

# Data Structures and Algorithms

*CS245-2015S-06*

## *Binary Search Trees*

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# 06-0: Ordered List ADT

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Operations:

- Insert an element in the list
- Check if an element is in the list
- Remove an element from the list
- Print out the contents of the list, in order

# 06-1: Implementing Ordered List

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Using an Ordered Array – Running times:

Check

Insert

Remove

Print

# 06-2: Implementing Ordered List

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Using an Ordered Array – Running times:

Check  $\Theta(\lg n)$

Insert  $\Theta(n)$

Remove  $\Theta(n)$

Print  $\Theta(n)$

# 06-3: Implementing Ordered List

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Using an *Unordered* Array – Running times:

Check

Insert

Remove

Print

# 06-4: Implementing Ordered List

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Using an *Unordered* Array – Running times:

Check  $\Theta(n)$

Insert  $\Theta(1)$

Remove  $\Theta(n)$  Need to find element first!

Print  $\Theta(n \lg n)$

(Given a fast sorting algorithm)

# 06-5: Implementing Ordered List

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Using an Ordered Linked List – Running times:

Check

Insert

Remove

Print

# 06-6: Implementing Ordered List

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Using an Ordered Linked List – Running times:

Check  $\Theta(n)$

Insert  $\Theta(n)$

Remove  $\Theta(n)$

Print  $\Theta(n)$



# 06-7: The Best of Both Worlds

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- Linked Lists – Insert fast / Find slow
- Arrays – Find fast / Insert slow
- The only way to examine  $n$ th element in a linked list is to traverse  $(n-1)$  other elements



- If we could leap to the middle of the list ...

# 06-8: The Best of Both Worlds

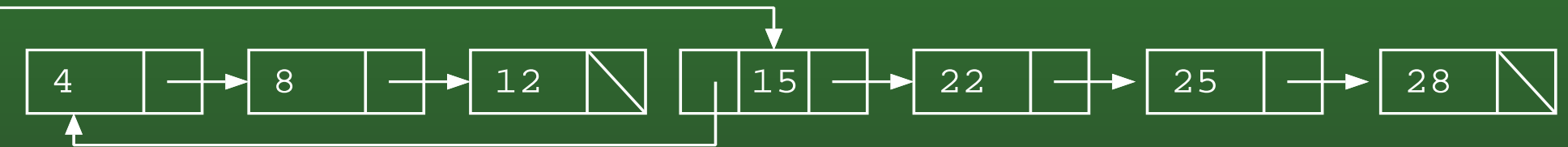
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# 06-9: The Best of Both Worlds



Move the initial pointer to the middle of the list:

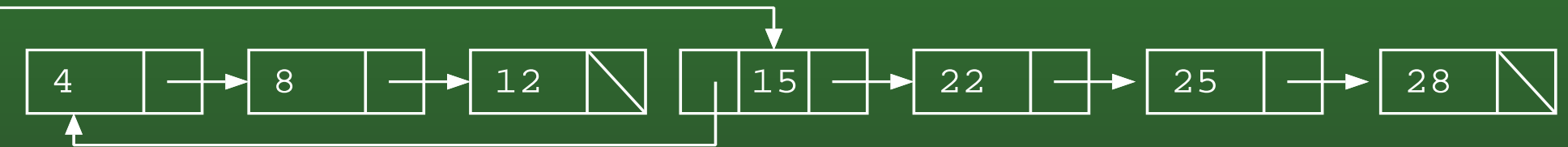


We've cut our search time in half! Have we changed the  $\Theta()$  running time?

