Computer Science 673
Fall 2016
Homework 10: Graphs III
Due Friday, November 11th

1. Problem 24-2 Nesting Boxes

   A d-dimensional box with dimensions $(x_1, x_2, \ldots, x_d)$ *nests* within another box with dimensions $(y_1, y_2, \ldots, y_d)$ if there exists a permutation π on $\{1, 2, \ldots d\}$ such that $x_{\pi(1)} < y_1, x_{\pi(2)} < y_2, \ldots, x_{\pi(d)} < y_d$.

   (a) (2 points) Argue that the nesting relation is transitive

   (b) (2 points) Describe an efficient method to determine whether or not one d-dimensional box nests inside another

   (c) (4 points) Suppose that you are given a set of n $d-$dimensional boxes $\{B_1, B_2, \ldots B_n\}$. Describe an efficient algorithm to determine the longest sequence $\langle B_{i_1}, B_{i_2}, \ldots, B_{i_k} \rangle$ of boxes such that B_{i_j} nests inside $B_{i_{j+1}}$ for $j = 1, 2, \ldots, k-1$. Express the running time of your algorithm in terms of n and d.

2. Exercise 25.1-10 (6 points) Give an efficient algorithm to find the length (number of edges) of a minimum-length negative-weight cycle in a graph.

3. (8 points) Exercise 26.1-9 Professor Adam's Children

   Professor Adam has two children who. unfortunately, dislike each other. The problem is so severe that not only do they refuse to walk to school together, but in fact each one refuses to walk on any block that the other child has stepped on that day. The children have no problem with their paths crossing at a corner. Fortunately both the professor's house and the school are on corners, but beyond that he is not sure if it is going to be possible to send both of his children to the same school. The professor has a map of his town. Show how to formulate the problem of determining if both his children can go the same school as a maximum flow problem

4. (8 points) Exercise 26.2-9 Edge connectivity

   The *edge connectivity* of an undirected graph is the minimum number k of edges that must be removed to disconnect the graph. For example, the edge connectivity of a tree is 1, and the edge connectivity of a cyclic chain of vertices is 2. Show how the edge connectivity of an undirected graph $G = (V, E)$ can be determined by running a maximum-flow algorithm on at most $|V|$ flow networks, each having $O(V)$ vertices and $O(E)$ edges.