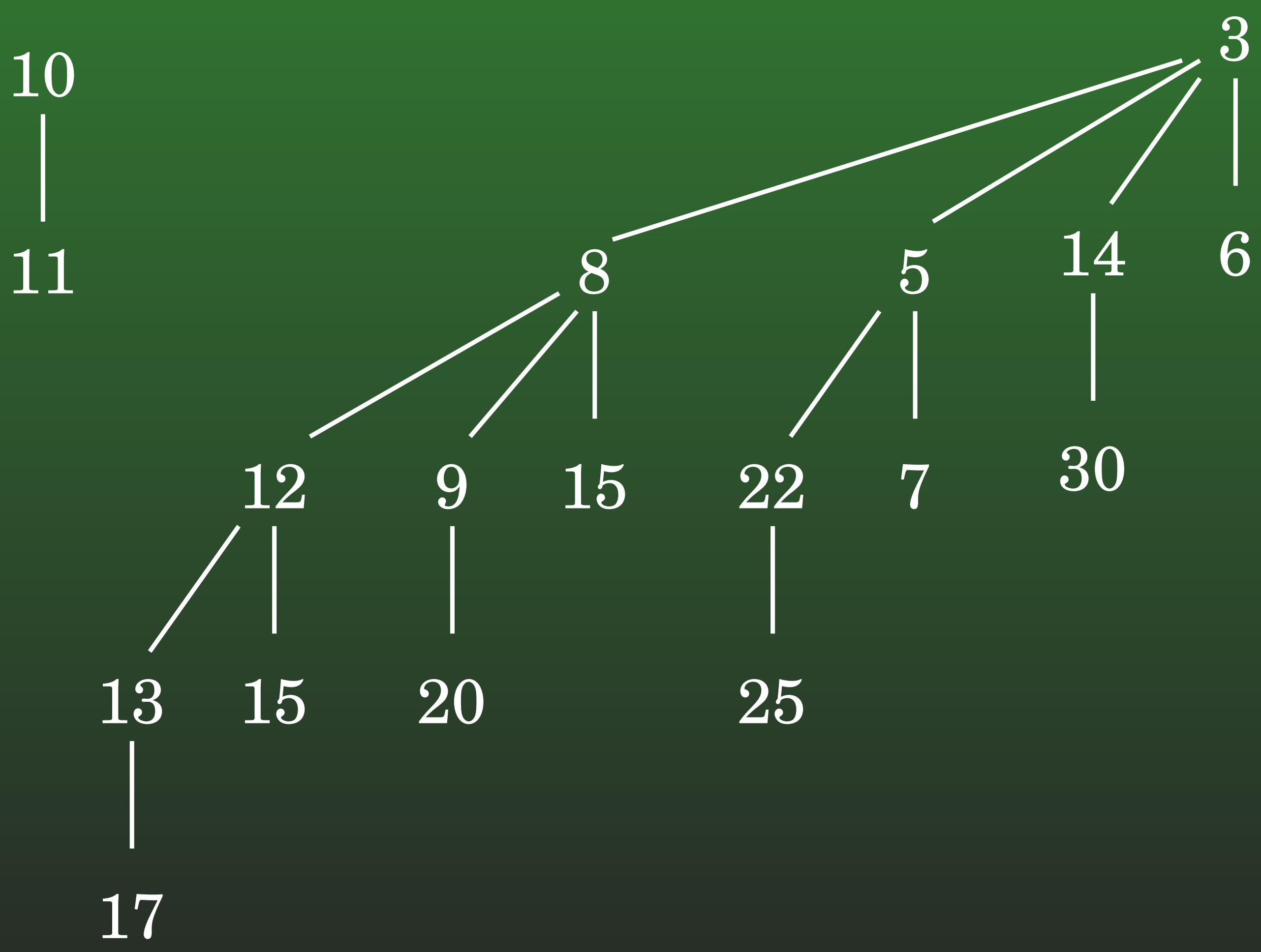


Binomial Heaps

- Removing minimum element
  - Find tree T that has minimum value at root, remove T from the list
  - Remove the root of T
    - Leaving a list of smaller trees
  - Reverse list of smaller trees
  - Merge two lists of trees together