# 06-10: The Best of Both Worlds



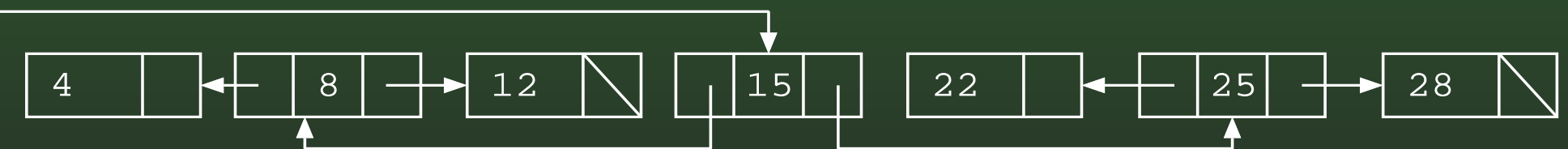
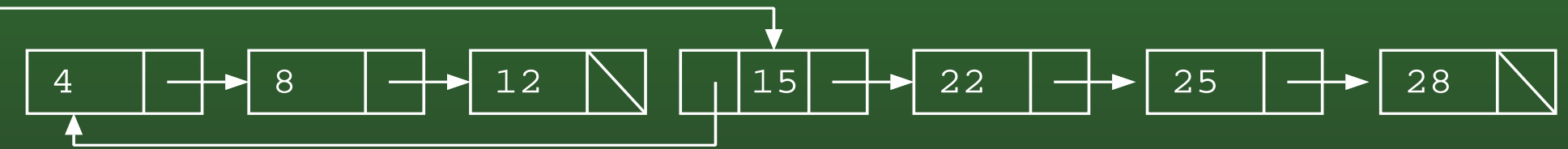
Move the initial pointer to the middle of the list:



We've cut our search time in half! Have we changed the  $\Theta()$  running time?

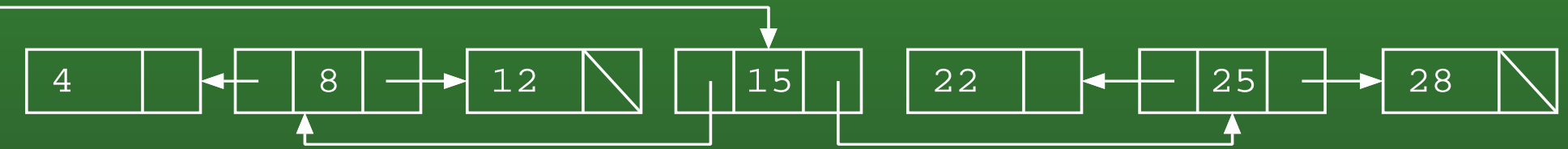
Repeat the process!

