Game Engineering

CS420-2014F-24

Board / Strategy Games

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24-0: Overview

- Example games (board splitting, chess, Othello)
- Min/Max trees
- Alpha-Beta Pruning
- Evaluation Functions
- Stopping the Search
- Playing with chance

24-1: Two player games

- Board-Splitting Game
  - Two players, V & H
  - V splits the board vertically, selects one half
  - H splits the board horizontally, selects one half
  - V tries to maximize the final value, H tries to minimize the final value

14	5	11	4
12	13	9	7
15	13	10	8
16	1	6	2

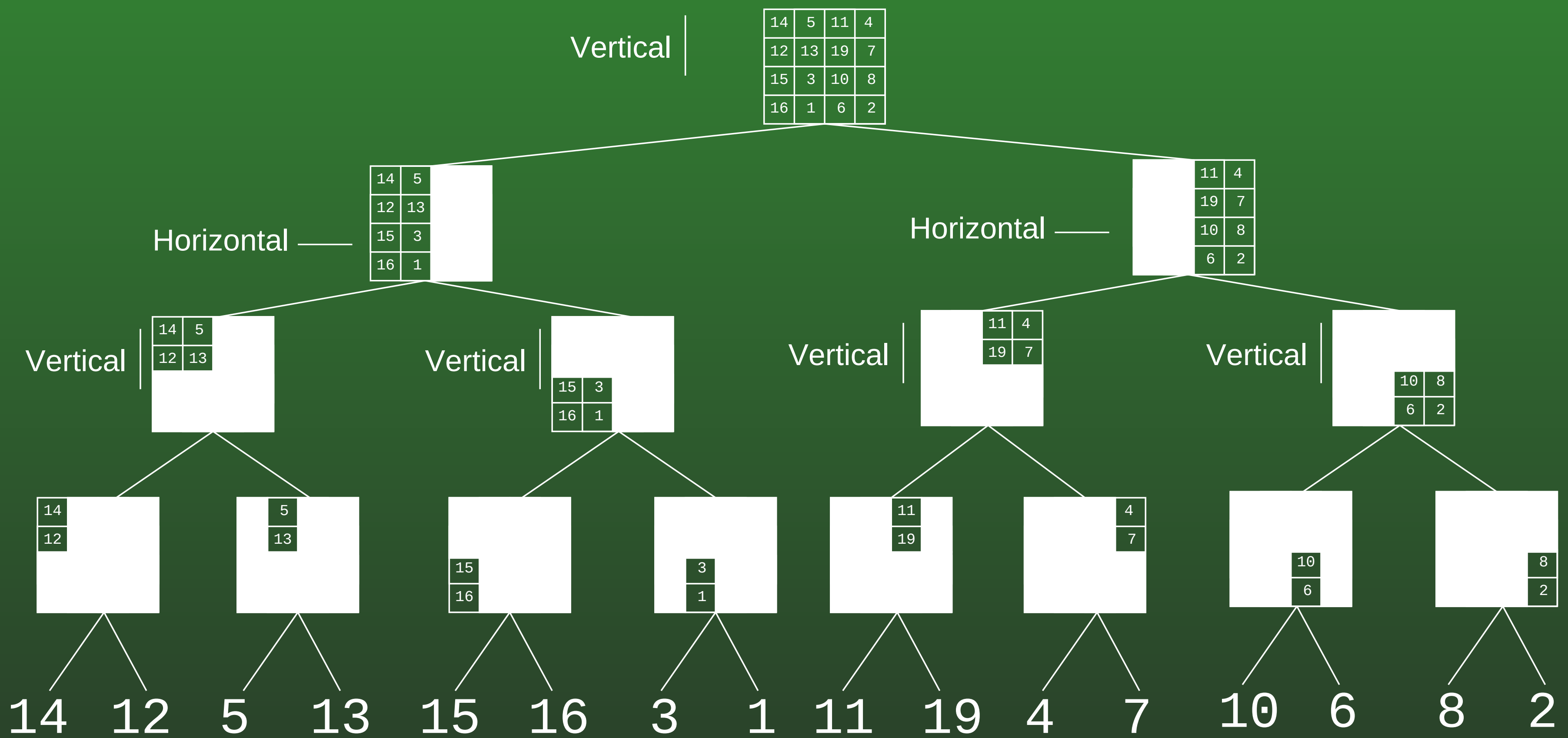
24-2: Two player games

- Board-Splitting Game
  - We assume that both players are rational (make the best possible move)
  - How can we determine who will win the game?

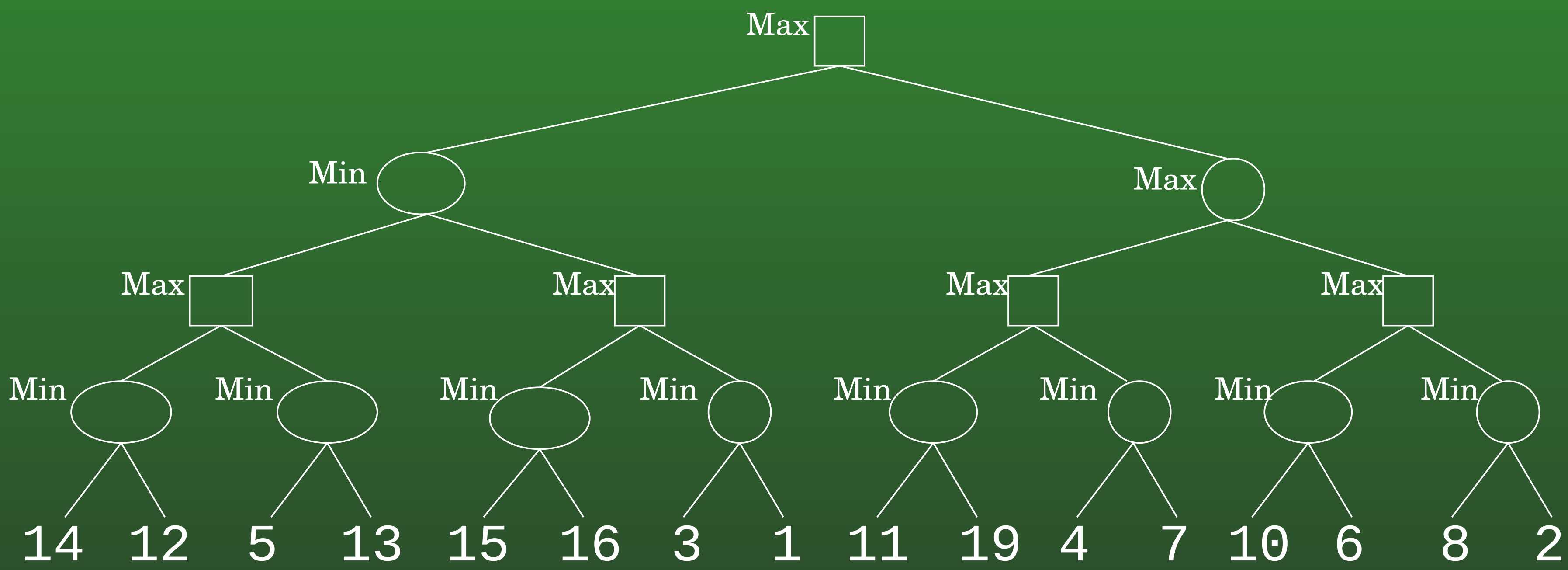
24-3: Two player games

- Board-Splitting Game
  - We assume that both players are rational (make the best possible move)
  - How can we determine who will win the game?
    - Examine all possible games!

24-4: Two player games



24-5: Two player games



24-6: Two player games

24-7: Two player games

- Game playing agent can do this to figure out which move to make
  - Examine all possible moves
  - Examine all possible responses to each move
  - ... all the way to the last move
  - Caclulate the value of each move (assuming opponent plays perfectly)

24-8: Two-Player Games

- Initial state
- Successor Function
  - Just like other Searches
- Terminal Test
  - When is the game over?
- Utility Function
  - Only applies to terminal states
  - Chess: +1, 0, -1
  - Backgammon: 192 . . . -192

24-9: Minimax Algorithm

```
Max(node)
  if terminal(node)
     return utility(node)
  maxVal = MIN_VALUE
  children = successors(node)
  for child in children
     maxVal = max(maxVal, Min(child))
  return maxVal

Min(node)
  if terminal(node)
     return utility(node)
  minVal = MAX_VALUE
  children = successors(node)
  for child in children
     minVal = min(minVal, Max(child))
  return minVal
```

Minimax Algorithm

- Branching factor of b, game length of d moves, what are the time and space requirements for Minimax?

24-11: Minimax Algorithm

- Branching factor of b, game length of d moves, what are the time and space requirements for Minimax?
  - Time: $O(b^d)$
  - Space: $O(d)$
- Not managable for any real games – chess has an average b of 35, can't search the entire tree
- Need to make this more managable

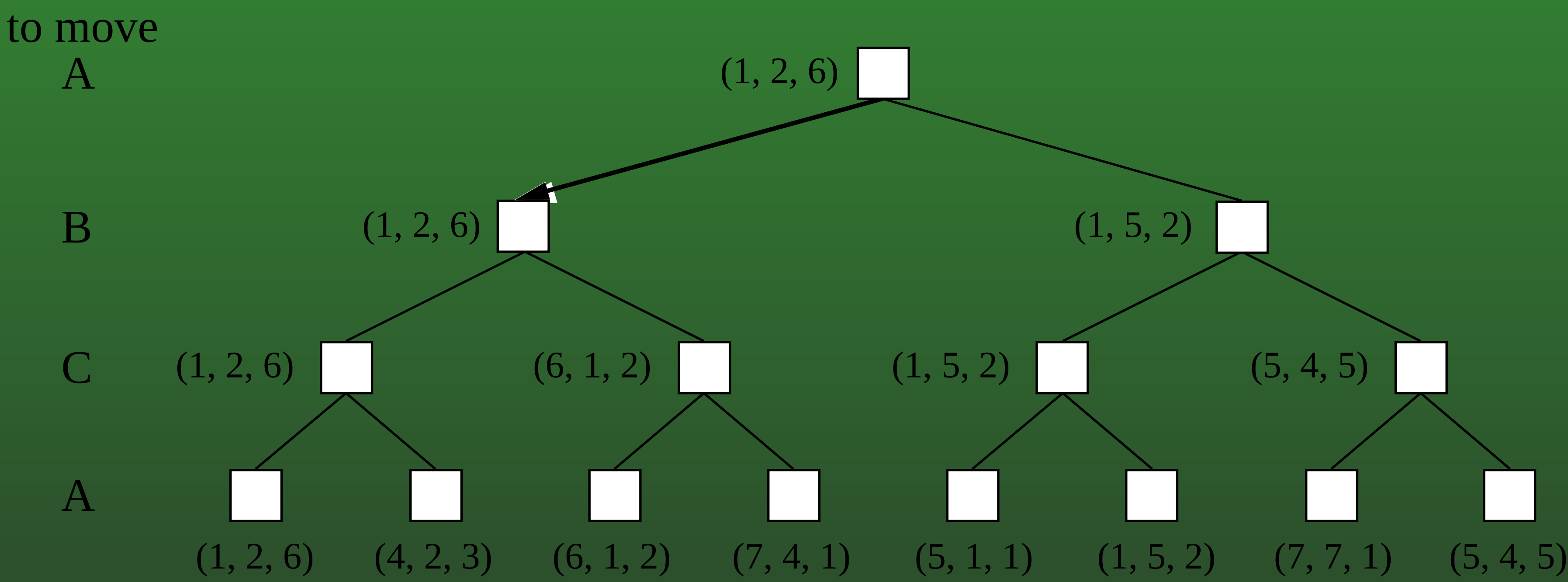
24-12: > 2 Player Games

- What if there are > 2 players?
- We can use the same search tree:
  - Alternate between several players
  - Need a different evaluation function