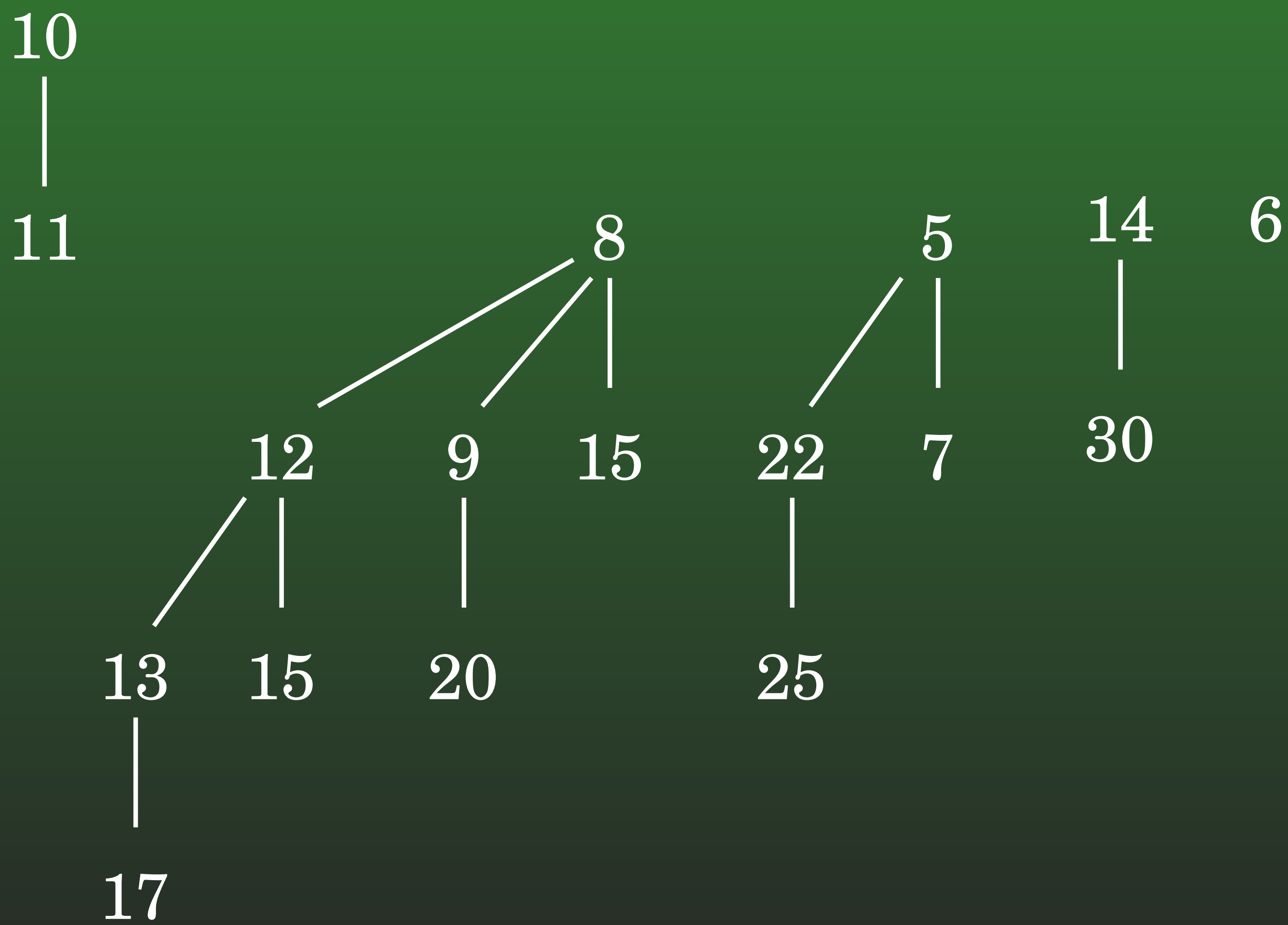
24-21: Binomial Heaps

- Removing minimum element



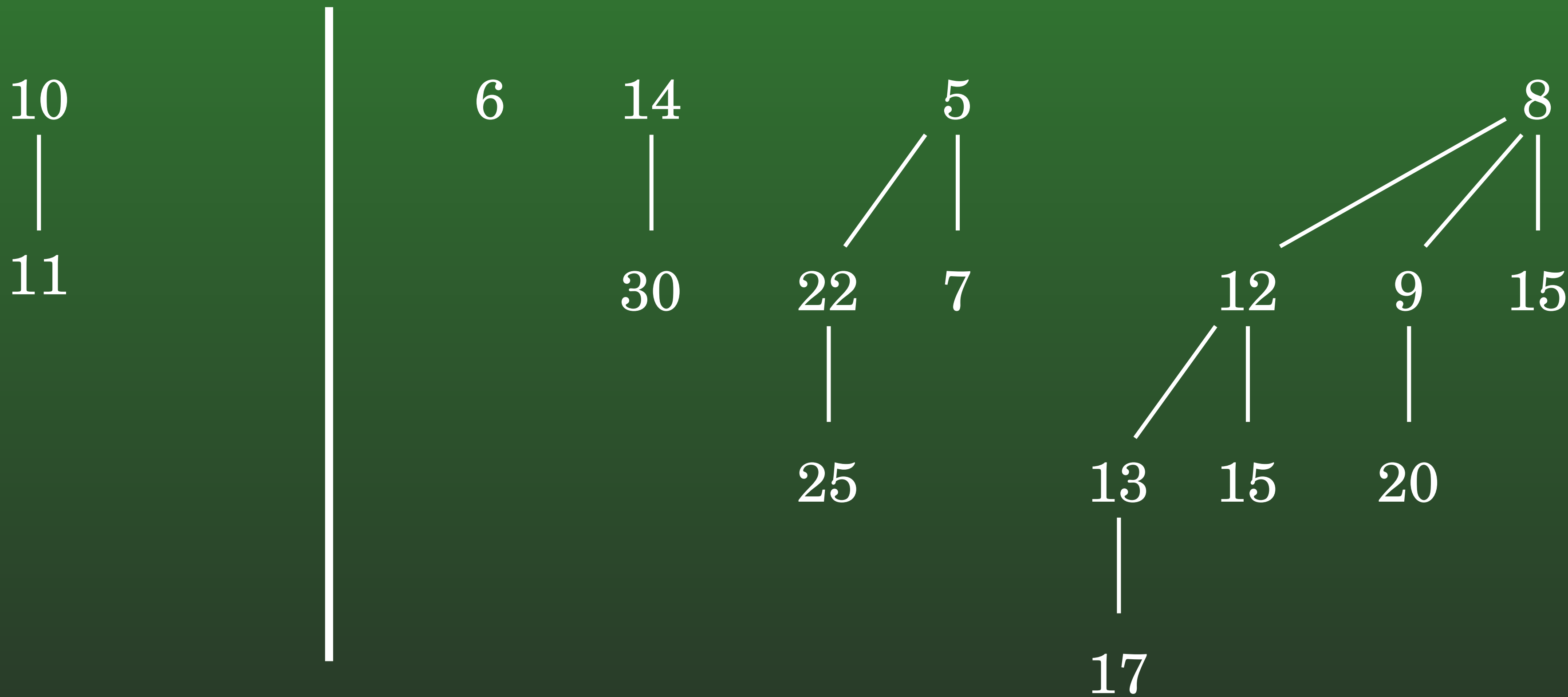