# 06-11: The Best of Both Worlds

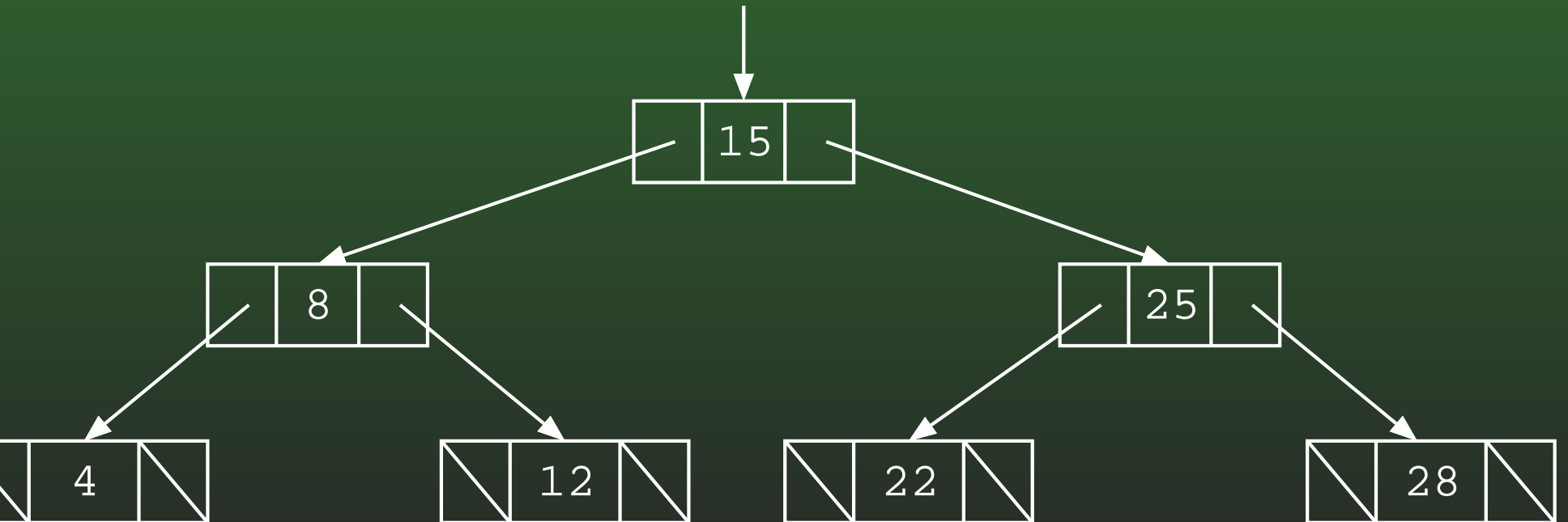


# 06-12: The Best of Both Worlds

Grab the first element of the list:



Give it a good shake -



# 06-13: Binary Trees

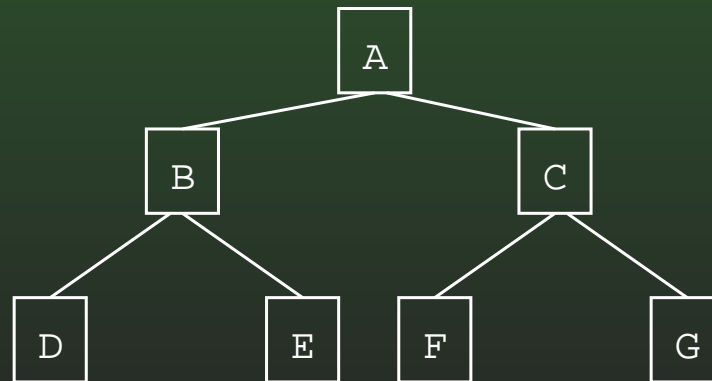
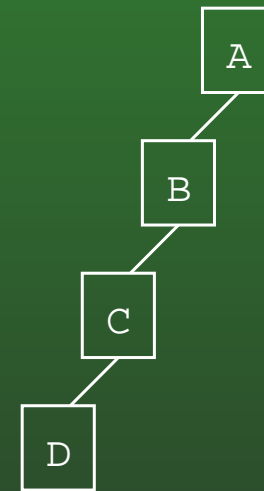
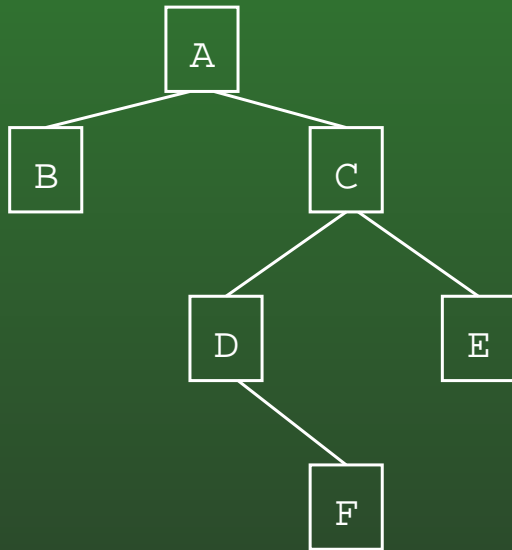
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Binary Trees are Recursive Data Structures

- Base Case: Empty Tree
- Recursive Case: Node, consisting of:
  - Left Child (Tree)
  - Right Child (Tree)
  - Data

# 06-14: Binary Tree Examples

The following are all Binary Trees (Though not Binary Search Trees)