24-13: > 2 Player Games

- Functions return a vector of utilities
  - One value for each player
  - Each player tries to maximize their utility
  - May or may not be zero-sum

24-14: > 2 Player Games

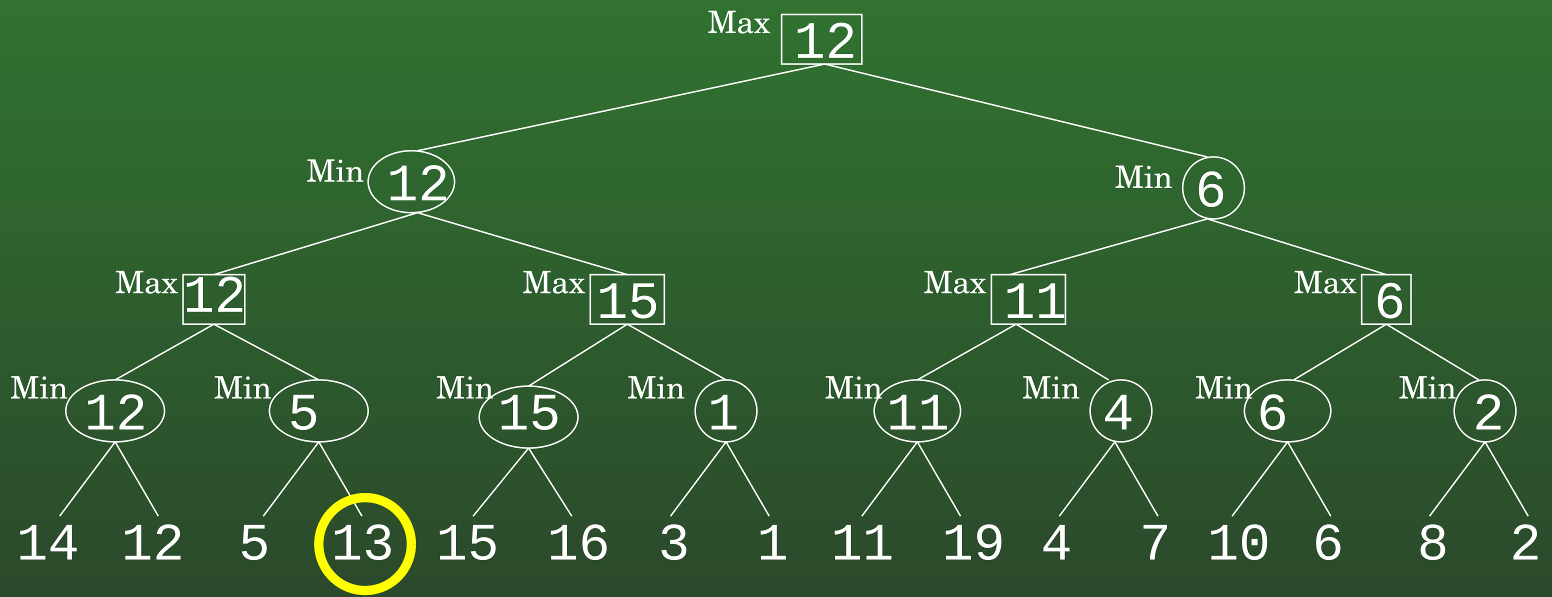


24-15: Non zero-sum games

- Even 2-player games don’t need to be zero-sum
  - Utility function returns a vector
  - Each player tries to maximize their utility
- If there is a state with maximal outcome for both players, rational players will cooperate to find it
- Minimax is rational, will find such a state

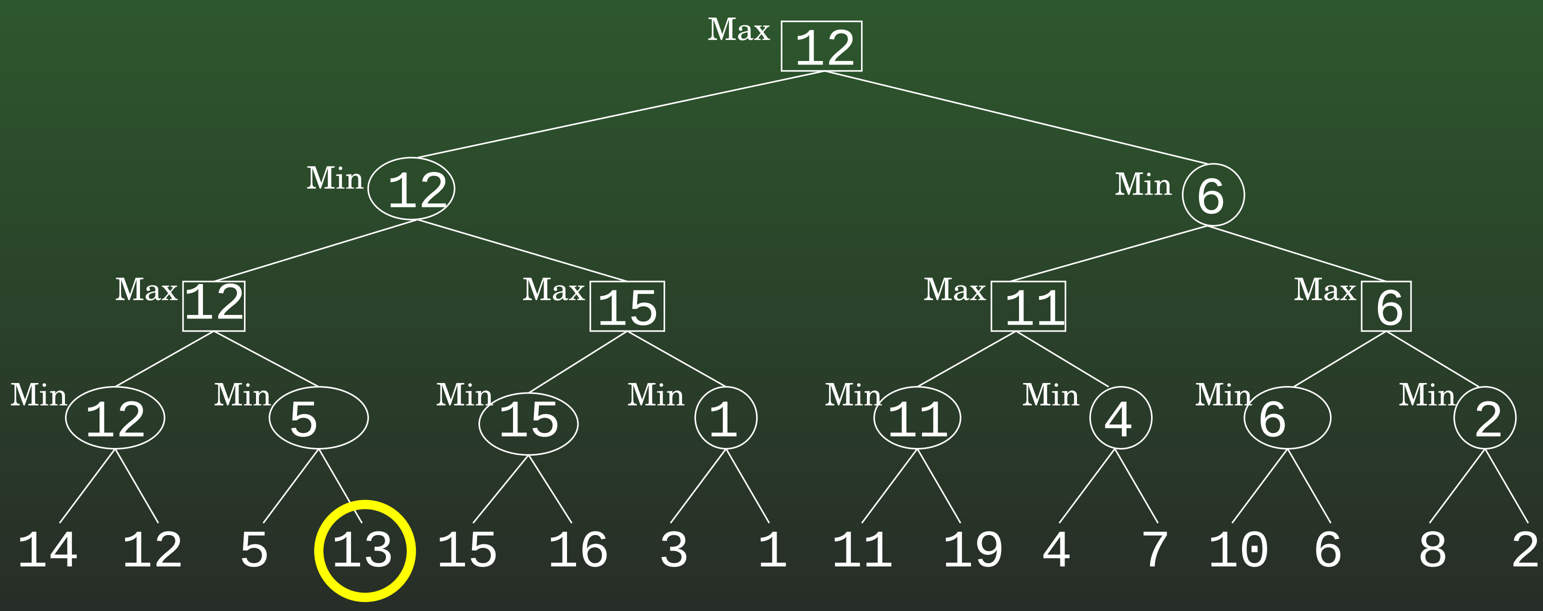
24-16: Alpha-Beta Pruning

- Does it matter what value is in the yellow circle?



24-17: Alpha-Beta Pruning

- If the yellow leaf has a value > 5, parent won't pick it
- If the yellow leaf has a value < 12, grandparent won't pick it
- To affect the root, value must be < 5 **and** > 12

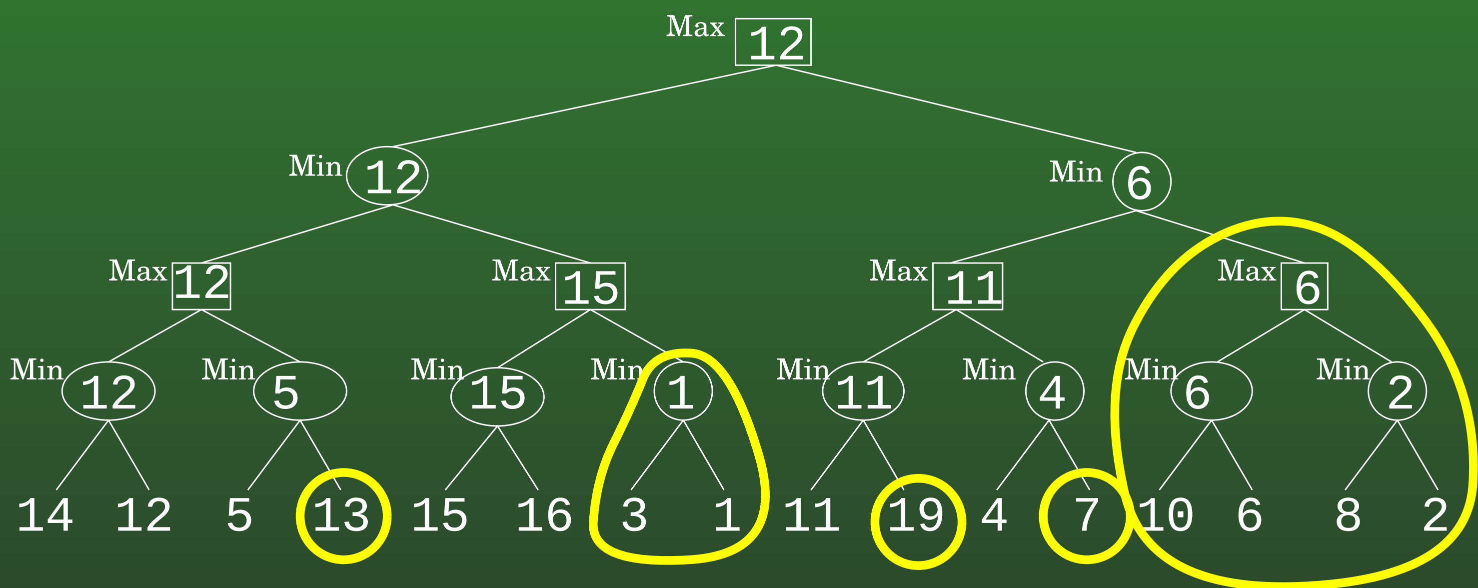


Alpha-Beta Pruning

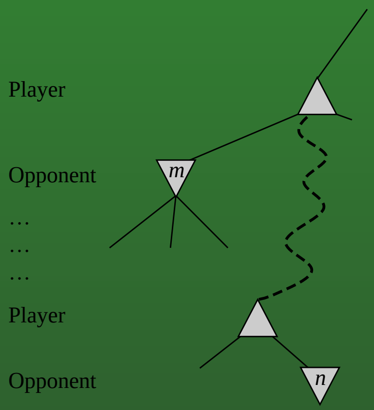
- Value of nodes in neither yellow circle matter. Are there more?

24-19: Alpha-Beta Pruning

- Value of nodes in none of the yellow circles matter.