24-22: Binomial Heaps

- Removing minimum element



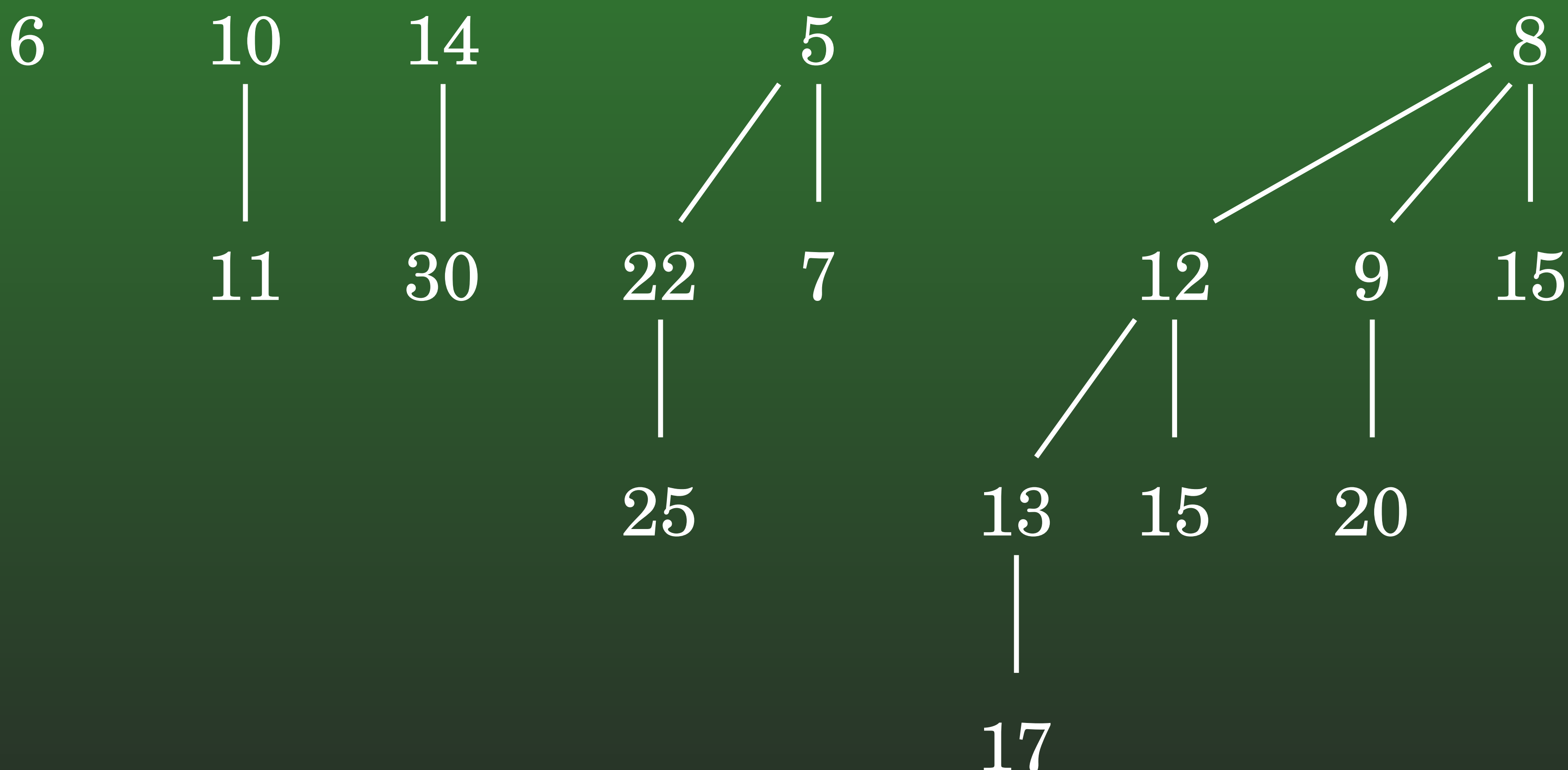