# 06-15: Tree Terminology

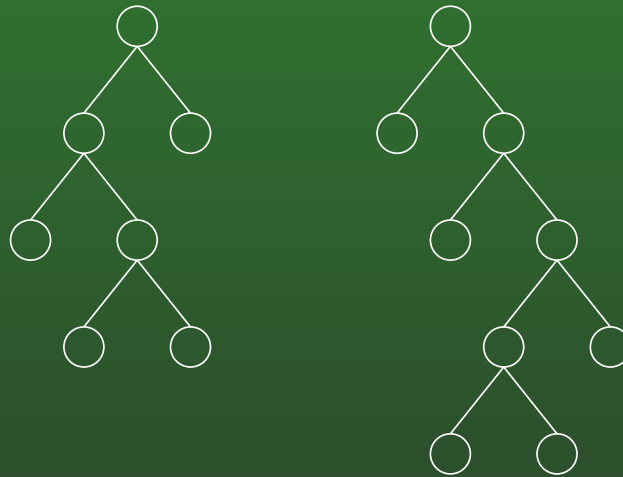
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- Parent / Child
- Leaf node
- Root node
- Edge (between nodes)
- Path
- Ancestor / Descendant
- Depth of a node  $n$ 
  - Length of path from root to  $n$
- Height of a tree
  - (Depth of deepest node) + 1

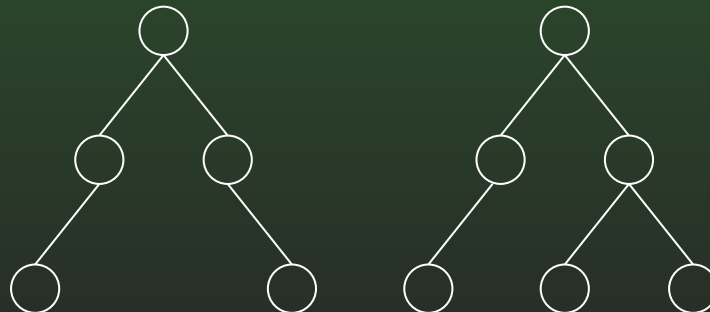
# 06-16: Full Binary Tree

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- Each node has 0 or 2 children
- Full Binary Trees

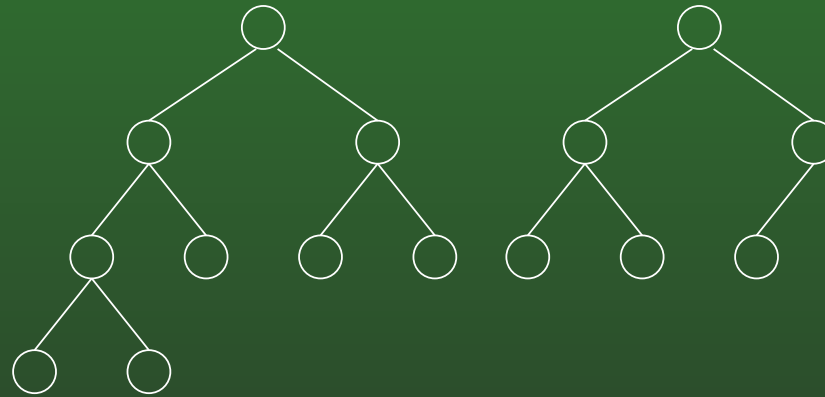


- *Not* Full Binary Trees

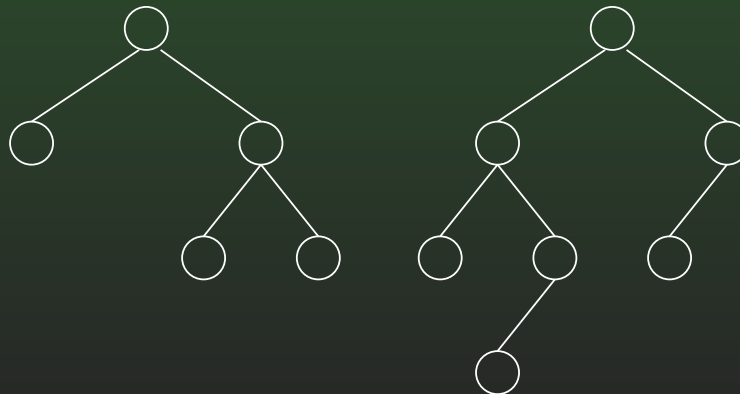


# 06-17: Complete Binary Tree

- Can be built by starting at the root, and filling the tree by levels from left to right
- Complete Binary Trees