24-20: Alpha-Beta Pruning

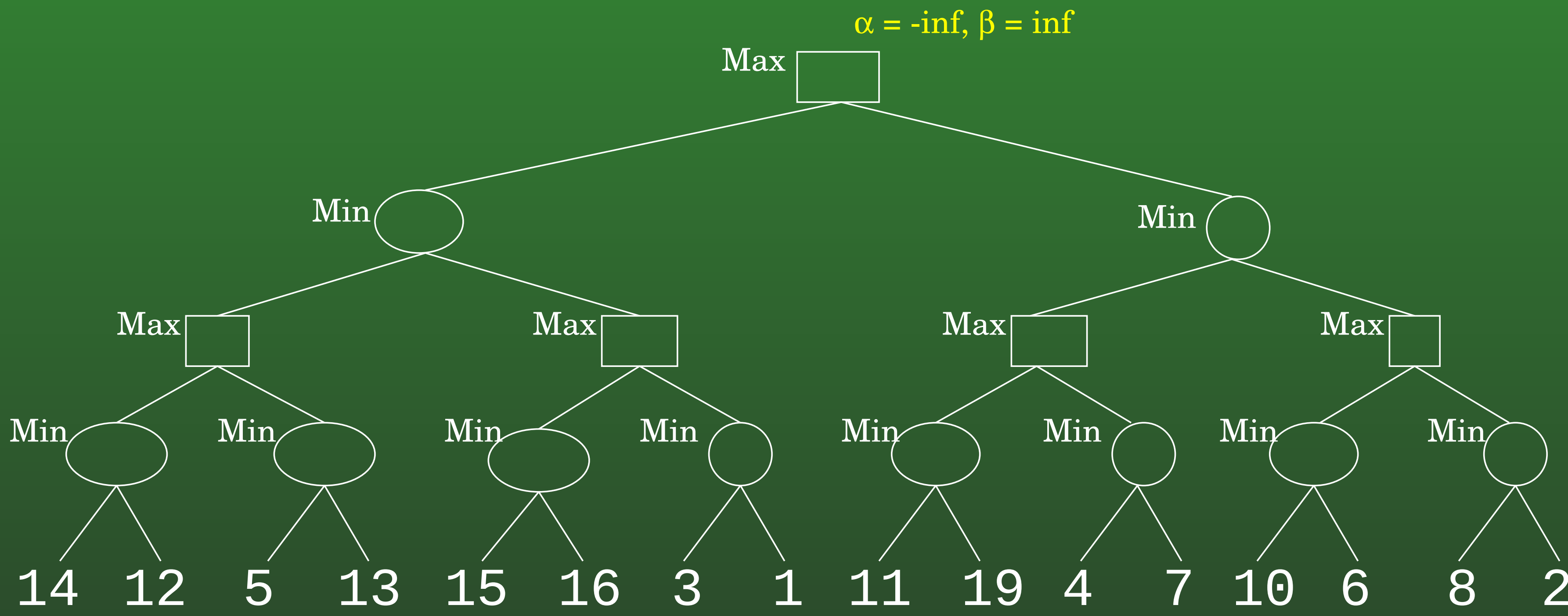


- If m is better than n for Player, we will never reach n
  - (player would pick m instead)

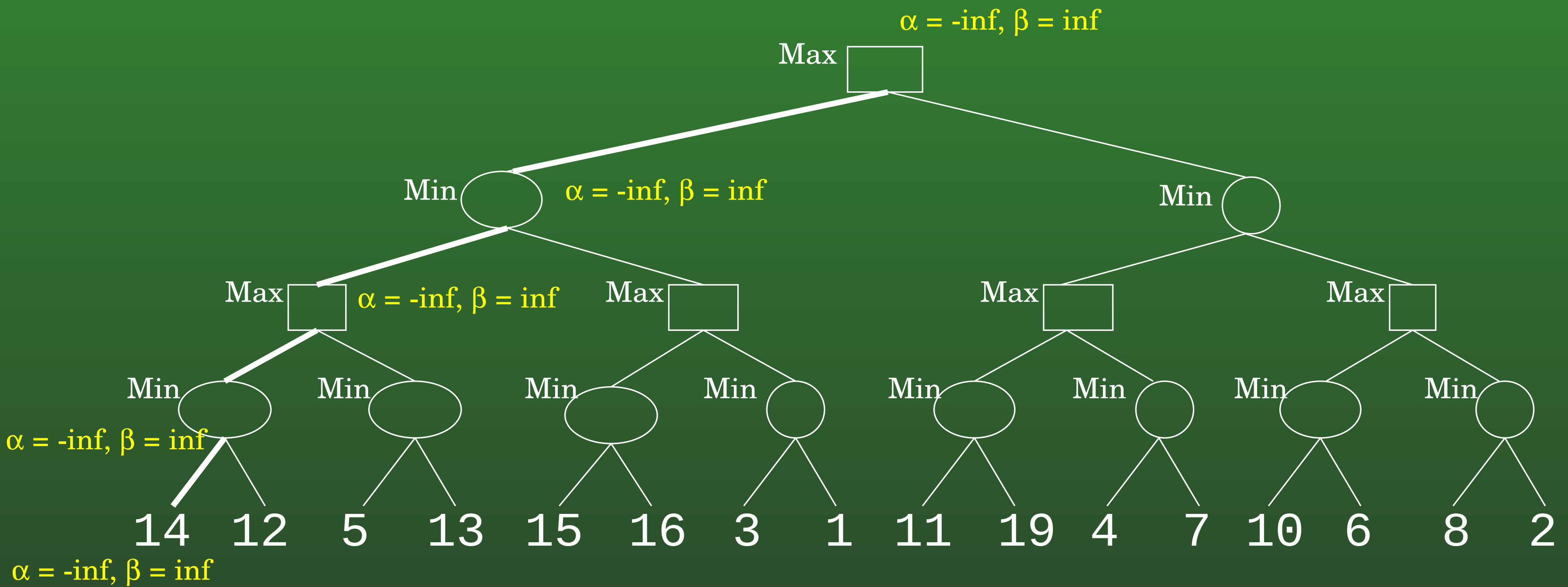
24-21: Alpha-Beta Pruning

- Maintain two bounds, lower bound α, and an upper bound β
  - Bounds represent the values the node must have to possibly affect the root
- As you search the tree, update the bounds
  - Max nodes increase α, min nodes decrease β
- If the bounds ever cross, this branch cannot affect the root, we can prune it.