24-23: Binomial Heaps

- Removing minimum element



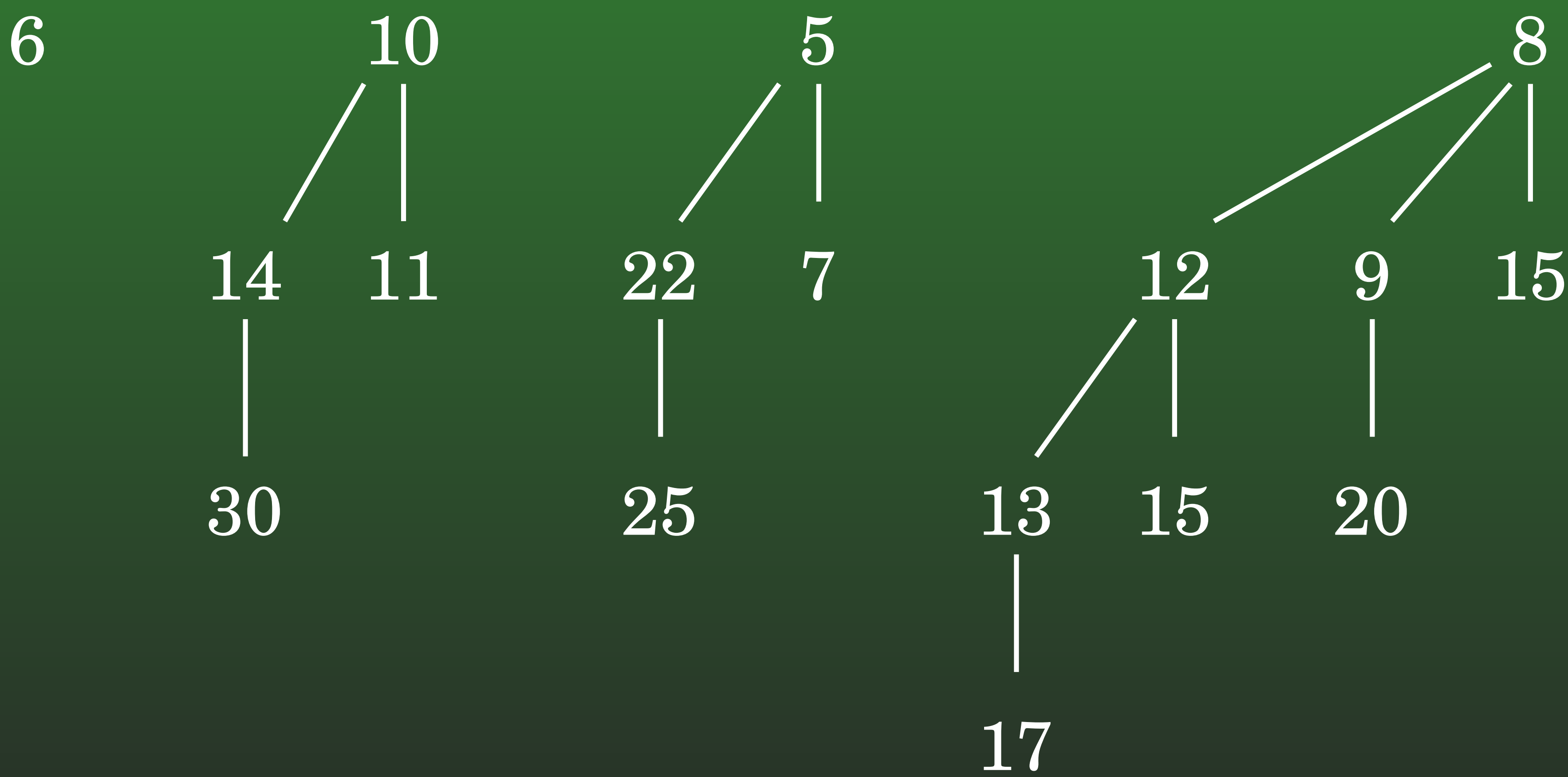
24-24: Binomial Heaps

