Department of Computer Science **University of San Francisco**

Computer Science 673
Fall 2016
Homework 1: Θ, O, Ω
Due Friday, September 2nd

All problem / exercise numbers are from the **3nd edition** of Introduction to Algorithms (the 1st and 2nd editions are different!) Note the difference between problems and exercises!

1. Finding the Minimum and Maximum values in a list

   (a) (4 points) Given a list of n elements, give an algorithm to find the smallest and largest elements in the list in the smallest number of comparisons, in the worst case. How many comparisons does your algorithm require, in terms of n, in the worst case? Note that I want the *exact* number of comparisons, not just a $\Theta()$ bound on the number of comparisons.

   (b) (4 points) Given a list of n elements, give an algorithm to find the smallest and largest elements in the list in the smallest number of comparisons, in the **best** case. Your algorithm needs to **always** give the correct answer, but you are trying to optimize the best-case performance, not the worst case performance. How many comparisons does your algorithm require, in terms of n, in the best case? How about in the worst case? Note I want the *exact* number of comparisons, not just a $\Theta()$ bound on the number of comparisons.

2. Use the substitution method to prove that the recurrene relation $T(n) = 9T(\lfloor n/3 \rfloor) + C_1 n$ is in $\Theta(n^2)$, as follows:

   - (2 points) First, use the substitution method to show that $T(n) > cn^2$
   - (2 points) Next, show that a substitution proof that $T(n) < cn^2$ fails
   - (2 points) Next, show how to subtract off a lower-order term to make the substitution work.

   You can assume that $T(0) = T(1) = T(2) = T(3) = C_2$

3. Prove the following bounds using the substitution method. Recall that to prove $T(n) \in \Theta(f(n))$, you must show that $T(n) \in O(f(n))$ **and** $T(n) \in \Omega(f(n))$. You can assume that $T(0) = T(1) = C$

   (a) (4 points) $T(n) = T(n-2) + 2n \in \Theta(n^2)$

   (b) (4 points) $T(n) = T(n/2) + 2n \in \Theta(n)$

   (c) (4 points) $T(n) = T(n/2) + 2 \in \Theta(\lg n)$

   (d) (4 points) $T(n) = 4T(n/4) + 4n \in \Theta(n \lg n)$