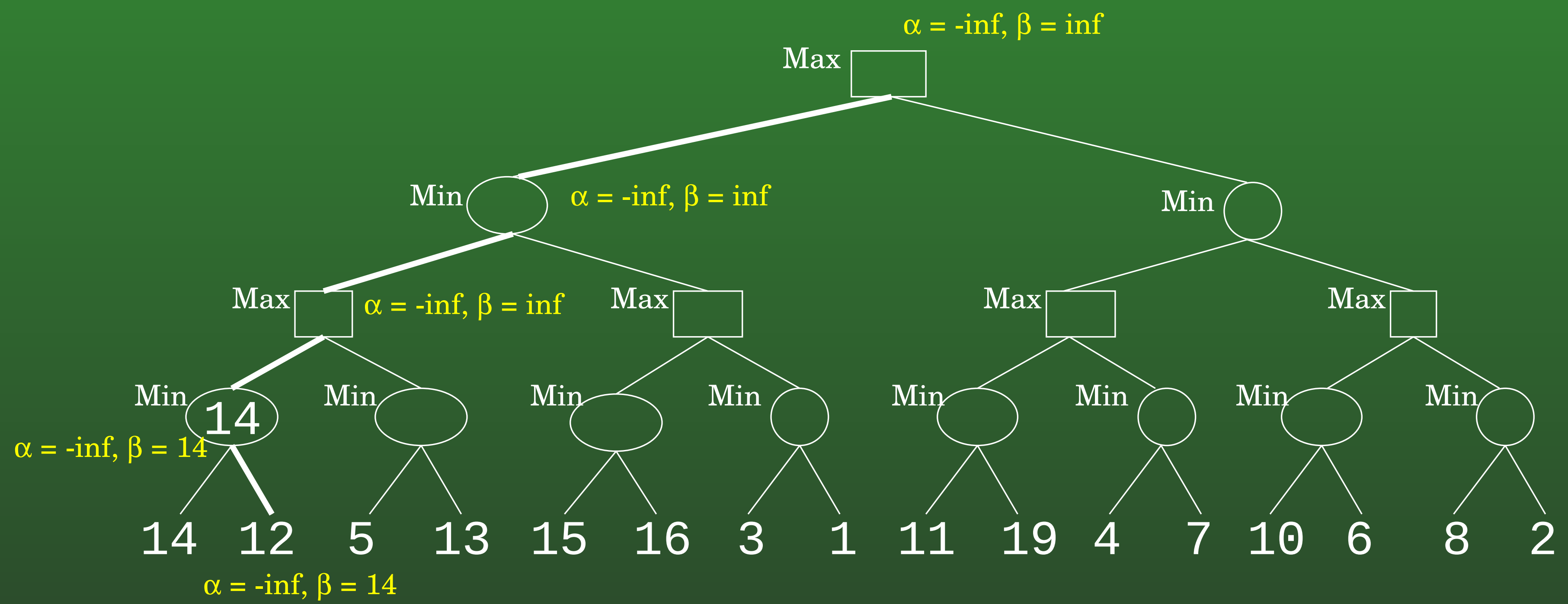
Alpha-Beta Pruning



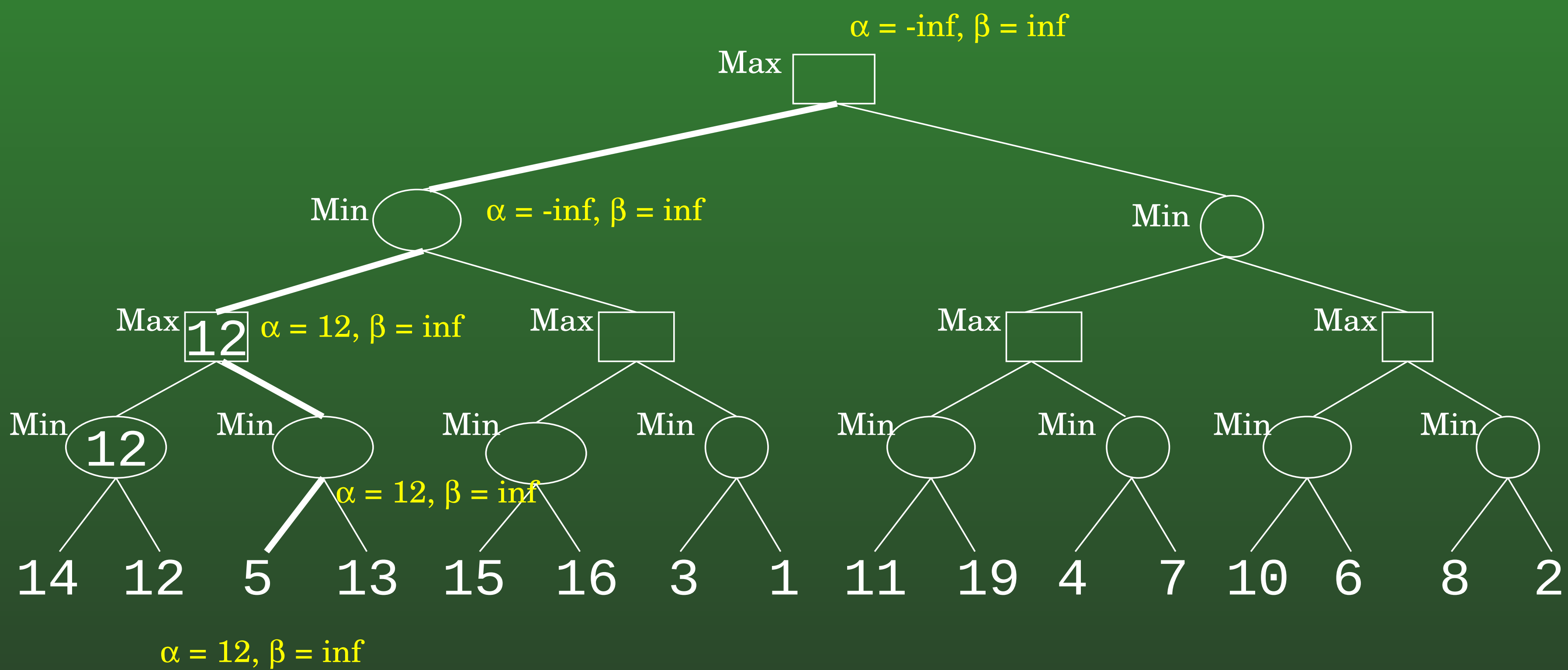
Alpha-Beta Pruning



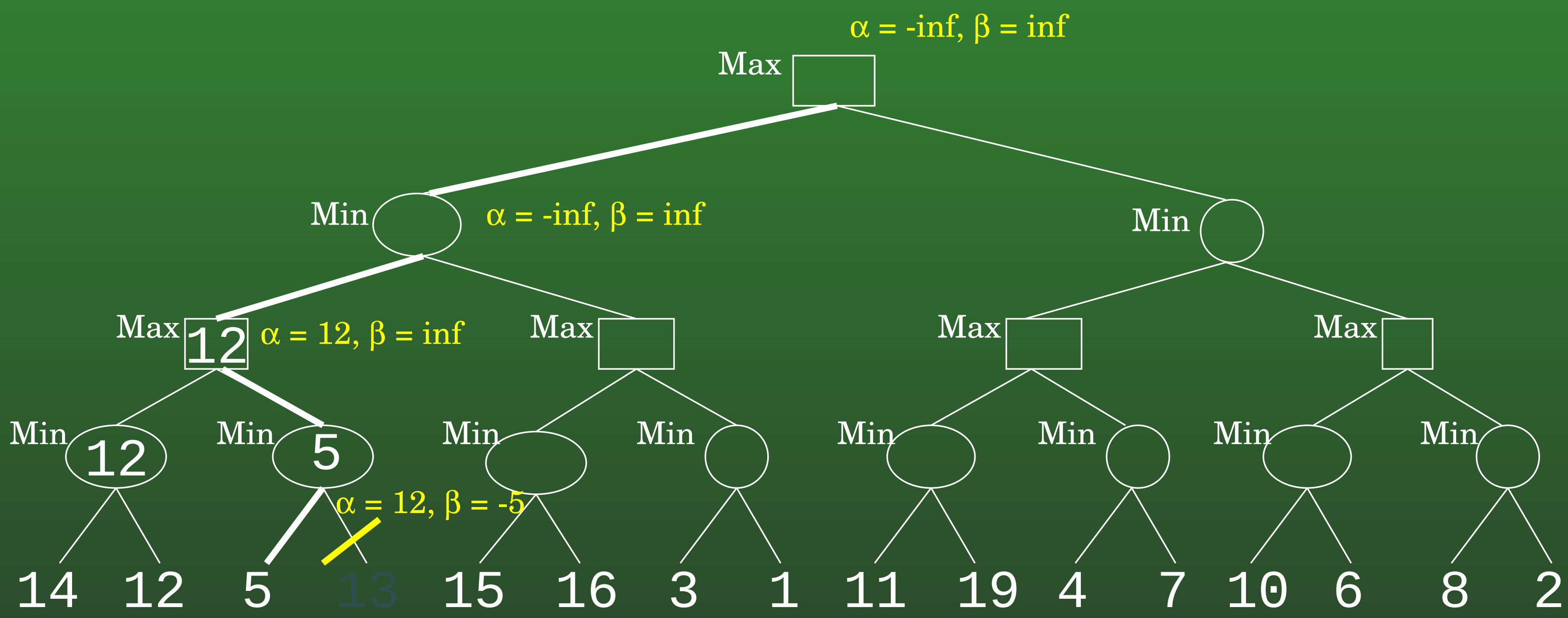
24-24: Alpha-Beta Pruning



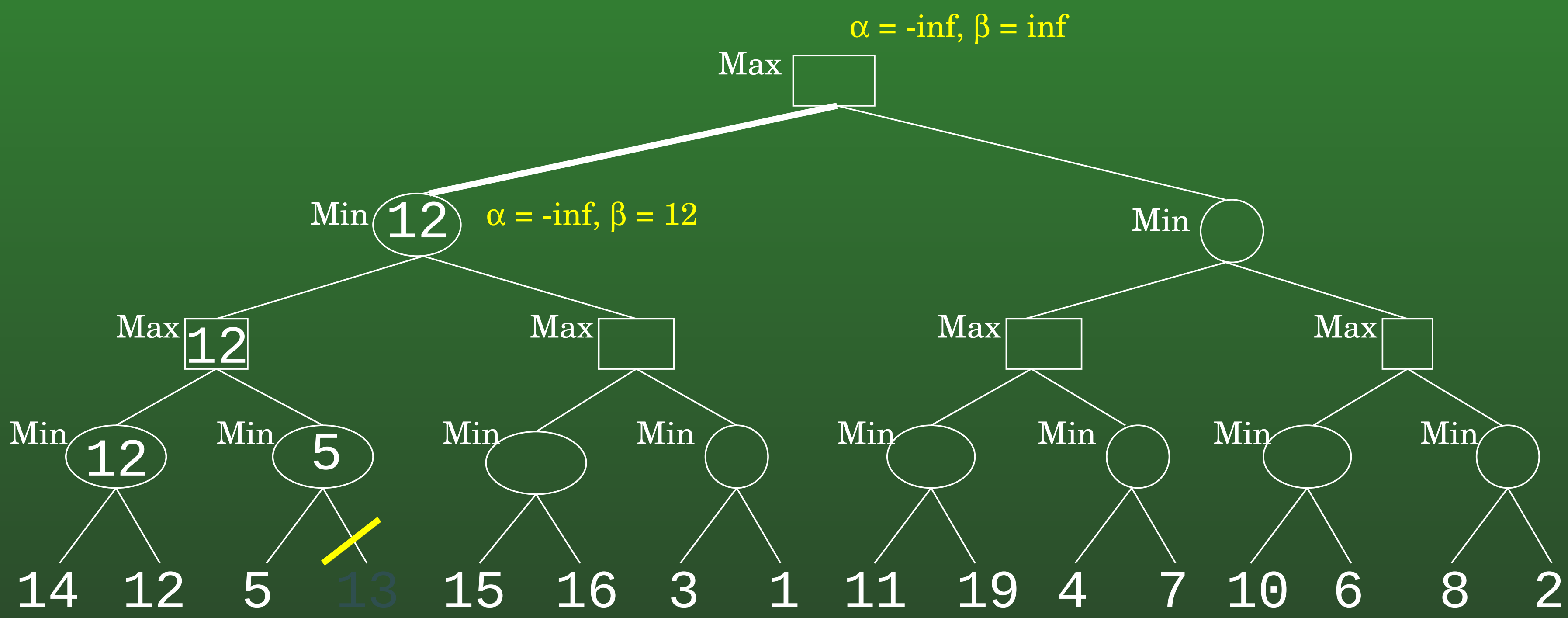
24-25: Alpha-Beta Pruning



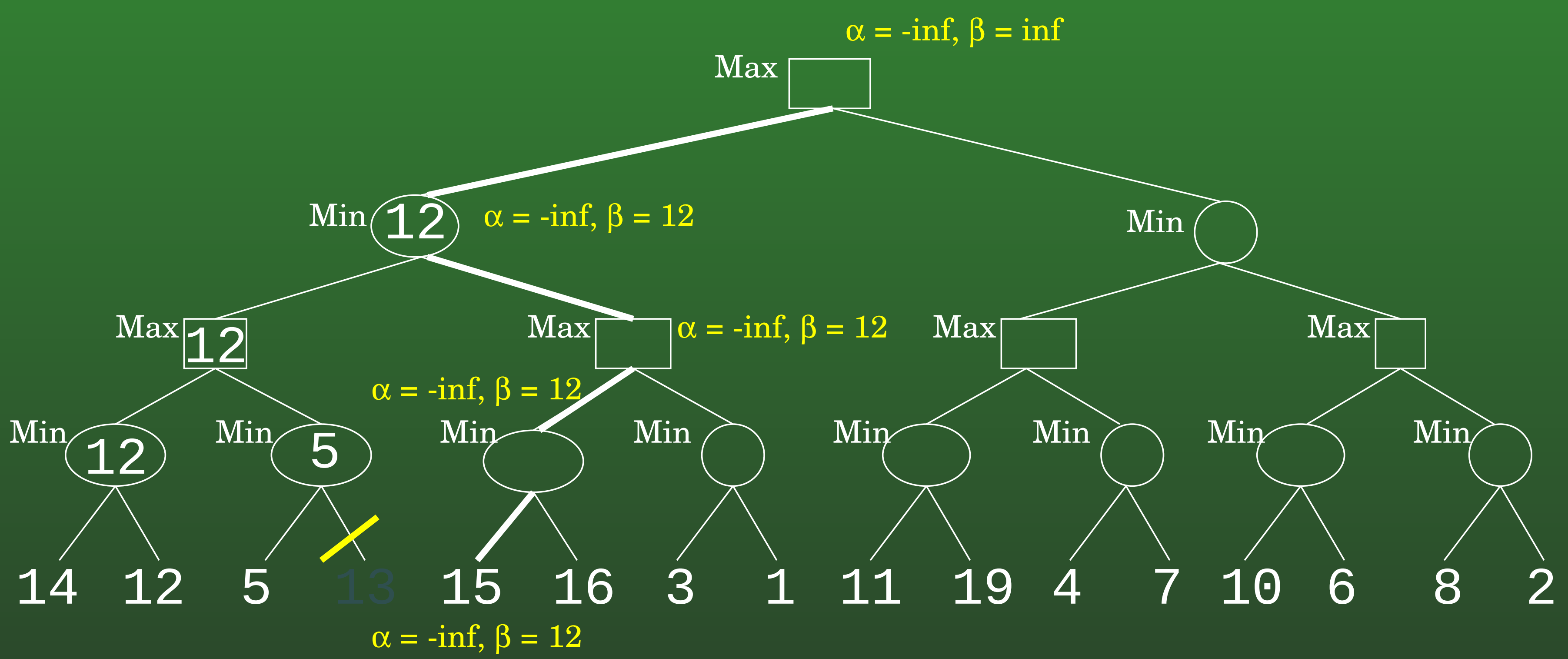
24-26: Alpha-Beta Pruning



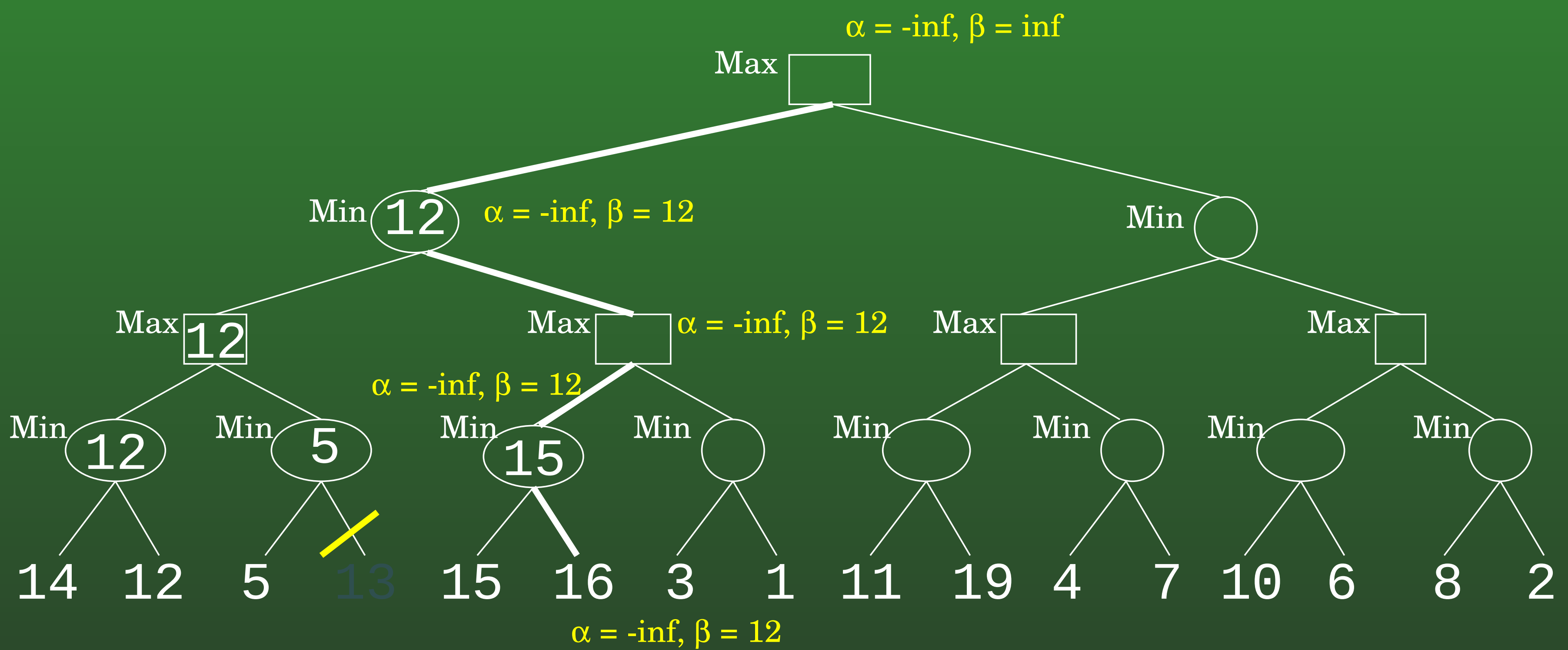
24-27: Alpha-Beta Pruning



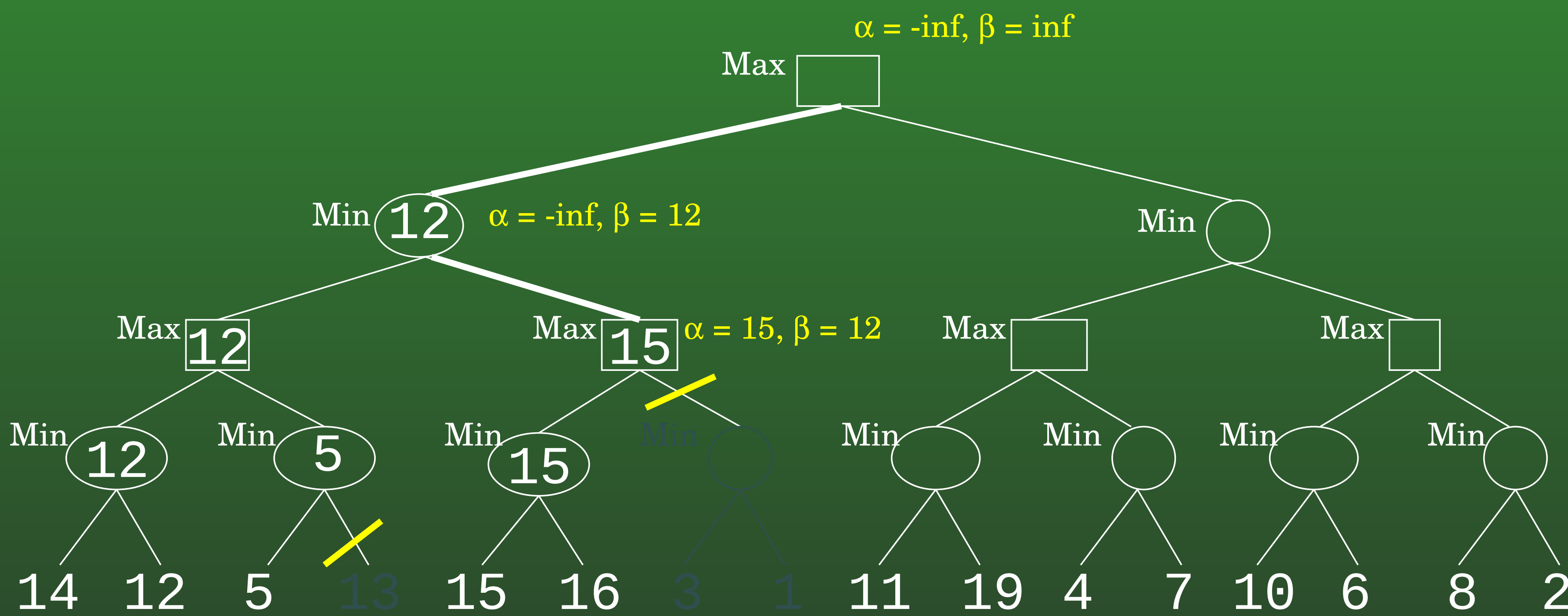
Alpha-Beta Pruning



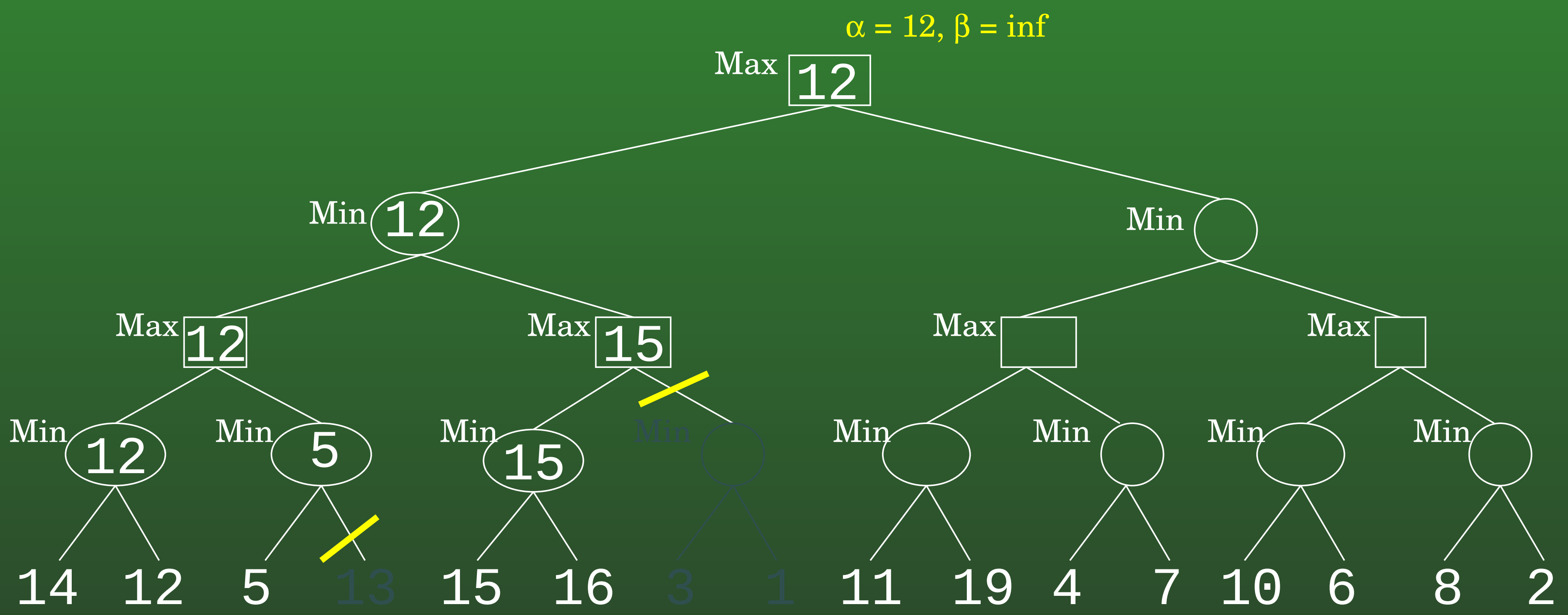
24-29: Alpha-Beta Pruning



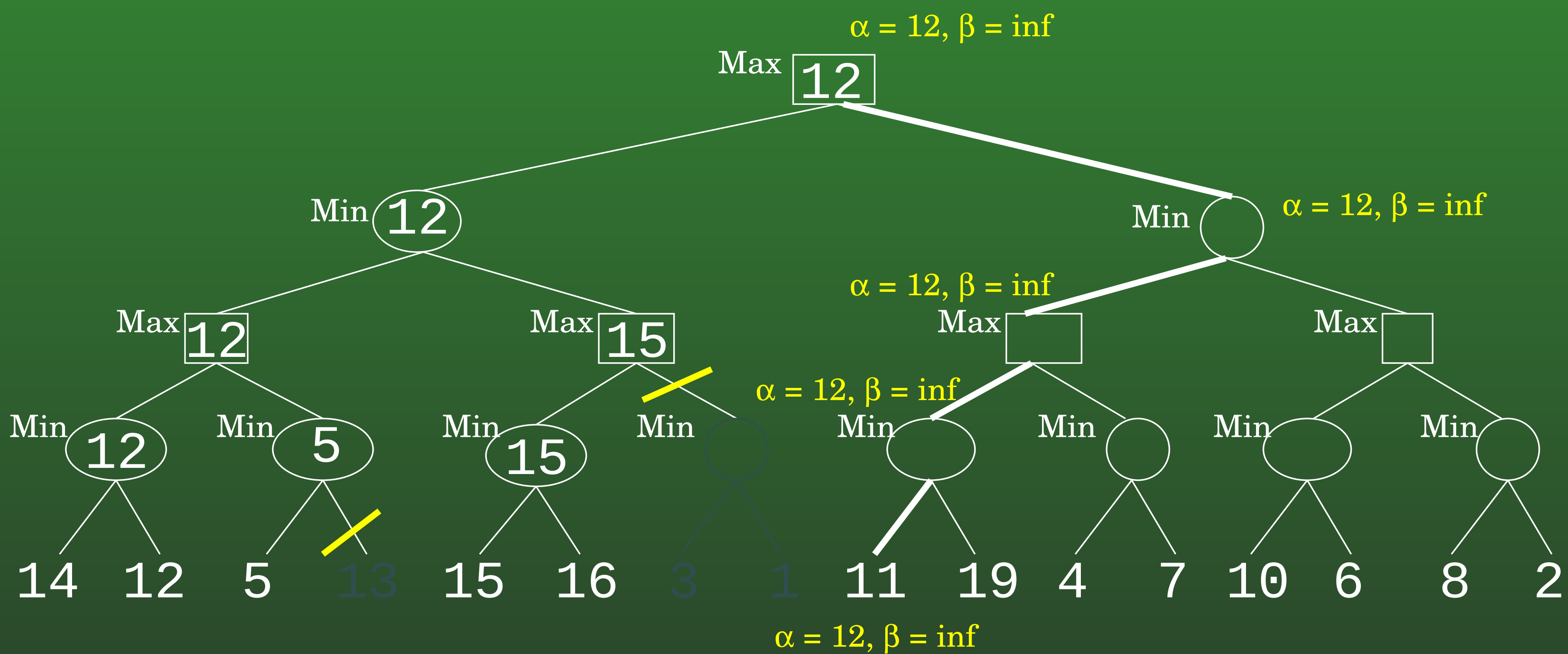
24-30: Alpha-Beta Pruning



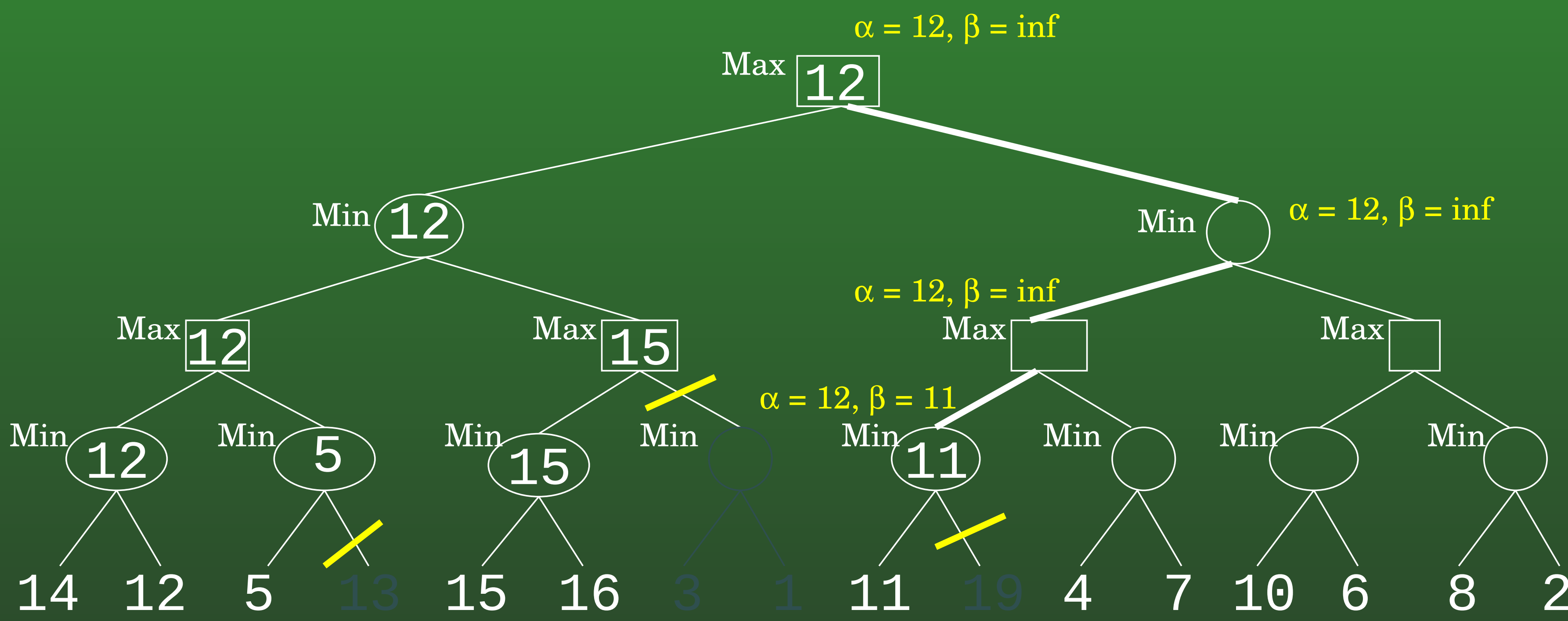
Alpha-Beta Pruning



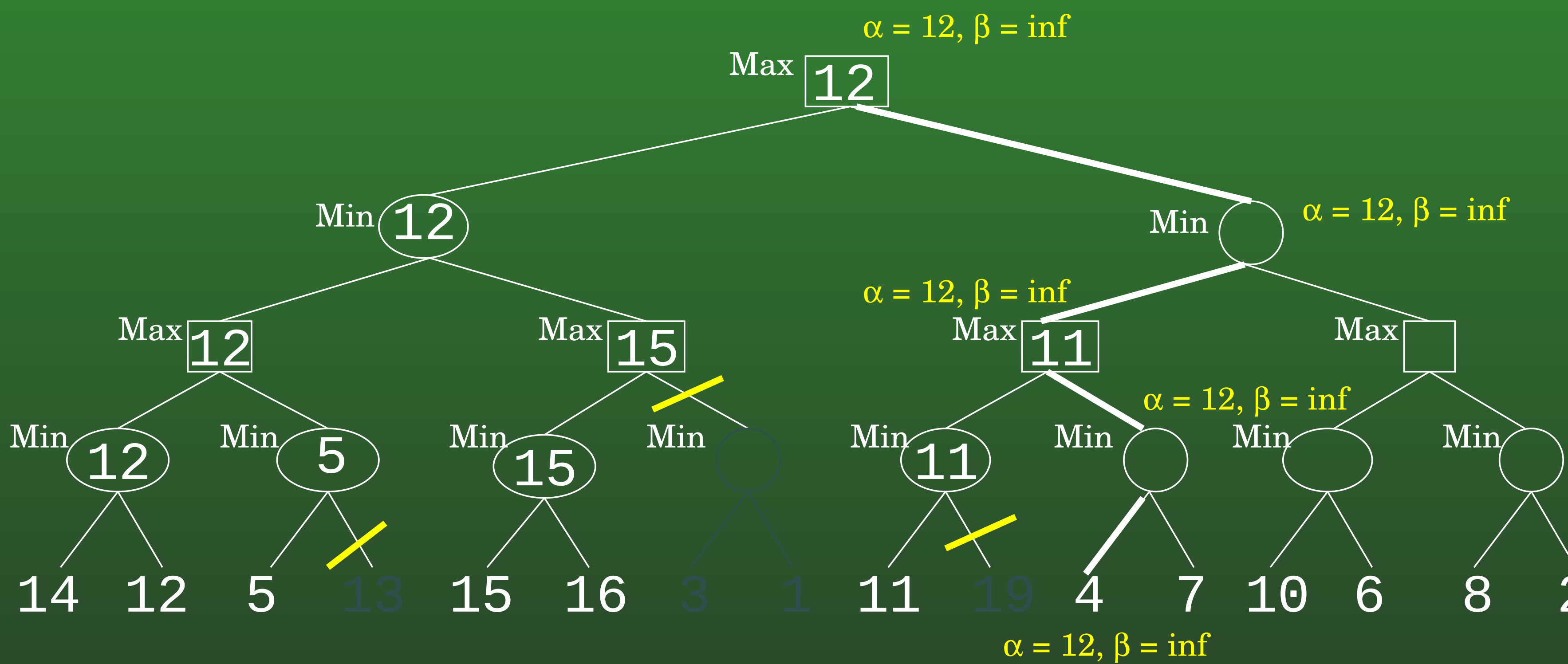
24-32: Alpha-Beta Pruning



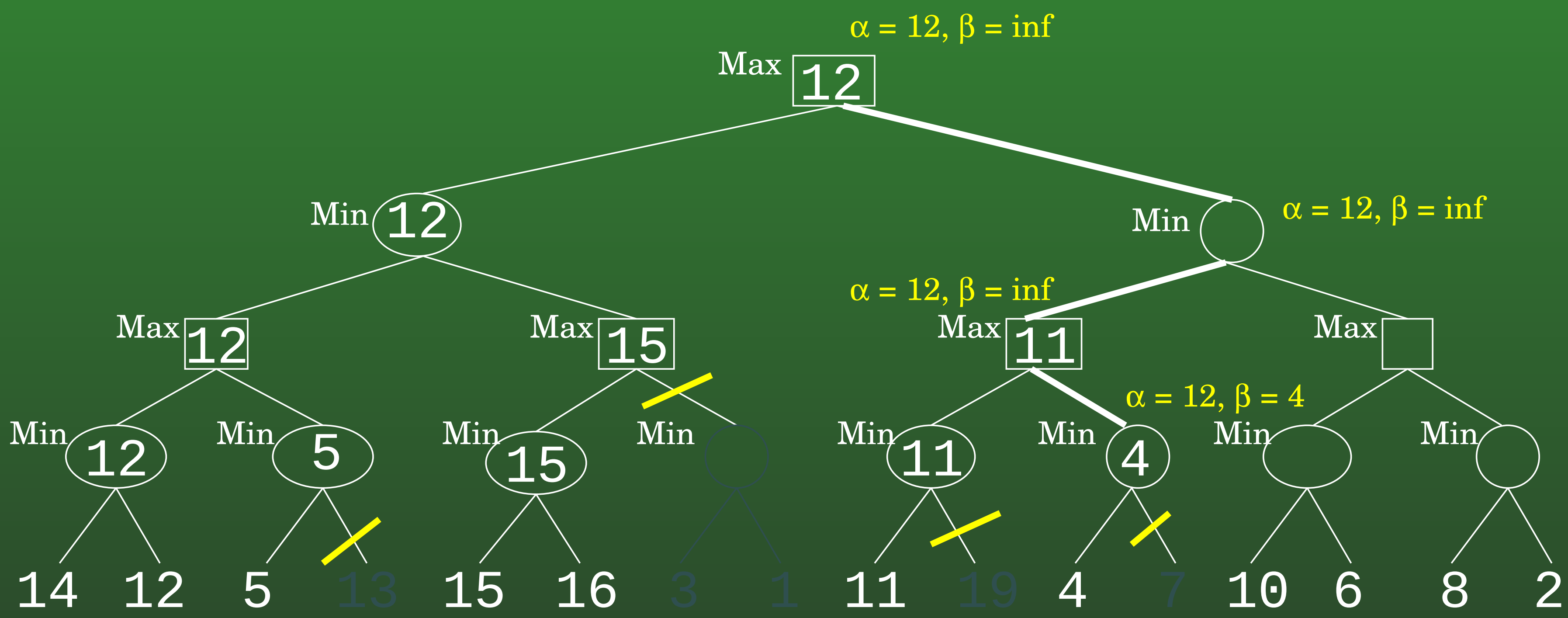
24-33: Alpha-Beta Pruning