- Removing minimum element



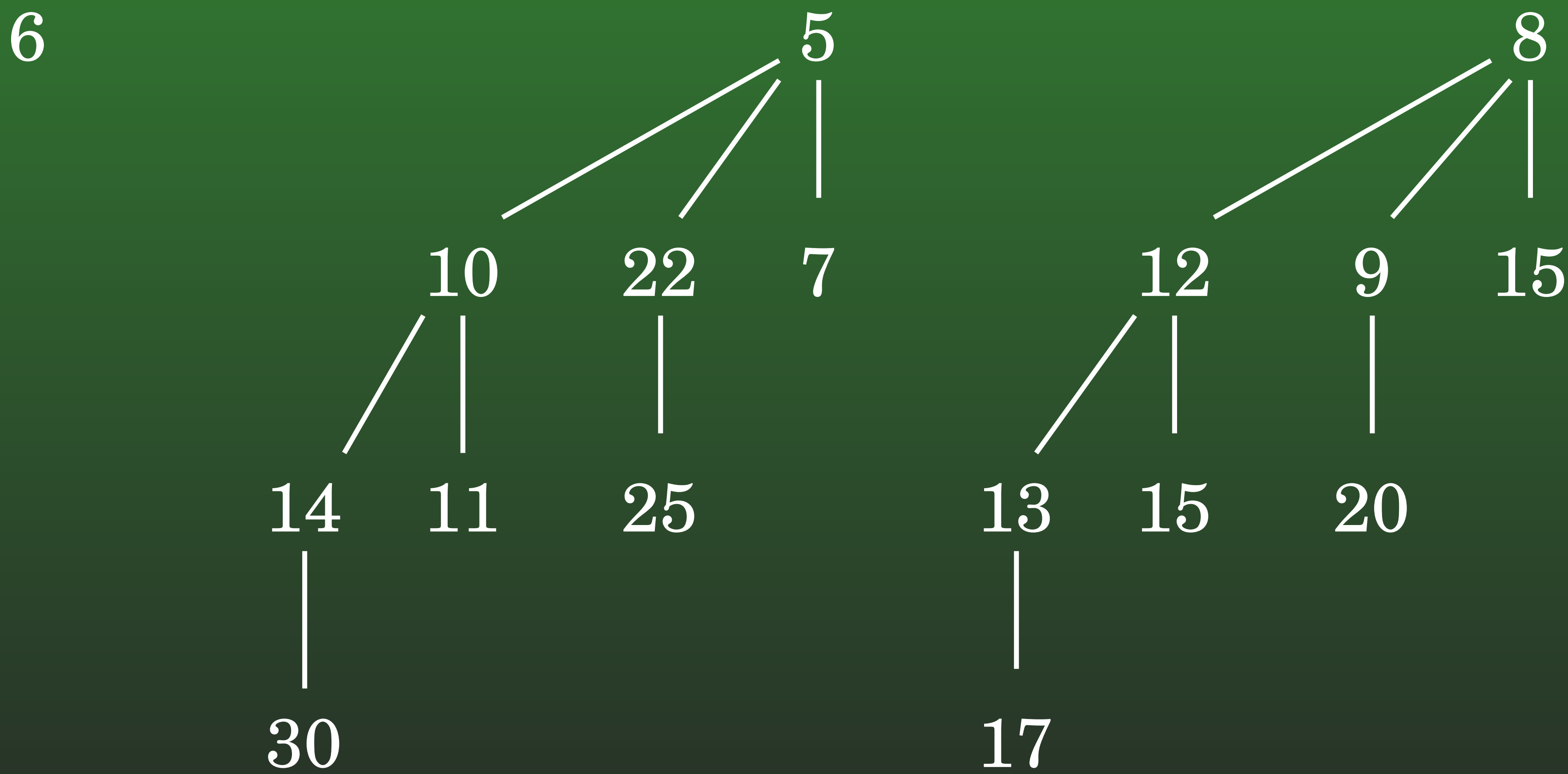
24-25: Binomial Heaps

- Removing minimum element