- *Not* Complete Binary Trees



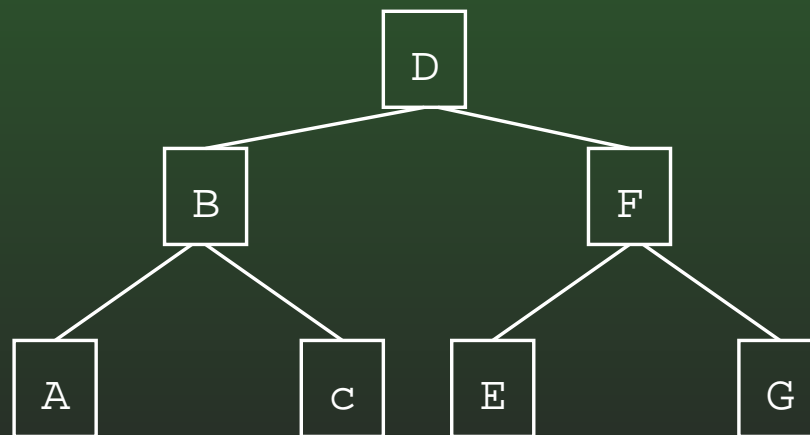
# 06-18: Binary Search Trees

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- Binary Trees
- For each node  $n$ , (value stored at node  $n$ )  $\geq$  (value stored in left subtree)
- For each node  $n$ , (value stored at node  $n$ )  $<$  (value stored in right subtree)

# 06-19: Example Binary Search Trees

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# 06-20: Implementing BSTs

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- Each Node in a BST is implemented as a class:

```
public class Node {  
    public Comparable data;  
    public Node left;  
    public Node right;  
}
```

# 06-21: Implementing BSTs

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```
public class Node {

    public Node(Comparable data, Node left, Node right) {
        this.data = data;
        this.left = left;
        this.right = right;
    }

    public Node left() {
        return left;
    }
    public Node setLeft(Node newLeft) {
        left = newLeft
    }
    ... (etc)

    private Comparable data;
    private Node left;
    private Node right;
}
```

# 06-22: Finding an Element in a BST

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- Binary Search Trees are recursive data structures, so most operations on them will be recursive as well
- Recall how to write a recursive algorithm ...



## 06-23: Writing a Recursive Algorithm

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- Determine a small version of the problem, which can be solved immediately. This is the *base case*
- Determine how to make the problem smaller
- Once the problem has been made smaller, we can assume that the function that we are writing *will work correctly on the smaller problem* (Recursive Leap of Faith)
  - Determine how to use the solution to the smaller problem to solve the larger problem

# 06-24: Finding an Element in a BST

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- First, the Base Case – when is it easy to determine if an element is stored in a Binary Search Tree?

## 06-25: Finding an Element in a BST

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- First, the Base Case – when is it easy to determine if an element is stored in a Binary Search Tree?
  - If the tree is empty, then the element can't be there
  - If the element is stored at the root, then the element is there

# 06-26: Finding an Element in a BST

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- Next, the Recursive Case – how do we make the problem smaller?

# 06-27: Finding an Element in a BST

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- Next, the Recursive Case – how do we make the problem smaller?
  - Both the left and right subtrees are smaller versions of the problem. Which one do we use?

## 06-28: Finding an Element in a BST

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- Next, the Recursive Case – how do we make the problem smaller?
  - Both the left and right subtrees are smaller versions of the problem. Which one do we use?
  - If the element we are trying to find is  $<$  the element stored at the root, use the left subtree. Otherwise, use the right subtree.

## 06-29: Finding an Element in a BST

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- Next, the Recursive Case – how do we make the problem smaller?
  - Both the left and right subtrees are smaller versions of the problem. Which one do we use?
  - If the element we are trying to find is  $<$  the element stored at the root, use the left subtree. Otherwise, use the right subtree.
- How do we use the solution to the subproblem to solve the original problem?

# 06-30: Finding an Element in a BST

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- Next, the Recursive Case – how do we make the problem smaller?
  - Both the left and right subtrees are smaller versions of the problem. Which one do we use?
  - If the element we are trying to find is  $<$  the element stored at the root, use the left subtree. Otherwise, use the right subtree.
- How do we use the solution to the subproblem to solve the original problem?
  - The solution to the subproblem *is* the solution to the original problem (this is not always the case in recursive algorithms)



# 06-31: Finding an Element in a BST

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To find an element  $e$  in a Binary Search Tree  $T$ :

- If  $T$  is empty, then  $e$  is not in  $T$
- If the root of  $T$  contains  $e$ , then  $e$  is in  $T$
- If  $e <$  the element stored in the root of  $T$ :
  - Look for  $e$  in the left subtree of  $T$

Otherwise

- Look for  $e$  in the right subtree of  $T$

## 06-32: Finding an Element in a BST

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```
boolean find(Node tree, Comparable elem) {
    if (tree == null)
        return false;
    if (elem.compareTo(tree.element()) == 0)
        return true;
    if (elem.compareTo(tree) < 0)
        return find(tree.left(), elem);
    else
        return find(tree.right(), elem);
}
```