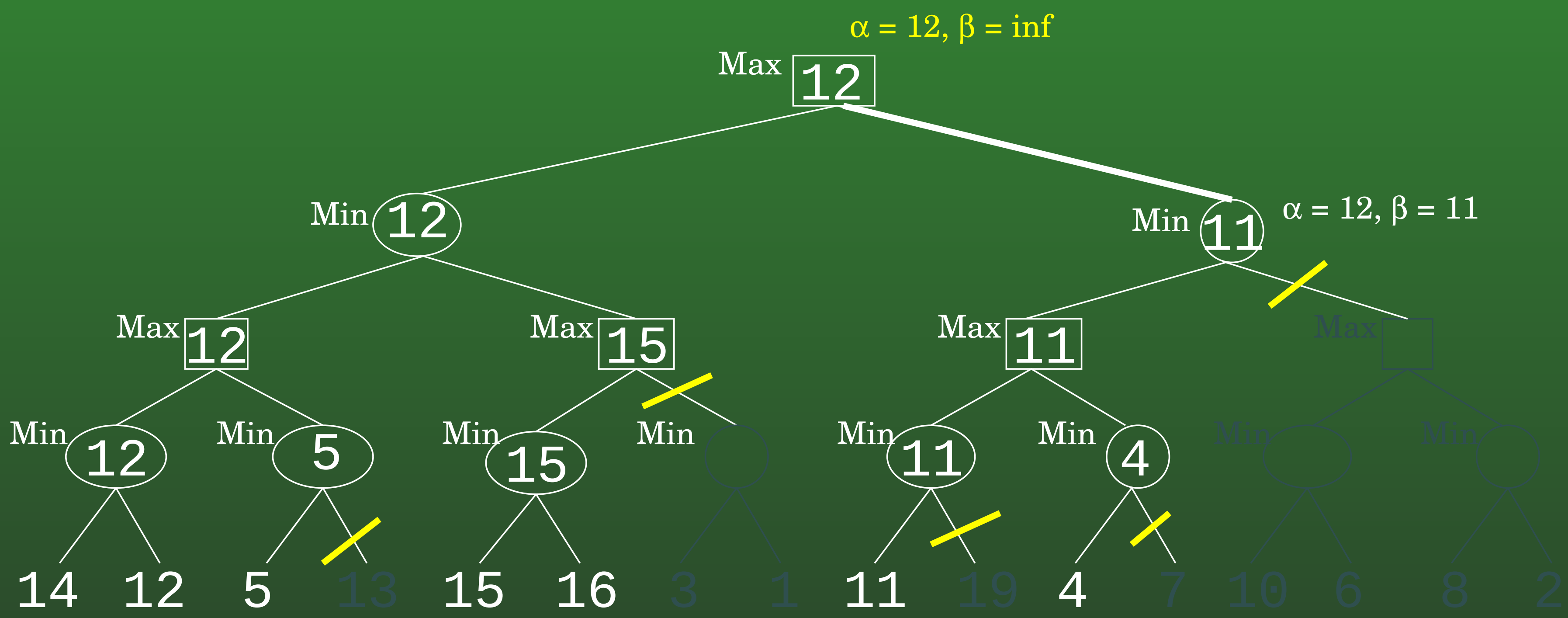
24-34: Alpha-Beta Pruning



24-35: Alpha-Beta Pruning



24-36: Alpha-Beta Pruning



24-37: Alpha-Beta Pruning

- We can cut large branches from the search tree
  - In the previous example, what would happen with similar values and a deeper tree?
- If we choose the order that we evaluate nodes (more on this in a minute ...), we can dramatically cut down on how much we need to search

Evaluation Functions

- We can't search all the way to the bottom of the search tree
  - Trees are just too big
- Search a few levels down, use an evaluation function to see how good the board looks at the moment
- Back up the result of the evaluation function, as if it was the utility function for the end of the game

24-39: Evaluation Functions

- Chess:
