

**P4-0: Overview**

- Example games (board splitting, chess, Network)
- Min/Max trees
- Alpha-Beta Pruning
- Evaluation Functions

**P4-1: Two player games**

- Board-Splitting Game
  - Two players,  $V$  &  $H$
  - $V$  splits the board vertically, selects one half
  - $H$  splits the board horizontally, selects one half
  - $V$  tries to maximize the final value,  $H$  tries to minimize the final value

14	5	11	4
12	13	9	7
15	13	10	8
16	1	6	2

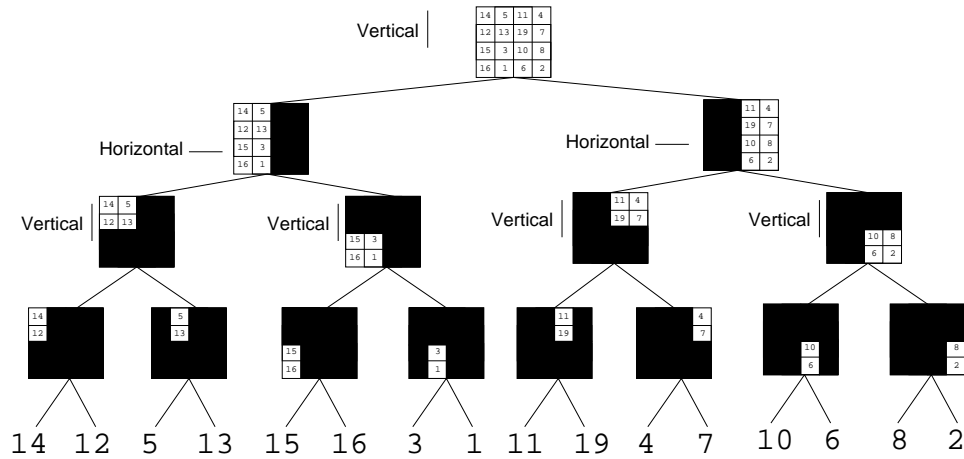
**P4-2: Two player games**

- Board-Splitting Game
  - We assume that both players are rational (make the best possible move)
  - How can we determine who will win the game?

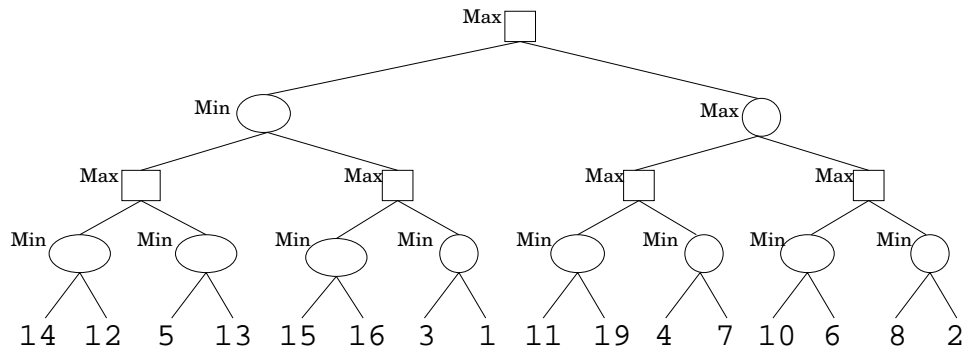
**P4-3: Two player games**

- Board-Splitting Game
  - We assume that both players are rational (make the best possible move)
  - How can we determine who will win the game?
    - Examine all possible games!

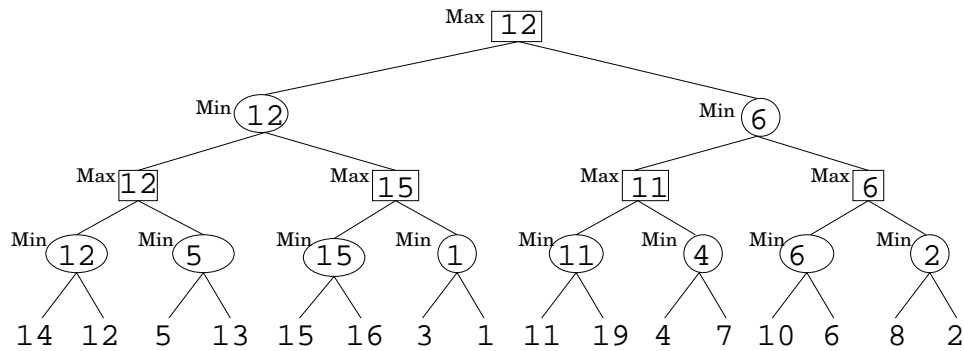
**P4-4: Two player games**



P4-5: Two player games



P4-6: Two player games



P4-7: Two player games

- Game playing agent can do this to figure out which move to make
  - Examine all possible moves
  - Examine all possible responses to each move
  - ... all the way to the last move
  - Calculate the value of each move (assuming opponent plays perfectly)
  -