# 06-33: Printing out a BST

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To print out all element in a BST:

- Print all elements in the left subtree, in order
- Print out the element at the root of the tree
- Print all elements in the right subtree, in order

## 06-34: Printing out a BST

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To print out all element in a BST:

- Print all elements in the left subtree, in order
- Print out the element at the root of the tree
- Print all elements in the right subtree, in order
  - Each subproblem is a smaller version of the original problem – we can assume that a recursive call will work!

## 06-35: Printing out a BST

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- What is the base case for printing out a Binary Search Tree – what is an easy tree to print out?

## 06-36: Printing out a BST

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- What is the base case for printing out a Binary Search Tree – what is an easy tree to print out?
- An empty tree is extremely easy to print out – do nothing!
- Code for printing a BST ...

## 06-37: Printing out a BST

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```
void print(Node tree) {  
    if (tree != null) {  
        print(tree.left());  
        System.out.println(tree.element());  
        print(tree.right());  
    }  
}
```

# 06-38: Printing out a BST

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Examples



# 06-39: Tree Traversals

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- PREORDER Traversal
  - Do operation on root of the tree
  - Traverse left subtree
  - Traverse right subtree
- INORDER Traversal
  - Traverse left subtree
  - Do operation on root of the tree
  - Traverse right subtree
- POSTORDER Traversal
  - Traverse left subtree
  - Traverse right subtree
  - Do operation on root of the tree

# 06-40: **PREORDER** Examples

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# 06-41: POSTORDER Examples

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# 06-42: INORDER Examples

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## 06-43: **BST Minimal Element**

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To find the minimal element in a BST:

- Base Case: When is it easy to find the smallest element in a BST?
- Recursive Case: How can we make the problem smaller?

How can we use the solution to the smaller problem to solve the original problem?

## 06-44: **BST Minimal Element**

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To find the minimal element in a BST:

Base Case:

- When is it easy to find the smallest element in a BST?

## 06-45: **BST Minimal Element**

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To find the minimal element in a BST:

Base Case:

- When is it easy to find the smallest element in a BST?
  - When the left subtree is empty, then the element stored at the root is the smallest element in the tree.

## 06-46: **BST Minimal Element**

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To find the minimal element in a BST:

Recursive Case:

- How can we make the problem smaller?



## 06-47: **BST Minimal Element**

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To find the minimal element in a BST:

Recursive Case:

- How can we make the problem smaller?
  - Both the left and right subtrees are smaller versions of the same problem
- How can we use the solution to a smaller problem to solve the original problem?

## 06-48: **BST Minimal Element**

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To find the minimal element in a BST:

Recursive Case:

- How can we make the problem smaller?
  - Both the left and right subtrees are smaller versions of the same problem
- How can we use the solution to a smaller problem to solve the original problem?
  - The smallest element in the left subtree is the smallest element in the tree

## 06-49: BST Minimal Element

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```
Comparable minimum(Node tree) {
    if (tree == null)
        return null;
    if (tree.left() == null)
        return tree.element();
    else
        return minimum(tree.left());
}
```

# 06-50: BST Minimal Element

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## Iterative Version

```
Comparable minimum(Node tree) {  
    if (tree == null)  
        return null;  
    while (tree.left() != null)  
        tree = tree.left();  
    return tree.element();  
}
```

## 06-51: Inserting $e$ into BST $T$

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- What is the base case – an easy tree to insert an element into?