  - Material - value for each piece (pawn = 1, bishop = 3, etc)
    - Sum of my material - sum of your material
  - Positional advantages
    - King protected
    - Pawn structure
- Othello:
  - Material – each piece has unit value
  - Positional advantages
    - Edges are good
    - Corners are better
    - “near” edges are bad

24-40: Evaluation Functions

- If we have an evaluation function that tells us how good a move is, why do we need to search at all?
  - Could just use the evaluation function
- If we are only using the evalution function, does search do us any good?

Evaluation Functions & α-β

- How can we use the evaluation function to maximize the pruning in alpha-beta pruning?

24-42: Evaluation Functions & α-β

- How can we use the evaluation function to maximize the pruning in alpha-beta pruning?
  - Order children of max nodes, from highest to lowest
  - Order children of min node, from lowest to highest
  - (Other than for ordering, eval function is not used for interior nodes)
- With perfect ordering, we need to search only $b^{d/2}$ (instead of b^d) to find the optimal move – can search up to twice as far

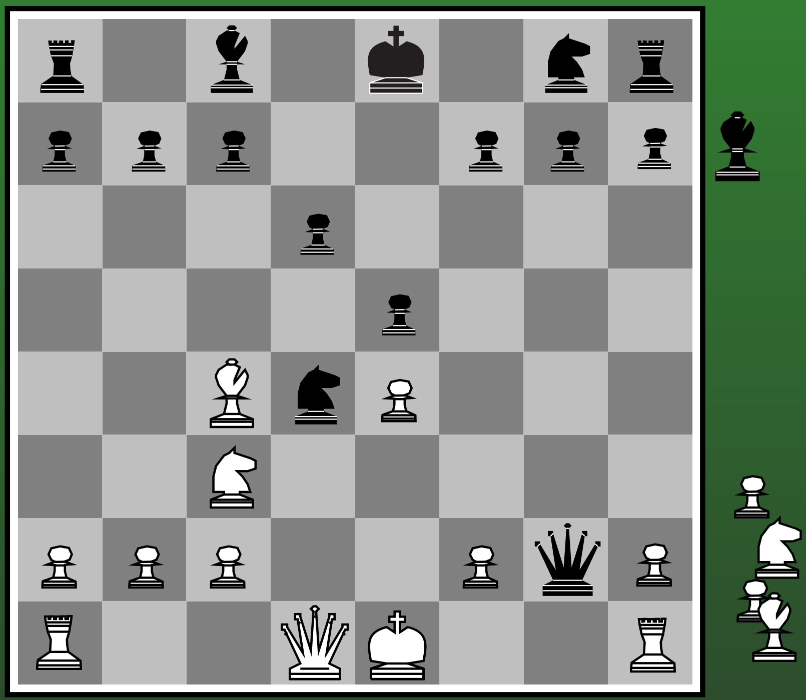
24-43: Stopping the Search

- We can’t search all the way to the endgame
  - Not enough time
- Search a set number of moves ahead
  - Problems?

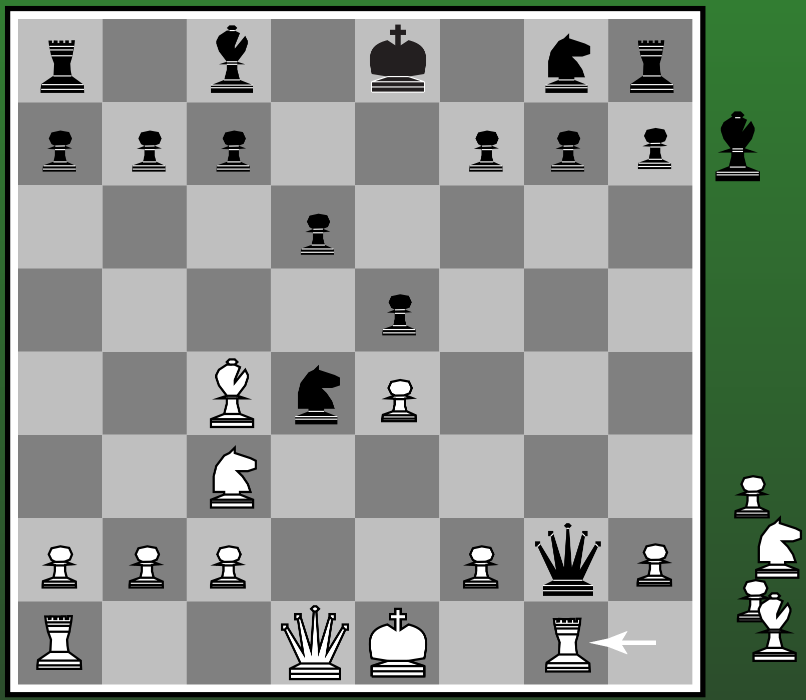
Stopping the Search

- We can’t search all the way to the endgame
  - Not enough time
- Search a set number of moves ahead
  - What if we are in the middle of a piece trade?
  - In general, what if our opponent is about to capture one of our pieces