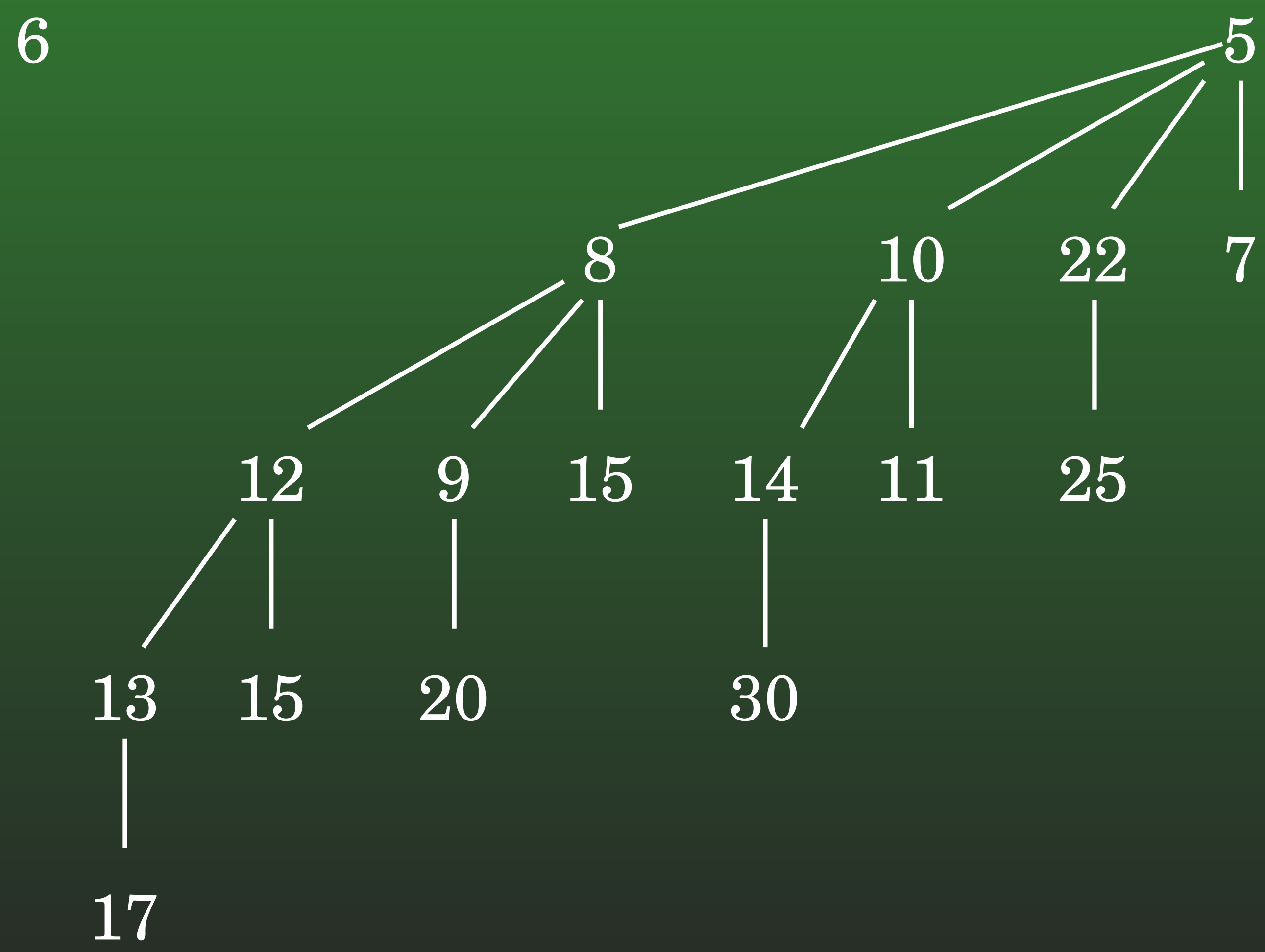
24-26: Binomial Heaps

- Removing minimum element



Binomial Heaps

