Computer Science 673
Fall 2016
Homework 2: Recurrence Relations, Probability
Due Friday, September 9th

All problem / exercise numbers are from the **3nd edition** of Introduction to Algorithms (the 1st and 2nd editions are different!) Note the difference between problems and exercises!

1. (4 points) Exercise 4.4-8 Use a recursion tree to give an asymptotically tight solution to the recurrence $T(n) = T(n - a) + T(a) + Cn$ where $a \geq 1$ and $C > 0$ are constants. You can assume that $T(C_1)$ is $\Theta(1)$ for any constant C_1.

2. (4 points) Exercise 4.4-9 Use a recursion tree to give an asymptotically tight solution to the recurrence $T(n) = T(\alpha n) + T((1 - \alpha)n) + Cn$ where α is a constant in the range $0 < \alpha < 1$, and $C > 0$ is also a constant. You can assume that $T(C_1)$ is $\Theta(1)$ for any constant C_1.

3. Give find a tight (Θ) bound for each of the following recurrence relations. Assume that $T(n)$ is a constant for sufficiently small n. Prove your solution is correct using either the master method or the substitution method.

   (a) (4 points) $T(n) = T(n/2) + T(n/3) + n$

   (b) (4 points) $T(n) = T(n/2) + T(n/3) + T(n/6) + n$

   (c) (4 points) $T(n) = 2T(n/2) + \lg n$

   (d) (4 points) $T(n) = 4T(n/4) + n$

   (e) (4 points) $T(n) = 3T(n/3) + n \lg n$

   (f) (4 points) $T(n) = 3T(n/9) + \sqrt{n}$

   (g) (4 points) $T(n) = 2T(n - 1) + n^2$

4. **Loyal Soldiers**.

   A general has n soldiers in his division. Each soldier is either honest or dishonest, and every soldier knows which soldiers are honest and which are dishonest. The general can ask a soldier if another soldier is honest or not. Honsest soldiers will always tell the truth, while dishonest solders can say whatever they like. So, If A and B are honest, and C and D are dishonest, if the gerenal asks A about B, then A will say 'honest', and if the general asks A about C, then A will say dishonest. If the general asks C about D, then C can say either honest or dishonest, and if the general asks C about A, then C can also say either honest or dishonest. The general wants to find a soldier who is guaranteed to be honest by asking as few questions as possible.

(a) (4 points) Show that if at least $\lceil n/2 \rceil$ soldiers are dishonest, it is impossible to know which soldiers are honest and which are dishonest, *even if the general knows exactly how many soldiers are honest*, no matter how many questions the gerneal askes. (You can assume that the dishonest soldiers can conspire to try to fool the general)

(b) (3 points) If there are more honest than dishonest soldiers, then the general can find one who is honest using a surprisingly small number of questions. Show how the general can find a single honest solder out of a set of size n soldiers (that contain more honest than dishonest soldiers) asking a minimal number of questions for the following values of n: 3, 4, 5.

(c) (6 points) The general decides he wants to use a recursive algorithm to solve the problem. Starting with a set A of n soldiers (more than half of whom are honest), he wants to create a subset B of A, such that:

- The number of soldiers in set B (written $|B|$) is no more than $\lceil |A|/2 \rceil$.
- The set B has more honest sodliers than dishonest soldiers

Show how the general can create the set B using exactly $\lfloor n/2 \rfloor$ comparisons.

(d) (4 points) Show how the general can find an honest soldier from a set of n soldiers, at least half of whom are honest, using a minimum number (worst-case) comparisons. What is the worst-case number of comparisons? You should use the answers in parts (b) and (c) as you base and recursive cases.