## 06-52: Inserting $e$ into BST $T$

---

- What is the base case – an easy tree to insert an element into?
  - An empty tree
  - Create a new tree, containing the element  $e$

## 06-53: Inserting $e$ into BST $T$

---

- Recursive Case: How do we make the problem smaller?

## 06-54: Inserting $e$ into BST $T$

---

- Recursive Case: How do we make the problem smaller?
  - The left and right subtrees are smaller versions of the same problem.
  - How do we use these smaller versions of the problem?



## 06-55: Inserting $e$ into BST $T$

---

- Recursive Case: How do we make the problem smaller?
  - The left and right subtrees are smaller versions of the same problem
  - Insert the element into the left subtree if  $e \leq$  value stored at the root, and insert the element into the right subtree if  $e >$  value stored at the root

## 06-56: Inserting $e$ into BST $T$

---

- Base case –  $T$  is empty:
  - Create a new tree, containing the element  $e$
- Recursive Case:
  - If  $e$  is less than the element at the root of  $T$ , insert  $e$  into left subtree
  - If  $e$  is greater than the element at the root of  $T$ , insert  $e$  into the right subtree

## 06-57: Tree Manipulation in Java

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- Tree manipulation functions return trees
- Insert method takes as input the old tree and the element to insert, and returns the new tree, with the element inserted
  - Old value (pre-insertion) of tree will be destroyed
- To insert an element  $e$  into a tree  $T$ :
  - `T = insert(T, e);`

## 06-58: Inserting $e$ into BST $T$

---

```
Node insert(Node tree, Comparable elem) {
    if (tree == null) {
        return new Node(elem);
    }
    if (elem.compareTo(tree.element()) <= 0) {
        tree.setLeft(insert(tree.left(), elem));
        return tree;
    }
    else {
        tree.setRight(insert(tree.right(), elem));
        return tree;
    }
}
```

# 06-59: Deleting From a BST

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- Removing a leaf:

# 06-60: Deleting From a BST

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- Removing a leaf:
  - Remove element immediately

# 06-61: Deleting From a BST

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- Removing a leaf:
  - Remove element immediately
- Removing a node with one child:

# 06-62: Deleting From a BST

---

- Removing a leaf:
  - Remove element immediately
- Removing a node with one child:
  - Just like removing from a linked list
  - Make parent point to child



# 06-63: Deleting From a BST

---

- Removing a leaf:
  - Remove element immediately
- Removing a node with one child:
  - Just like removing from a linked list
  - Make parent point to child
- Removing a node with two children:

## 06-64: Deleting From a BST

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- Removing a leaf:
  - Remove element immediately
- Removing a node with one child:
  - Just like removing from a linked list
  - Make parent point to child
- Removing a node with two children:
  - Replace node with largest element in left subtree, or the smallest element in the right subtree

## 06-65: Comparable vs. .key() method

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- We have been storing “Comparable” elements in BSTs
- Alternately, could use a “key()” method – elements stored in BSTs must implement a key() method, which returns an integer.
- We can combine the two methods
  - Each element stored in the tree has a key() method
  - key() method returns Comparable class

# 06-66: BST Implementation Details

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- Use BSTs to implement Ordered List ADT
- Operations
  - Insert
  - Find
  - Remove
  - Print in Order
- The specification (interface) should not specify an implementation
  - Allow several different implementations of the same interface

# 06-67: BST Implementation Details

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- BST functions require the root of the tree be sent in as a parameter
- Ordered list functions should *not* contain implementation details!
- What should we do?

## 06-68: BST Implementation Details

---

- BST functions require the root of the tree be sent in as a parameter
- Ordered list functions should *not* contain implementation details!
- What should we do?
  - Private variable, holds root of the tree
  - Private recursive methods, require root as an argument
  - Public methods call private methods, passing in private root