- Removing minimum element



24-28: Binomial Heaps

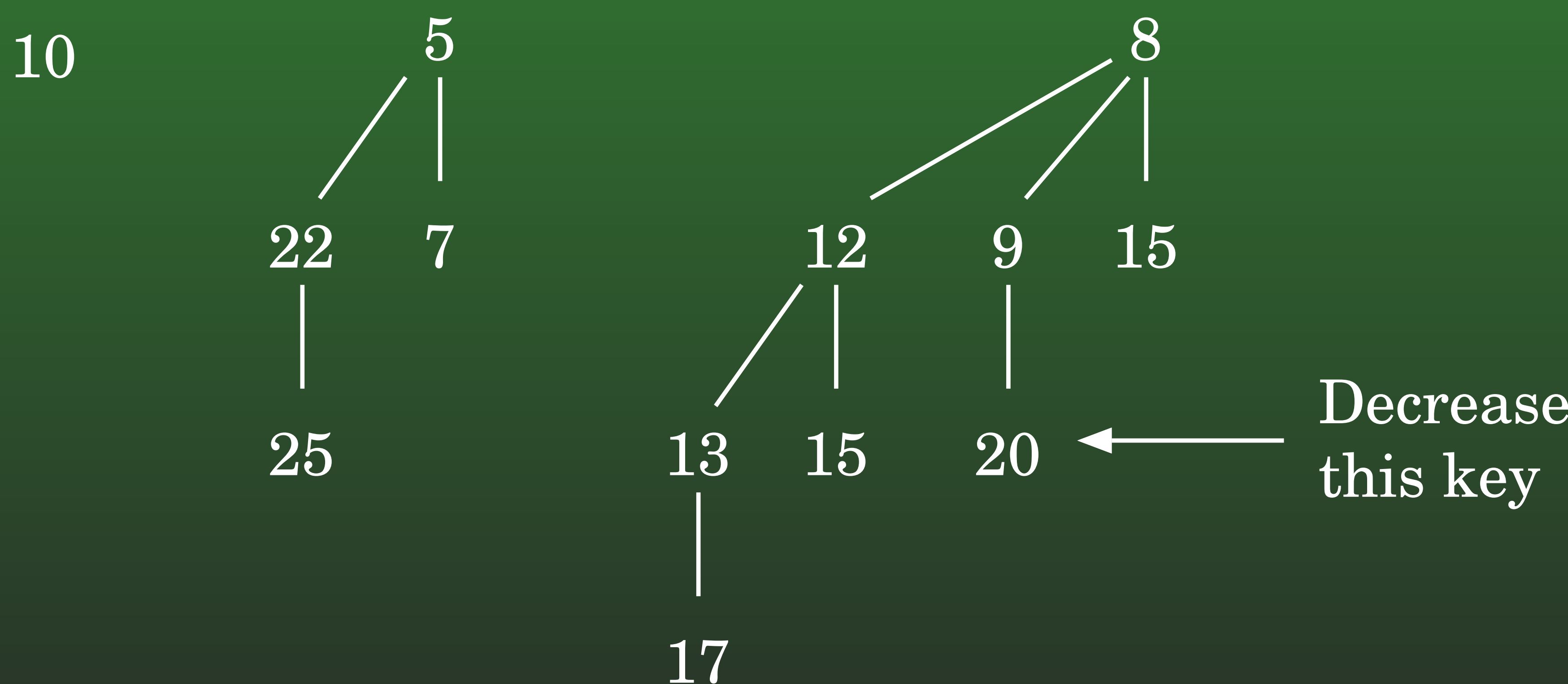
- Removing minimum element
  - Time?