24-45: Stopping the Search



(a) White to move



(b) White to move

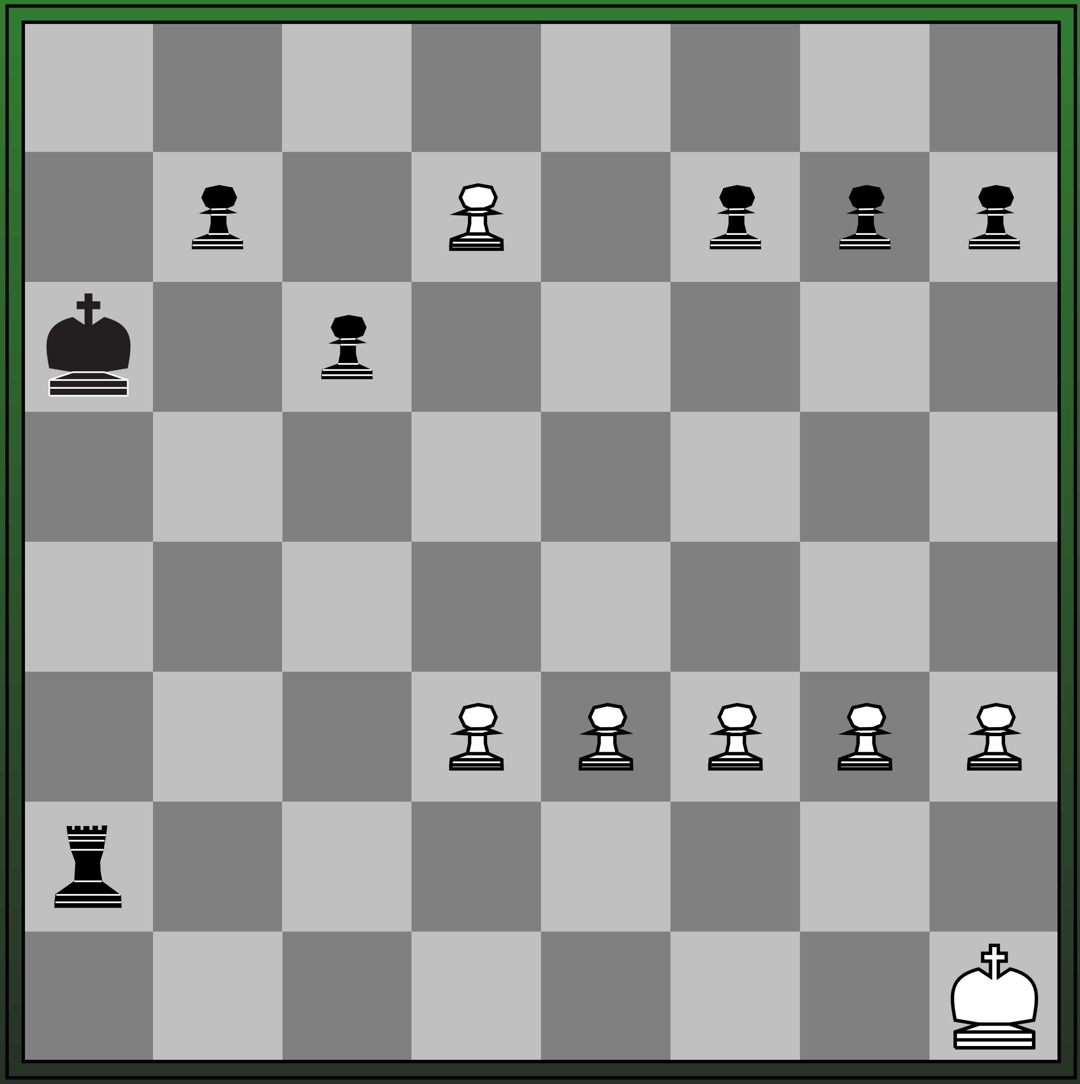
24-46: Stopping the Search

- Quiescence Search
  - Only apply the evaluation function to nodes that do not swing wildly in value
  - If the next move makes a large change to the evaluation function, look ahead a few more moves
  - Not increasing the search depth for the entire tree, just around where the action is
  - To prevent the search from going too deep, may restrict the kinds of moves (captures only, for instance)

24-47: Stopping the Search

- Horizon Problem
  - Sometimes, we can push a bad move past the horizon of our search
  - Not preventing the bad move, just delaying it
  - A position will look good, even though it is utlimately bad

24-48: Horizon Problem



Black to move

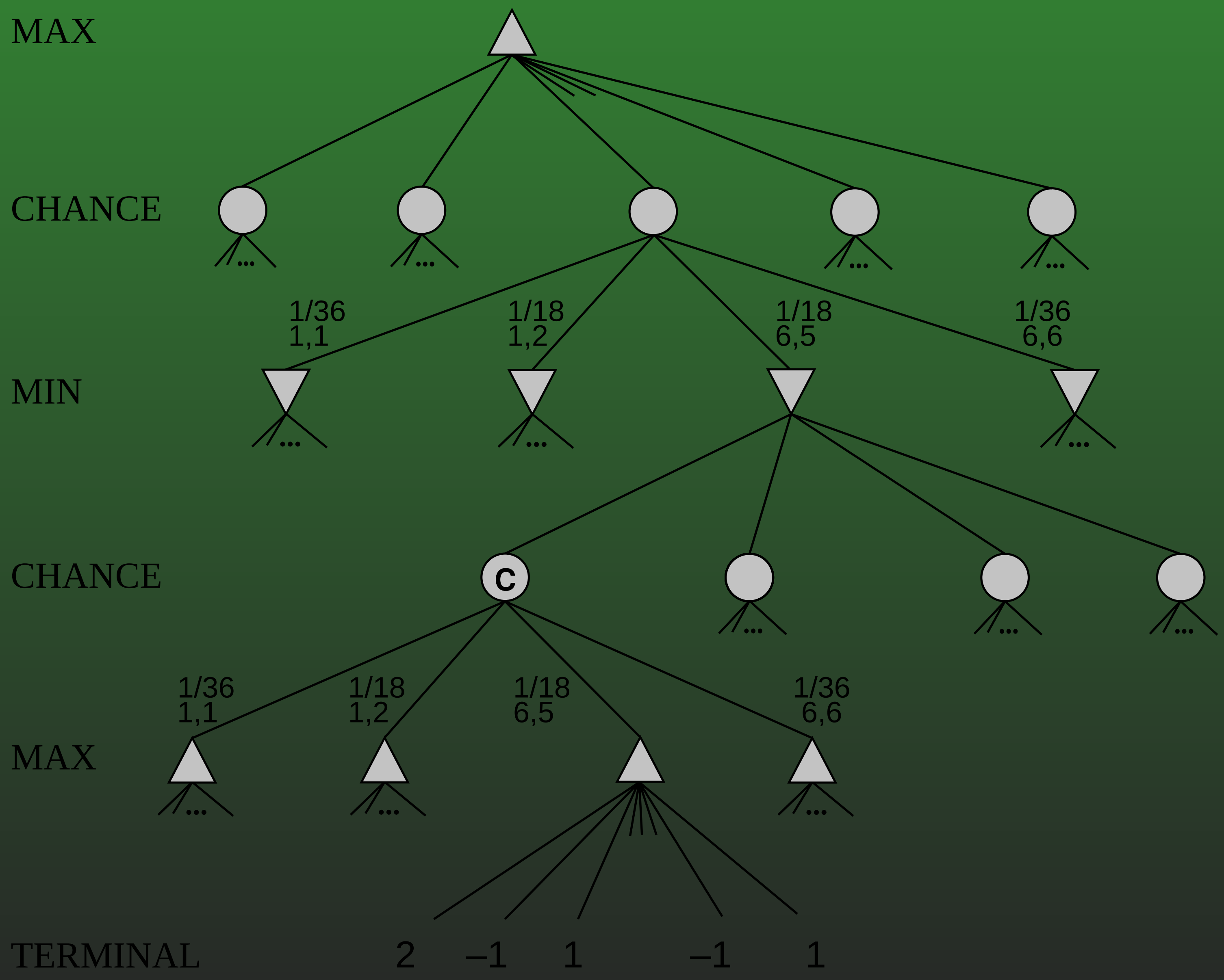
24-49: Horizon Problem

- Singular Extensions
  - When we are going to stop, see if there is one move that is clearly better than all of the others.
  - If so, do a quick “search”, looking only at the best move for each player
  - Stop when there is no “clearly better” move
  - Helps with the horizon problem, for a series of forced moves
- Similar to quiescence search

24-50: Adding Chance

- What about games that have an element of chance (backgammon, poker, etc)
- We can add chance nodes to our search tree
  - Consider "chance" to be another player
- How should we back up values from chance nodes?

Adding Chance



Adding Chance

- For Max nodes, we backed up the largest value:

$$\max_{s \in Sucessors(n)} Val(s)$$

- For Min nodes, we backed up the smallest

$$\max_{s \in Sucessors(n)} Val(s)$$

- For chance nodes, we back up the expected value of the node

$$\sum_{s \in Sucessors(n)} P(s) Val(s)$$

Adding Chance

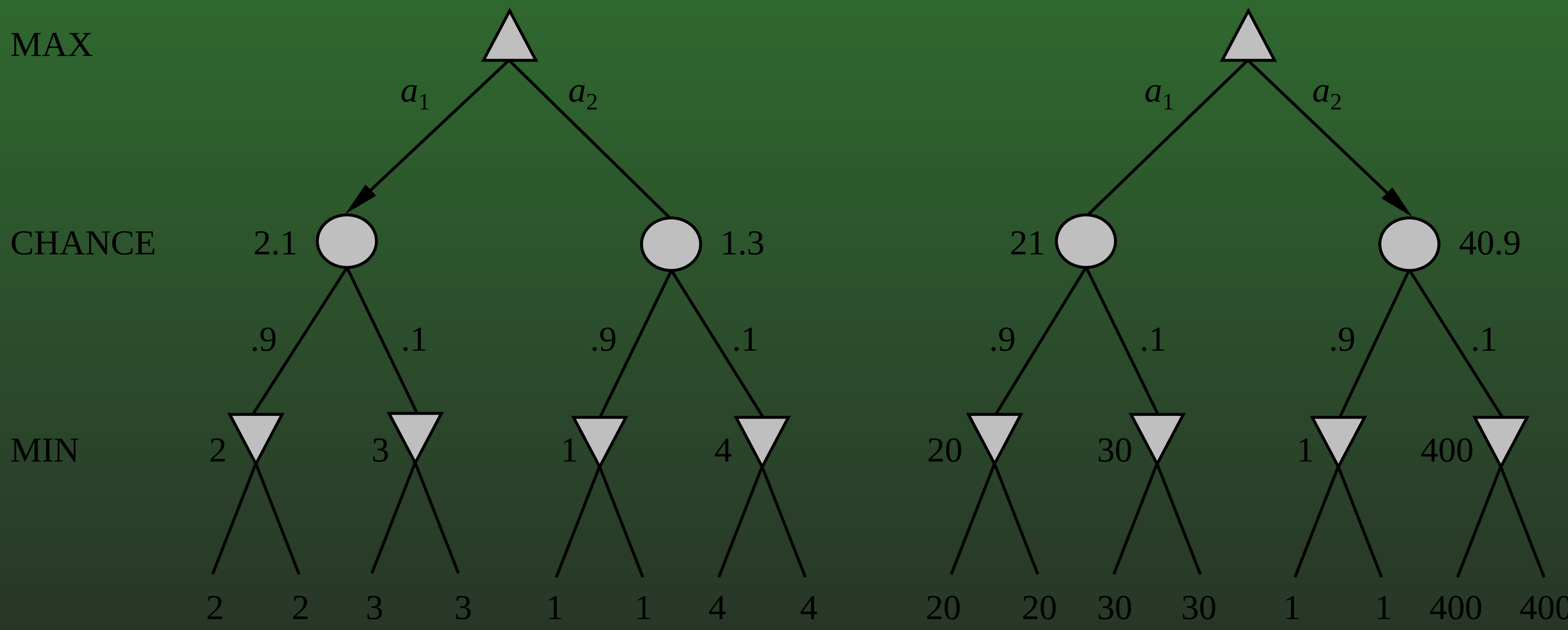
- Adding chance dramatically increases the number of nodes to search
  - Braching factor b (ignoring die rolls)
  - n different dice outcomes per turn
  - Time to search to level m?

Adding Chance

- Adding chance dramatically increases the number of nodes to search
  - Braching factor b (ignoring die rolls)
  - n different dice outcomes per turn
  - Time to search to level m: $b^m n^m$

24-55: Adding Chance

- Because we are using expected value for chance nodes, need to be more careful about choosing the evaluation function



24-56: Summary

- Min/Max trees
- Alpha-Beta Pruning
- Evaluation Functions
- Stopping the Search
- Playing with chance