14-0: **Overview**

- Example games (board splitting, chess, Othello)
- Min/Max trees
- Alpha-Beta Pruning
- Evaluation Functions
- Stopping the Search
- Playing with chance

14-1: **Two player games**

- Board-Splitting Game
  - Two players, V & H
  - V splits the board vertically, selects one half
  - H splits the board horizontally, selects one half
  - V tries to maximize the final value, H tries to minimize the final value

14	5	11	4
12	13	9	7
15	13	10	8
16	1	6	2

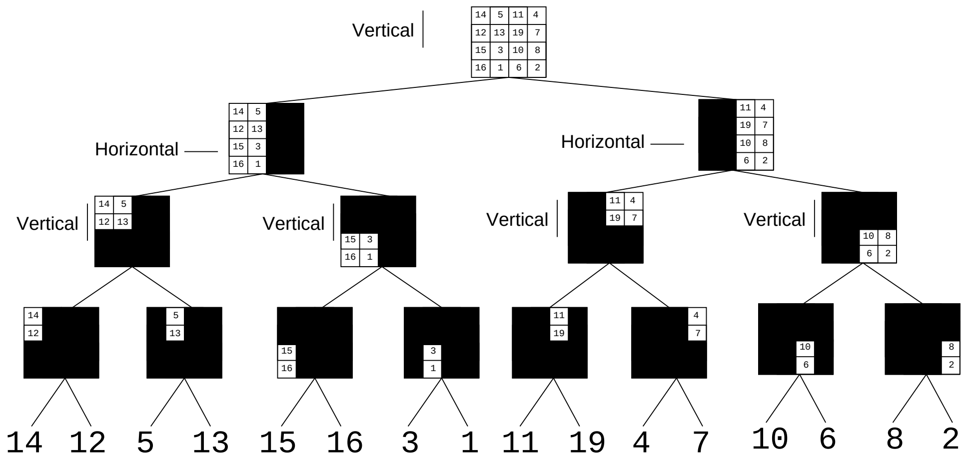
14-2: **Two player games**

- Board-Splitting Game
  - We assume that both players are rational (make the best possible move)
  - How can we determine who will win the game?

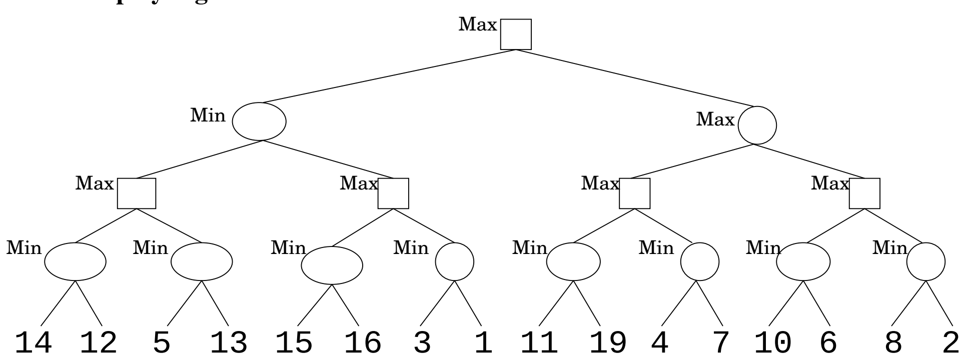
14-3: **Two player games**

- Board-Splitting Game
  - We assume that both players are rational (make the best possible move)
  - How can we determine who will win the game?
    - Examine all possible games!

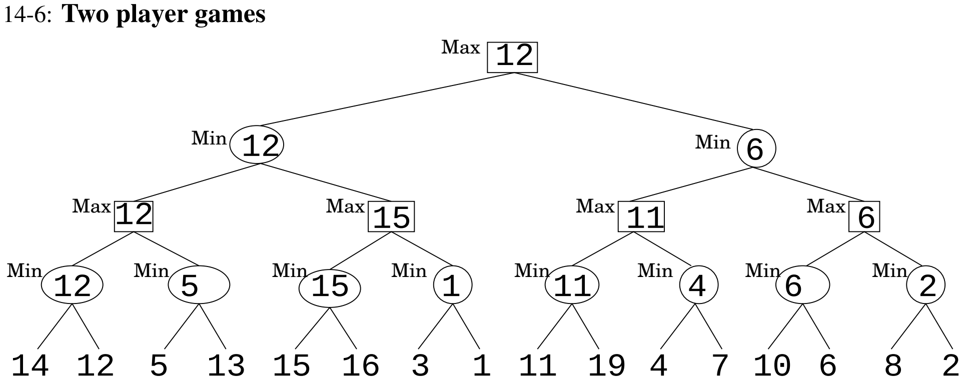
14-4: **Two player games**



14-5: **Two player games**



14-6: **Two player games**



14-7: **Two player games**

- Game playing agent can do this to figure out which move to make
  - Examine all possible moves
  - Examine all possible responses to each move
  - ... all the way to the last move
  - Caclulate the value of each move (assuming opponent plays perfectly)
  -

14-8: **Two-Player Games**

- Initial state
- Successor Function
  - Just like other Searches
- Terminal Test
  - When is the game over?
- Utility Function
  - Only applies to terminal states
  - Chess: +1, 0, -1
  - Backgammon: 192 . . . -192

14-9: **Minimax Algorithm**

```
Max(node)
  if terminal(node)
     return utility(node)
  maxVal = MIN_VALUE
  children = successors(node)
  for child in children
     maxVal = max(maxVal, Min(child))
  return maxVal

Min(node)
  if terminal(node)
     return utility(node)
  minVal = MAX_VALUE
  children = successors(node)
  for child in children
     minVal = min(minVal, Max(child))
  return minVal
```

14-10: **Minimax Algorithm**

- Branching factor of b, game length of d moves, what are the time and space requirements for Minimax?

14-11: **Minimax Algorithm**

- Branching factor of b, game length of d moves, what are the time and space requirements for Minimax?
  - Time: $O(b^d)$
  - Space: $O(d)$
- Not managable for any real games – chess has an average b of 35, can't search the entire tree
- Need to make this more managable

14-12: > 2 **Player Games**

- What if there are > 2 players?
- We can use the same search tree:
  - Alternate between several players
  - Need a different evaluation function

14-13: > 2 **Player Games**

- Functions return a vector of utilities
  - One value for each player

- Each player tries to maximize their utility
- May or may not be zero-sum

14-14: > 2 **Player Games**