Binomial Heaps

- Removing minimum element
  - Time?
    - Find the smallest element: $O(\lg n)$
    - Reverse list of children $O(\lg n)$
    - Merge heaps $O(\lg n)$

Binomial Heaps

- Decreasing the key of an element (assuming you have a pointer to it)

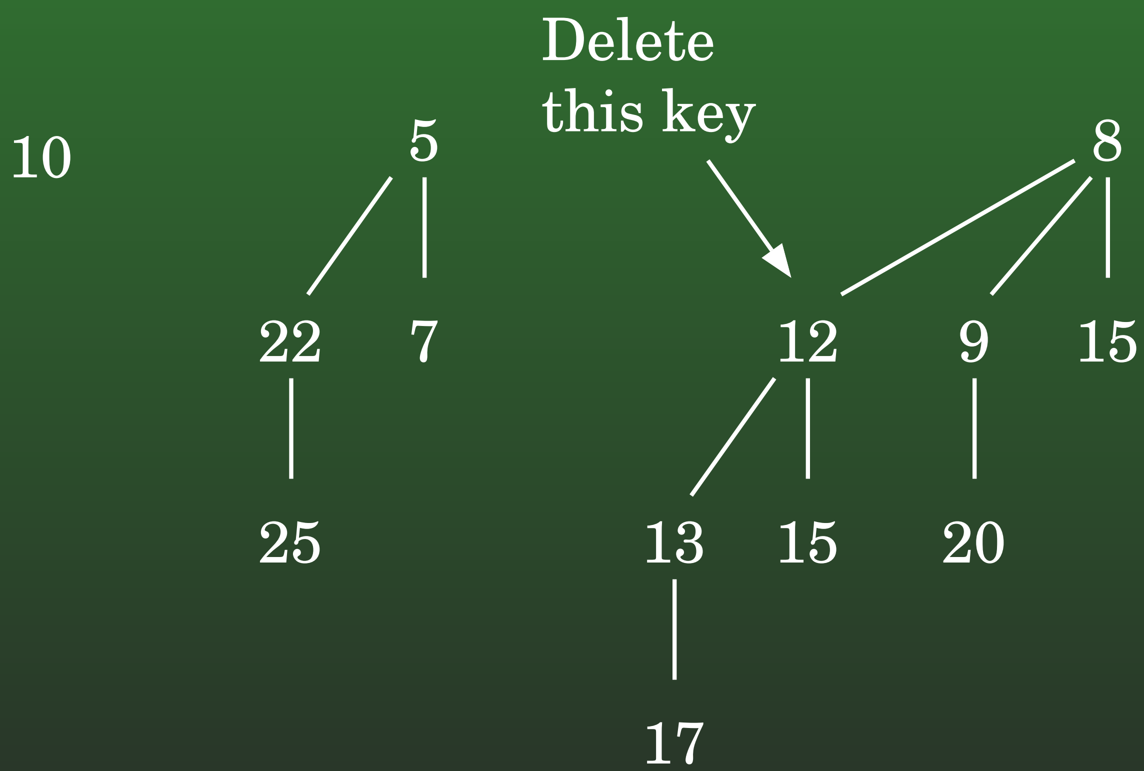


24-31: Binomial Heaps

- Decreasing the key of an element (assuming you have a pointer to it)
  - Decrease key value
  - While value $<$ parent, swap with parent
    - Exactly like standard, binary heaps
- Time: $O(\lg n)$

Binomial Heaps

- How could we delete an arbitrary element (assuming we had a pointer to this element)?



24-33: Binomial Heaps

- How could we delete an arbitrary element (assuming we had a pointer to this element)?
  - Decrease key to $-\infty$, Time $O(\lg n)$
  - Remove smallest, Time $O(\lg n)$