P4-8: Minimax Algorithm

```

Max(node)
  if terminal(node)
    return utility(node)
  maxVal = MIN_VALUE
  children = successors(node)
  for child in children
    maxVal = max(maxVal, Min(child))
  return maxVal

Min(node)
  if terminal(node)
    return utility(node)
  minVal = MAX_VALUE
  children = successors(node)
  for child in children
    minVal = min(minVal, Max(child))
  return minVal
    
```

**P4-9: Minimax Algorithm**

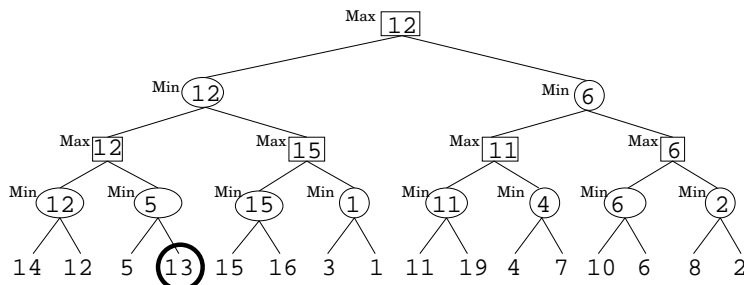
- Branching factor of  $b$ , game length of  $d$  moves, what are the time and space requirements for Minimax?

**P4-10: Minimax Algorithm**

- Branching factor of  $b$ , game length of  $d$  moves, what are the time and space requirements for Minimax?
  - Time:  $O(b^d)$
  - Space:  $O(d)$
- Not manageable for any real games – chess has an average  $b$  of 35, can't search the entire tree
- Need to make this more manageable

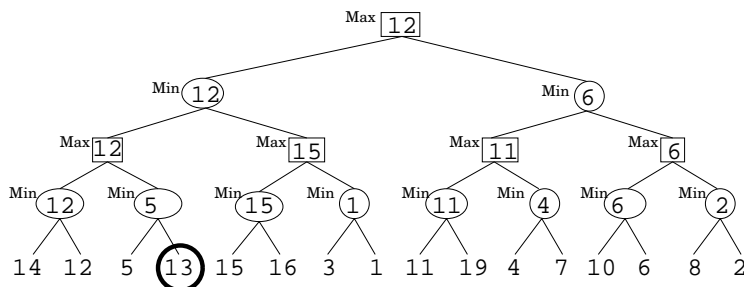
**P4-11: Alpha-Beta Pruning**

- Does it matter what value is in the yellow circle?



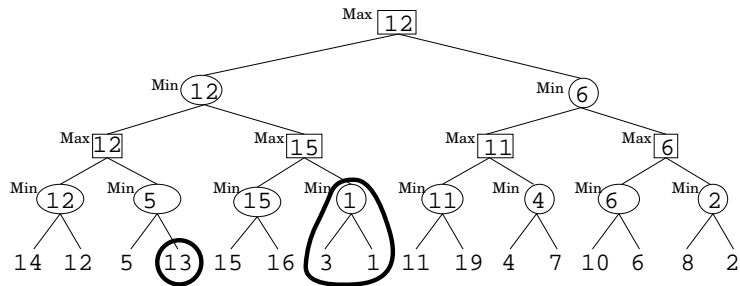
**P4-12: Alpha-Beta Pruning**

- If the yellow leaf has a value  $> 5$ , parent won't pick it
- If the yellow leaf has a value  $< 12$ , grandparent won't pick it
- To affect the root, value must be  $< 5$  **and**  $> 12$



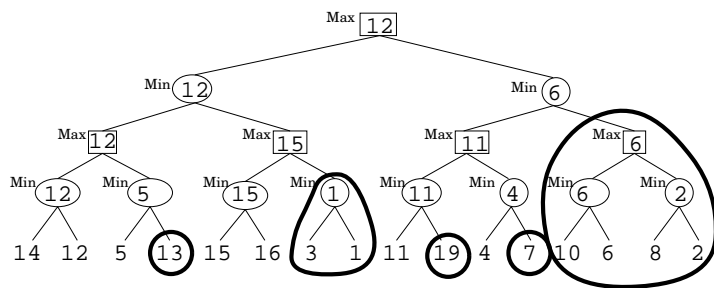
**P4-13: Alpha-Beta Pruning**

- Value of nodes in neither yellow circle matter. Are there more?



P4-14: Alpha-Beta Pruning

- Value of nodes in none of the yellow circles matter.



P4-15: Alpha-Beta Pruning