

**04-0: Abstract Data Types**

- An Abstract Data Type is a definition of a type based on the operations that can be performed on it.
- An ADT is an *interface*
- Data in an ADT cannot be manipulated directly – only through operations defined in the interface

**04-1: Abstract Data Types**

- To define an ADT, give the operations that can be performed on it
- The ADT says nothing about *how* the operations are performed
- Could have different implementations of the same ADT
- First ADT for this class: Stack

**04-2: Stack**

A Stack is a Last-In, First-Out (LIFO) data structure.

Stack Operations:

- Add an element to the top of the stack
- Remove the top element
- Check if the stack is empty

**04-3: Stack Implementation**

Array:

**04-4: Stack Implementation**

Array:

- Stack elements are stored in an array
- Top of the stack is the *end* of the array
  - If the top of the stack was the beginning of the array, a push or pop would require moving all elements in the array
- Push: `data[top++] = elem`
- Pop: `elem = data[--top]`

**04-5: Stack Implementation**

- See code & Visualizaion

**04-6:  $\Theta()$  For Stack Operations**

Array Implementation:

```
push
pop
empty()
```

**04-7:  $\Theta()$  For Stack Operations**

Array Implementation:

push  $\Theta(1)$   
pop  $\Theta(1)$   
empty()  $\Theta(1)$

**04-8: Stack Implementation**

Linked List:

**04-9: Stack Implementation**

Linked List:

- Stack elements are stored in a linked list
- Top of the stack is the *front* of the linked list
- push: `top = new Link(elem, top)`
- pop: `elem = top.element(); top = top.next()`

**04-10: Stack Implementation**

- See code & Visualization

**04-11:  $\Theta()$  For Stack Operations**

Linked List Implementation:

push  
pop  
empty()

**04-12:  $\Theta()$  For Stack Operations**

Linked List Implementation:

push  $\Theta(1)$   
pop  $\Theta(1)$   
empty()  $\Theta(1)$

**04-13: Queue**

A Queue is a First-In, First-Out (FIFO) data structure.

Queue Operations:

- Add an element to the end (tail) of the Queue
- Remove an element from the front (head) of the Queue
- Check if the Queue is empty

**04-14: Queue Implementation**

Linked List:

**04-15: Queue Implementation**

Linked List:

- Maintain a pointer to the first and last element in the Linked List
- Add elements to the back of the Linked List
- Remove elements from the front of the linked list
- Enqueue: `tail.setNext(new link(elem,null)); tail = tail.next()`
- Dequeue: `elem = head.element(); head = head.next();`

04-16: **Queue Implementation**

- See code & visualization

04-17: **Queue Implementation**

Array:

04-18: **Queue Implementation**

Array:

- Store queue elements in a circular array
- Maintain the index of the first element (head) and the next location to be inserted (tail)
- Enqueue: `data[tail] = elem;`  
`tail = (tail + 1) % size`
- Dequeue: `elem = data[head];`  
`head = (head + 1) % size`

04-19: **Queue Implementation**

- See code & visualization

04-20: **Modifying Stacks**

“Minimum Stacks” have one additional operation:

- minimum: return the minimum value stored in the stack

Can you implement a  $O(n)$  minimum?

04-21: **Modifying Stacks**

“Minimum Stacks” have one additional operation:

- minimum: return the minimum value stored in the stack

Can you implement a  $O(n)$  minimum?

Can you implement a  $\Theta(1)$  minimum?

push, pop must remain  $\Theta(1)$  as well! 04-22: **Modifying Queues**

- We’d like our array-based queues and our linked list-based queues to behave in the same way
- How do they behave differently, given the implementation we’ve seen so far?

04-23: **Modifying Queues**

- We’d like our array-based queues and our linked list-based queues to behave in the same way
- How do they behave differently, given the implementation we’ve seen so far?
  - Array-based queues can get full
  - How can we fix this?

04-24: **Modifying Queues**

- Growing queues

- If we do a Enqueue on a full queue, we can:
  - Create a new array, that is twice as big as the old array
  - Copy all of the data across to the new array
  - Replace the old array with a new array
- Why is this a little tricky?

#### 04-25: Modifying Queues

- Growing queues
  - If we do a Enqueue on a full queue, we can:
    - Create a new array, that is twice as big as the old array
    - Copy all of the data across to the new array
    - Replace the old array with a new array
  - Why is this a little tricky?
    - Queue could wrap around the end of the array (examples!)

#### 04-26: Modifying Queues

- Growing stacks/queues
  - Why do we double the size of the queue when it gets full, instead of just increasing the size by a constant amount
  - Hint – think about running times

#### 04-27: Modifying Queues

- Growing stacks/queues
  - What is the running time for a single enqueue/push, if we allow the stack to grow? (by doubling the size of the stack/queue when it is full)
  - What is the running time for  $n$  enqueues/pushes?

#### 04-28: Modifying Queues

- Growing stacks/queues
  - What is the running time for a single enqueue/push, if we allow the stack to grow? (by doubling the size of the stack/queue when it is full)
    - $O(n)$ , for a stack size of  $n$
  - What is the running time for  $n$  enqueues/pushes?
    - $O(n)$  – so each push/enqueue takes  $O(1)$  on average

#### 04-29: Modifying Queues

- Growing stacks/queues
  - What is the running time for a single enqueue/push, if we allow the stack to grow? (by adding  $k$  elements when the stack/queue is full)
  - What is the running time for  $n$  enqueues/pushes?

04-30: **Modifying Queues**

- Growing stacks/queues
  - What is the running time for a single enqueue/push, if we allow the stack to grow? (by adding  $k$  elements when the stack/queue is full)
    - $O(n)$ , for a sack size of  $n$
  - What is the running time for  $n$  enqueues/pushes?
    - $O(n * n/k) = O(n^2)$ , if  $k$  is a constant
    - Each enqueue/dequeue takes time  $O(n)$  on average!

04-31: **Amortized Analysis**

- Figuring out how long an algorithm takes to run on average, by adding up how long a sequence of operations takes, is called *Amortized Analysis*
- Washing Machine example
  - Cost of a washing machine
  - Amortized cost per wash
- We'll take quite a bit about amortized analysis, complete with some more formal mathematics, later in the semester.